OBSERVER-CONTROLLER DESIGN FOR CRANES VIA POLE PLACEMENT AND GAIN-SCHEDULING

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In this paper we address the design of a controller-observer for a mechanical crane. We consider a linear model of the crane where the lenght of the suspending rope is a time-varying parameter. The set of models given by frozen values of the rope lenght can be reduced to a single time-invariant reference model using a suitable time scaling. We construct a controller and an observer for the reference model assigning the desired closed loop eigenvalues for the both system and estimation error. The time scaling relation can be inverted to derive the corresponding controller and observer law for the time-varying system. This law takes the form of a gain-scheduling as a function of the rope length. The proposed procedure leads to the computation of the desired time-varying gains for controller and observer design in a symbolic parameterized form. Using a Lyapunov-like theorem, it is possible to find relative upper bounds for the rate of change of the time-varying parameter that ensure the stability of the original system.

1. Introduction

The control of a mechanical crane during cargo handling aims to optimize its dynamic performance reducing the swing of the load while moving it to the desired position as fast as possible. Software tools, as reported in [4], have been developed for this purpose, and different control methodologies [1, 5] have been presented in the literature.

In this paper we follow the approach presented in [3] that uses a linear parameter-varying model of the crane. The varying parameter is the length of the rope that sustains the load. The idea is that of considering the set of frozen models given by different constant values of the rope length. Using a suitable time scaling, all these models can be reduced to a single time-invariant reference model [8] that does not depend on the value of the rope length.

In [3] a feedback controller design has been proposed: the control problem for the time-invariant reference model was posed as an LQR, and the corresponding constant feedback gains were computed. By inverting the time scaling, these constant feedback gains gave the corresponding time-varying gains that implement an implicit gain-scheduling. In this paper we use a similar approach, but we design the controller for the reference model by assigning the desired poles of the closed loop reference model. Pole assignment seems a more natural way of computing the controller for the following reasons. Firstly, pole placement allows one to directly assign the damping coefficients of the poles of the reference model that — by a property of the time scaling — can be shown to be the same of the damping coefficients of the poles of all frozen models. Secondly, we are able to derive a closed form expression of the controller gains as a function of the desired closed loop

poles, that assume the role of design parameters. Thirdly, we observed that finding by trial-and-error "good" poles — both in terms of performance and of stability — was easier than tuning the coefficient of the weight matrices used in [3] to compute the LQR controller.

The physical realization of such a gain-scheduling controller requires the knowledge of all state variables, of the rope length, and of the load weight. In this paper we address the problem of designing an observer to estimate the unknown system state, while we still assume that the rope length and the load weight are known or measurable. The observer uses as system output the measure of the trolley position, and is implemented, as the controller, by implicit gain-scheduling.

There are two important aspects in the approach we propose. First of all, we use the same framework to design both observer and controller. Secondly, the state-feedback gains and the observer gains are expressed in a parametrized form, as a symbolic function of the desired closed loop dynamics (i.e., the eigenvalues of the reference closed loop system and observer), rope length, rope velocity, trolley and load mass. As these parameters vary, the gains need not be recomputed by reapplying the whole design procedure but can simply be obtained by function evaluation.

Studying the stability of a time-varying system is usually a difficult task. It is well known that the stability of the set of frozen models does not ensure the stability of the time-varying system unless the parameter variation is sufficiently slow [6]. It is often the case that the bounds on parameter variations that give sufficient conditions for stability are too restrictive to be of any practical interest.

We propose to use the general methodology of [3], based on a Lyapunov-like theorem [6], and show that in an ap-
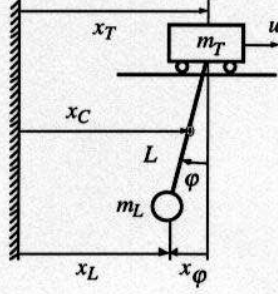
Figure 1: Model of the crane.

plicative example this approach gives sufficiently large bounds on the rope velocity to ensure stability of the time-varying system in all nominal operating conditions. Since this procedure is founded on the results of numerical simulations, we also propose a test to ensure that the computational error cannot invalidate the stability result.

2. Time-varying model and time scaling

We will consider a planar crane, whose model is shown in Figure 1. The following notation is used: m_T, m_L are the mass of the trolley and that of the load, respectively; L is the length of the suspending rope; x_T, x_L are, respectively, the displacement of the trolley, and that of the load with respect to (wrt) a fixed coordinate system; $x_C = (m_T x_T + m_L x_L)/(m_L + m_T)$ is the displacement of the center of gravity of the overall system wrt a fixed coordinate system; φ is the angle between the suspending rope and the vertical taken as positive in the clockwise direction (see figure); $x_\varphi = x_T - x_L = L \sin \varphi$ is the displacement of the load wrt the vertical; u is the control force, applied to the trolley; g is the gravitation constant.

We take as measurable variable the trolley position x_T.

