

# Control of Hybrid Petri Nets using Max-Plus Algebra

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*Abstract:* - In this paper, *Hybrid Petri Nets* are proposed as a modelling methodology to control production in a large class of manufacturing processes. A hybrid Petri net (HPN) is a combination of a 'classical' Petri net and a continuous Petri net, the 'fluid' version of a timed Petri net. The two parts of an HPN can affect each other. In the particular model proposed, the discrete part of the HPN 'regulates' the continuous Petri net. In fact, whenever an event occurs which changes the present system behavior, the discrete Petri net manages the provoked changes adapting the production process functioning, that is, the continuous Petri net functioning, so as to cope with the new conditions and still fulfil the production requirements. The introduced model allows to realize the control of production by solving suitable optimization problems. The characteristic of the model that makes it specially interesting for analysis and optimization purposes is the special structure of its discrete part, which turns out to be a *Timed Event Graph* (TEG). TEG's are Petri nets with peculiar structural characteristics, which makes them be analytically represented by means of state equations linear in the *Max-Plus algebra*. This capability is exploited to state the constraints imposed by the manufacturing system behavior on the cost functionals to be optimized to solve the control problems of interest.

*Key-Words:* - Hybrid Petri nets, modelling, manufacturing systems, Timed Event Graphs, Max-plus algebra, control

## 1 Introduction

The growing attention that researchers have been paying to hybrid systems since some years has resulted in the development of several approaches to their modelling, analysis, and control. A large interesting class of hybrid systems consists of those systems made up of continuous-time and discrete-event components. In such a hybrid system, the presence of a discrete-event subsystem results in the possibility to design and implement control strategies embedded in the system itself. Significant works of survey are [1,2,3].

On the other hand, a growing interest is focused on using Petri nets to model, analyze, and control Discrete Event Systems (DES) [4,5,6], considered to be standing-alone systems or components of more complex systems, e.g., hybrid systems. In fact, Petri nets are able to capture the precedence relations and interactions among the concurrent and asynchronous events typical of DES.

As a result, Petri nets provide not only advantageous modelling capabilities and helpful structural properties, but also powerful tools for design, analysis, and control.

This paper makes the two above mentioned research streams join by proposing to use a special class of Petri nets, the *Hybrid Petri Nets* (HPN's) [7],[8], to model large-scale systems, and in particular manufacturing processes, which can be considered to be hybrid systems. In such systems, the functioning is meant 'regular' whenever no variation in the system behavior occurs, whereas the changes in the system behavior are modelled by means of discrete events. As a result, the system functioning is always regular between the occurrences of two subsequent events.

A Hybrid Petri net, as first introduced in [7], is composed of a discrete part and a continuous part, so that it can be viewed as a combination of an usual (discrete) Petri net and a continuous Petri net, the

‘fluid’ version of usual timed Petri nets [9]. The states of the two Petri nets making up a HPN, i.e., the continuous one and the discrete one, can affect each other. HPN’s are used in this paper to define a modelling framework that can be effectively applied to those large-scale systems in which not only is it important to model suitably any deviation from the ‘nominal’ system behavior, but it is also necessary to act somehow so as to recover from degraded situations and restore the nominal functioning of the system as soon as possible. Special attention is paid to the application of the proposed model to the description of manufacturing processes with the aim of controlling production [10,11].

The characteristic of the proposed model that makes it specially interesting for analysis and optimization purposes is the peculiar structure of the discrete-event part of the HPN, which turns out to be a *Timed Event Graph* (TEG). A TEG is a particular kind of a Petri net essentially characterized by the property that every place has a unique input transition and a unique output transition. A major feature of TEG’s lies in the fact that they can be analytically represented by means of state equations linear in the particular algebra called *Max-Plus algebra* [12,13].

A variety of optimization problems can be considered within the proposed hybrid modelling framework. The example treated in this paper consists in an on-line control problem regarding a manufacturing process, with the objective of minimizing the deviation between the actual system behavior and a specified nominal behavior, while taking into account some contrasting cost proportional to the production speed. The capabilities of TEG’s are exploited to state the constraints imposed by the manufacturing system behavior on the cost functional in a compact, convenient way. The resulting optimization problem is a quadratic programming problem, the solution of which provides time after time the production speeds of all the resources involved in the manufacturing process.

## 2 Modelling Large-Scale Systems via Hybrid Petri Nets

To introduce the modelling capabilities peculiar to hybrid Petri nets (HPN’s), some basic issues are reported in the following. The reader can refer to [7] and [8] to find details.

A *marked hybrid Petri net* is formally defined as a triplet  $H = \langle H^*, h, M_0 \rangle$  in which:

- $H^*$  is an unmarked Petri net, i.e., an oriented bipartite graph  $H^* = \langle P, T, Pre, Post \rangle$  where:
  - $P$  is a finite set of places;
  - $T$  is a finite set of transitions;
  - $Pre : P \times T \rightarrow \{0,1\}$  is the input incidence

function defining the existence of an arc joining a place with a transition;

- $Post : P \times T \rightarrow \{0,1\}$  is the output incidence function defining the existence of an arc joining a transition with a place;
- $h : P \times T \rightarrow \{D, C\}$  is the so-called *hybrid function* which indicates if a node is a discrete node ( $D$ ) or a continuous one ( $C$ );
- $M_0$  is the initial marking of the net.

It is worth noting that  $Pre(p_i, t_j)$  and  $Post(p_i, t_j)$  are positive integer numbers if  $p_i$  is a  $D$ -place and positive real numbers if  $p_i$  is a  $C$ -place. The set of places is split into two subsets  $CP$  and  $DP$  gathering the  $C$ -places and the  $D$ -places, respectively,  $CP \cup DP = P$ . Moreover, also transitions are divided into  $C$ -transitions and  $D$ -transitions, making up the sets  $CT$  and  $DT$ , respectively,  $CT \cup DT = T$ . In timed Petri nets delays can be associated with places and/or transitions; in the same way, it is possible to assign delays to places and/or transitions in HPN’s. In the proposed model, the timing is associated with  $D$ -transitions, being their firing times stochastic (depending on given probability distributions) or immediate.

The states of the two Petri nets making up a HPN, i.e., the continuous one and the discrete one, can affect each other. HPN’s are used in this paper to define a modelling framework that can be effectively applied to those large-scale systems in which not only is it important to model suitably any deviation from the ‘nominal’ system behavior, but it is also necessary to act somehow so as to recover from degraded situations and restore the nominal functioning of the system as soon as possible.

To this end, in the proposed model the continuous part of the HPN represents the functioning of the system whenever no changes occur in its behavior, that is, in time intervals between two subsequent event occurrences, whereas the changes in the system operating framework are represented and managed through the discrete part of the net. In particular, an event corresponds to a variation in the system behavior that can require the execution of a suitable control procedure, with the objectives of modifying the functioning of the continuous part of the system in order to take into account the degraded behavior, reducing its impact on the overall system performance, and restoring the nominal system functioning. In this sense, it is possible to state that the discrete part of the HPN ‘regulates’ the functioning of the continuous part, through a particular definition of  $D$ -transitions.

More specifically, the set  $DT$  of  $D$ -transitions can be divided into three subsets,  $DT = DT_e \cup DT_c \cup DT_r$ , according to the different functions assigned: occurrences of events are represented by  $D$ -transitions belonging to set  $DT_e$ ,  $D$ -transitions corresponding to the executions of

control actions make up set  $DT_C$ , whereas  $D$ -transitions gathered in set  $DT_T$  represent the resuming of the system functioning after an event occurrence or after the completion of the control procedure. According to the definition of the different types of  $D$ -transitions, only those representing the occurrences of events, which take place based on a-priori given probability distributions, have stochastic firing times. The firing of other  $D$ -transitions is immediate.

To describe a large-scale system using the proposed hybrid model, consider it to be divided into  $N$  sections, being  $N$  the number of its parts that need separate modelling, due to different characteristics. In a manufacturing system, for instance, a section can model a resource of any kind.