If the load is heavy enough, it is possible to consider the suspending rope as a rigid rod. Under the assumptions reported in [3] (namely, small angles and force applied by the rope equal to the weight of the load), choosing the following state vector:

$$\vec{x}_t = \begin{bmatrix} x_\varphi(t) \ x_C(t) \ \dot{x}_\varphi(t) \ \dot{x}_C(t) \end{bmatrix}^T \tag{1}$$

and denoting

$$\omega_t \equiv \omega_t(L(t)) = \left(\frac{g(m_T + m_L)}{m_T L(t)} \right)^{0.5} \tag{2}$$

we get the following state variable system

$$\begin{cases} \dot{\vec{x}}_t = A_t \vec{x}_t + B_t u_t \\ y_t = C_t \vec{x}_t \end{cases} \tag{3}$$

with

$$A_t = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_t^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \ B_t = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{m_T} \\ \dfrac{1}{m_T + m_L} \end{bmatrix};$$

$$C_t = [\frac{m_L}{m_T + m_L} \ 1 \ 0 \ 0].$$

The subscript t has been introduced to recall that the model given by (3) is time-varying because ω_t is a function of $L(t)$.

If we consider a given constant value of ω_t, i.e., if we consider the system (3) for a frozen value of L, we can consider the following transformation:

$$\tau = \omega_t t. \tag{4}$$

This transformation defines a time scaling that enables us to rewrite (3) as:

$$\begin{aligned} x_\varphi(t) &= x_\varphi(\tau(t)) = x_\varphi(\tau) \\ x_C(t) &= x_C(\tau(t)) = x_C(\tau) \\ \dot{x}_\varphi(t) &= \frac{dx_\varphi(\tau(t))}{dt} = \omega_t \frac{dx_\varphi(\tau)}{d\tau} = \omega_t \dot{x}_\varphi(\tau) \\ \dot{x}_C(t) &= \frac{dx_C(\tau(t))}{dt} = \omega_t \frac{dx_C(\tau)}{d\tau} = \omega_t \dot{x}_C(\tau). \end{aligned} \tag{5}$$

According to (5), variables x_C and x_φ can be taken as functions of t or τ, while their derivatives are changed by the time scaling. We can write (5) as:

$$\vec{x}_t = N \vec{x}_\tau \tag{6}$$

where $x_\tau = \begin{bmatrix} x_\varphi(\tau) \ x_C(\tau) \ \dot{x}_\varphi(\tau) \ \dot{x}_C(\tau) \end{bmatrix}^T$ and

$$N = diag\{1, 1, \omega_t, \omega_t\}; \tag{7}$$

According to (5) we may also write

$$\dot{\vec{x}}_t = \omega_t N \dot{\vec{x}}_\tau \tag{8}$$

where $\dot{\vec{x}}_\tau$ is the derivative of $\vec{x}_\tau$ wrt τ.

Using equations (6) and (8), it is possible to rewrite system (3) as

$$\begin{cases} \dot{\vec{x}}_\tau = A_\tau \vec{x}_\tau + B_\tau u_\tau \\ y_\tau = C_\tau \vec{x}_\tau \end{cases} \tag{9}$$

with (here $\mu = \frac{m_L}{m_T}$)

$$A_\tau = \omega_t^{-1} N^{-1} A_t N = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

$$B_\tau = \omega_t N^{-1} B_t M_T = \begin{bmatrix} 0 & 0 & 1 & \dfrac{1}{1+\mu} \end{bmatrix}^T; \tag{11}$$

$$C_\tau = C_t N = \left[\frac{\mu}{1+\mu}\ 1\ 0\ 0\right] \qquad (12)$$

$$u_\tau = \frac{u_t}{\omega_t^2 m_T}. \qquad (13)$$

The representation given by (9) is a time-invariant reference model and does not depend on the frozen value of L in equation (3).

3. Controller Design

As shown in [3], it is possible to express the relashionships between the eigenvalues and the eigenvectors of the matrices A_t and A_τ. Let Δ_t (Δ_τ) be the diagonal matrix of the eigenvalues of A_t (A_τ); then from (10) we obtain:

$$\Delta_t = \omega_t \Delta_\tau \qquad (14)$$

while between the eigenvector matrices V_t and V_τ

$$V_t = N V_\tau \qquad (15)$$

holds, i.e., given a matrix of eigenvectors V_τ for A_τ it is possible to compute one of the possible matrices of eigenvectors V_t for A_t.

A regulator can be designed by imposing the closed loop poles to system (9), finding a control law of the form

$$u_\tau = -K_\tau \vec{x}_\tau \qquad (16)$$

where K_τ is a constant matrix and does not depend on the value of L. The above equation can be transformed, using (6) and (13), into a corresponding law for the frozen system (6) that gives:

$$u_t = -K_t \vec{x}_t \qquad (17)$$

where

$$K_t = m_T \omega_t^2 K_\tau N^{-1}. \qquad (18)$$

The feedback laws (16) and (17) lead to closed loop systems whose characteristic matrices are:

$$\bar{A}_\tau = A_\tau - B_\tau K_\tau \qquad \bar{A}_t = A_t - B_t K_t. \qquad (19)$$

Equations (10), (14) and (15), written for the open loop systems, still hold for the corresponding closed loop systems. The poles of the frozen closed loop system in t depend on the value of L, and thus on ω_t, but they have the same damping factor for all values of L (see [3]).