Consider the modelling of the generic section  $i$ ,  $i=1, \dots, N$ , as depicted in Fig. 1. In this figure, the  $C$ -places are represented by dotted circles, the  $C$ -transitions by gray rectangles, the  $D$ -places by white circles, and the  $D$ -transitions by black lines.  $C$ -transition  $Ct_i$ ,  $Ct_i \in CT$ , represents the flow through section  $i$ ,  $i=1, \dots, N$ . The input  $C$ -place with respect to  $Ct_i$ ,  $Cp_{i,in}$ , represents the enabling for section  $i$  to work. In analogy with continuous Petri nets [9], the firing speed  $v_i$  associated with  $C$ -transition  $Ct_i$  models the relevant working speed.

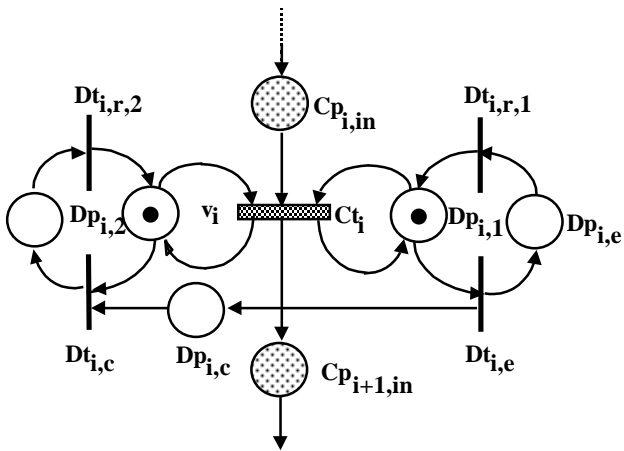


Fig. 1

A token is present in  $D$ -places  $Dp_{i,1}$  and  $Dp_{i,2}$  as far as no event occurs, that is, the corresponding  $D$ -transition  $Dt_{i,e}$  does not complete a firing. The end of its firing means the occurrence of an event, which makes the token in  $Dp_{i,1}$  move to places  $Dp_{i,e}$  and  $Dp_{i,c}$ . The token in place  $Dp_{i,e}$  is immediately available for the firing of transition  $Dt_{i,r,1}$  and, thus, one token is immediately restored in place  $Dp_{i,1}$ . Instead, the token in place  $Dp_{i,c}$  enables the firing of transition  $Dt_{i,c}$  which removes the token from place  $Dp_{i,2}$ , thus temporarily stopping the functioning of the continuous part of

section  $i$ . Moreover, transition  $Dt_{i,c}$  represents the execution of a suitable control procedure which, on the basis of the event occurred, modifies the firing speeds of  $C$ -transitions optimizing some predefined cost functional. As soon as the control procedure has completed,  $D$ -transition  $Dt_{i,r,2}$  is enabled and its firing moves one token to  $Dp_{i,2}$ , thus resuming the system functioning. It is worth remarking that the execution time of the control procedure is definitely negligible.

With reference to Fig.1, two different discrete parts of the HPN can actually be identified. The right part models the event occurrences whereas the left part is the structure implementing the control procedure. The presence of these two different portions of the discrete part of the net is motivated by the fact that the control structure of the generic section  $i$  can also be activated by an event occurrence in a section  $j$ ,  $j \neq i$ . In this way, it is possible to define the portion of the system that is affected by a perturbation occurring in a generic section. Each continuous section included in such portion of the net is stopped by the occurrence of the considered perturbation and the corresponding control procedure is run to define a new firing speed. The part of the net influenced by each perturbation must be identified when designing the structure of the HPN modelling the considered system and, of course, a perturbation can also affect the whole net. Thus, the presence of two discrete parts in the proposed model is necessary to correctly consider, and especially recover from, significant perturbations to the regular system functioning, as it will be clarified in next section dealing with the application of the proposed model to manufacturing systems.

### 3 Application to Manufacturing Processes

To characterize the above introduced model to the description of manufacturing processes, consider each section  $i$ ,  $i=1, \dots, N$ , to represent a resource  $M_i$ ,  $i=1, \dots, N$ , of the system. Then, the firing speed associated with  $C$ -transition  $Ct_i$  models the working speed of the resource.

The proposed model is here described by means of a simple example relevant to a manufacturing system composed of two failure-prone machines  $M_1$  and  $M_2$  processing one product class. The HPN relevant to the considered system is depicted in Fig.2.

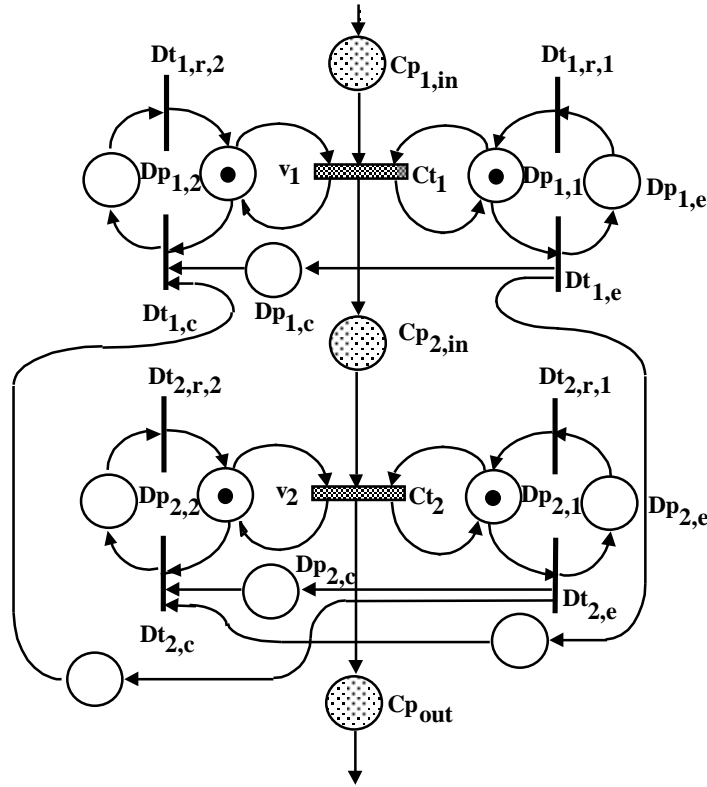


Fig. 2

The manufacturing system functioning is defined by the production speeds of the two machines. It is assumed that two ‘nominal’ speeds  $v_1^*$  and  $v_2^*$  are defined by means of an off-line planning phase of the system behavior. The regular system functioning is, thus, identified by the machines working at their nominal speeds. Moreover, the continuous place  $Cp_{out}$  corresponds to final products exiting the system.

For the sake of simplicity, in this case it is assumed that the only events affecting the system functioning are related to machine failures. The time interval between two subsequent failures and the duration of each failure are stochastic variables with known probability density functions. The firing times of  $D$ -transitions  $Dt_{1,e}$  and  $Dt_{2,e}$  are just defined by such stochastic times. Moreover, it can be noted that each machine failure enables the firing of both transitions  $Dt_{1,c}$  and  $Dt_{2,c}$ , thus executing the control procedure for both resources. This means that, as it is necessary in this case, the speeds of the two machines are actually changed whenever a failure event occurs in the system. The way in which the control procedure can be defined when dealing with manufacturing processes is defined in next section.

#### 4 Control Issues: Use of Max-Plus Algebra

This section is devoted to the definition of a control problem aiming at restoring the nominal functioning of the system fulfilling some contrasting constraints. To state this problem, it is worth noting that the discrete part of the HPN representing the proposed model has the structure of a particular kind of Petri net, a Timed Event Graph (TEG). From a structural point of view, a TEG is a Petri net with unitary arcs in which every place has a unique input transition and a unique output transition.

A major feature of TEG's lies in the fact that they can be analytically represented by means of equations formally similar to those of linear systems, provided that some tools peculiar to *Max-Plus algebra* [12],[13] are used. The fundamental operations in this algebraic structure are maximum and sum. For each generic transition  $i$ ,  $i=1,\dots,NT$ , ( $NT$ = number of transitions in the discrete part of the HPN),  $x_i(k)$  denotes the time of its  $k$ -th firing. A TEG with no input nor output transitions as the one representing the discrete part of the HPN can be described by state equations of the type

$$\underline{x} = A \otimes \underline{x} \quad (1)$$

where:

- the operator  $\otimes$  implements the sum operation;
- $\underline{x}$  is the state vector,  $\underline{x} = \text{col}[x_i, i=1, \dots, NT]$ ;
- matrix  $A$ ,  $\dim A = (NT \times NT)$ , represents the relations between each pair of transitions.

It is important to remark that equations (1) depend not only on the TEG topological structure, but also on the firing times of transitions. Moreover, to make the TEG representation unambiguous, every token in the initial marking is assumed to be immediately available.

It is known that the application of the algebraic tools peculiar to Max-Plus algebra makes workable a compact analysis of TEG's [12,13]. In particular, it is here described a suitable control procedure to be applied in the TEG representing the discrete part of the HPN modelling manufacturing systems as described in previous section. The control procedure to be executed at the occurrence of each perturbation consists in an on-line control problem. The solution of such a control problem yields, at the occurrence of a machine failure, the new values of the production speeds of all the machines affected by the failure, i.e., related to the one which has failed. The considered cost functional takes into account the square deviation from the nominal speeds, upper bounds to be imposed on the machine speeds, and a cost depending on the machine speeds and on the length of the time interval during which such speeds must be maintained.

The control problem to be solved at the occurrence of the  $h$ -th failure of a generic machine  $M_k$  can be stated as follows.

### Problem

Find

$$\min \left\{ \sum_{i=1}^N \left\{ \alpha_i (v_i - v_i^*)^2 + \beta_i [x_j(2h) - x_j(2h-1)] v_i \right\} \right\} \quad (2)$$

subject to constraints

$$\underline{x}(2h) = A(2h-1) \otimes \underline{x}(2h-1) \quad (3)$$

$$v_i \leq v_i^{\max} \quad i = 1, \dots, N \quad (4)$$

where:

- $\alpha_i$  and  $\beta_i$ ,  $i=1, \dots, N$ , are suitable weighting coefficients which take on constant values;
- $j$  is the index associated with transition  $Dt_{k,e}$ ;
- $v_i^{\max}$ ,  $i=1, \dots, N$ , are the maximum values of the machine speeds in the new perturbed system conditions.

□

In the above problem formalization,  $x_j(2h-1)$  and  $x_j(2h)$  are the time instants of beginning and end of the  $h$ -th failure of machine  $M_k$ , respectively. Note that the control procedure is run at the beginning of the failure and, thus,  $x_j(2h-1)$  is known, whereas  $x_j(2h)$  is determined through constraint (3).

Moreover, the optimization problem addressed is a quadratic programming problem, since constraints (3), which contain maximum operations in the standard algebra, can be suitably rewritten as linear disequalities. Actually, Max-Plus algebra is here used only as a formalism to represent in an aggregate form the constraints on the system behavior, and not to take advantage of the mathematical tools arising from the related theory.

The above formalized control problem refers to the occurrence of a machine failure. It is quite obvious that the control procedure consists in the solution of such a problem only at the beginning of a failure. The event corresponding to the end of a perturbation is processed by the control part of the net simply by restoring the nominal speed values on all the machines previously affected by the failure event.

## 5 Conclusions

Joining some results from the two research streams relevant to hybrid systems and Petri nets, this work has introduced the use of a special class of Petri nets, the *Hybrid Petri Nets* (HPN's), to model large-scale systems, and in particular manufacturing processes, which can be considered to be hybrid systems. In such systems, the functioning is meant 'regular' whenever no variation in system behavior occurs, whereas the changes in the system behavior are modelled by means of discrete events.

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