For a SISO stationary system it is easy to find a feedback control law by imposing the closed loop eigenvalues [2]. Let us denote as $q(s) = s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$ and $\bar{q}(s) = s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0$ the open loop and the desired closed loop characteristic polynomial relative

to system (9), respectively. Therefore the time-invariant control law is:

$$K_\tau = D_p P_c^{-1}$$

where $D_p = \left[(p_0 - a_0)\ (p_1 - a_1)\ (p_2 - a_2)\ (p_3 - a_3)\right]$ and

$$P_c = \left[\begin{array}{c} (A_\tau^3 + a_3 A_\tau^2 + a_2 A_\tau + a_1 I) B_\tau \\ \hline (A_\tau^2 + a_3 A_\tau + a_2 I) B_\tau \\ \hline (A_\tau + a_3 I) B_\tau \\ \hline B_\tau \end{array}\right]^T$$

is an equivalence transformation that brings the initial system into a controllable canonical form. So the time-varying control law, according to (18), is:

$$K_t(t) = \left[\begin{array}{c} (p_1 - p_3 - 1) m_T \omega_t^2 \\ p_3 (m_T + m_L) \omega_t^2 \\ (p_0 - p_2) m_T \omega_t \\ p_2 (m_T + m_L) \omega_t \end{array}\right]^T.$$

4. Observer Design

It is possible to construct a Luemberger observer for (9) by finding the matrix G_τ which imposes the desired closed loop poles to the reference error system:

$$\dot{\vec{e}}_\tau = (A_\tau - G_\tau C_\tau)\vec{e}_\tau = \bar{E}_\tau \vec{e}_\tau \qquad (20)$$

where $\vec{e}_\tau = \vec{x}_\tau - \hat{\vec{x}}_\tau$ and $\hat{\vec{x}}_\tau$ is the reference state estimate. If we denote $\hat{\vec{x}}_t$ the frozen system estimate and $\vec{e}_t = \vec{x}_t - \hat{\vec{x}}_t$ the corresponding error, it is easy to observe that:

$$\vec{e}_t = N \vec{e}_\tau, \qquad (21)$$

and

$$\dot{\vec{e}}_t = (A_t - G_t C_t)\vec{e}_t = \bar{E}_t \vec{e}_t = \omega_t N \dot{\vec{e}}_\tau \qquad (22)$$

where

$$G_t = \omega_t N G_\tau. \qquad (23)$$

It is worth noting that (10), (14) and (15) still hold for the error closed loop system.

The assignement of the eigenvalues is done as in the controller case, by first transforming the system into an observable canonical form by means of the equivalence transformation:

$$P_o = \left[\begin{array}{c} C_\tau (A_\tau^3 + a_3 A_\tau^2 + a_2 A_\tau + a_1 I) \\ \hline C_\tau (A_\tau^2 + a_3 A_\tau + a_2 I) \\ \hline C_\tau (A_\tau + a_3 I) \\ \hline C_\tau \end{array}\right]$$

where the coefficients a_i are defined as above and $s^4 + q_3 s^3 + q_2 s^2 + q_1 s + q_0$ is the desired closed loop characteristic polynomial for the error system. In such a case if we call $D_q = \left[(q_0 - a_0)\ (q_1 - a_1)\ (q_2 - a_2)\ (q_3 - a_3)\right]^T$:

$$G_\tau = P_o^{-1} D_q$$

and, according to (23),

$$G_t(t) = \begin{bmatrix} (q_3 - q_2) \dfrac{m_T + m_L}{m_L} \omega_t \\ q_2 \omega_t \\ (q_1 - q_3 - 1) \dfrac{m_T + m_L}{m_L} \omega_t^2 \\ q_3 \omega_t^2 \end{bmatrix}.$$

5. Stability

The matrices $\bar{A}_t$ and $\bar{E}_t$ have eigenvalues with negative real parts for all values of $L(t)$. However, this is not enough to ensure stability of the time-varying closed-loop model unless the rate of change of the time-varying parameter $L(t)$ is sufficiently slow.

We propose to apply as in [3] a Lyapunov-like theorem reported in [6], to determine upper bounds for the rate of change of $L(t)$ that ensure stability.

Theorem 1 (Shamma [6]). Given the time-varying system:

$$\dot{\vec{x}}(t) = A(t)\vec{x}(t) \tag{24}$$

where $A(t)$ is bounded and globally Lipschitz continuous, let there exist matrices $P(t)$ and $Q(t)$, symmetric and positive definite, such that:

1. $P(t)$ is continuously differentiable for all $t \geq 0$;

2. there exist constants α_1, α_2 and $\alpha_3 > 0$ such that, for all $t \geq 0$:

$$\alpha_1 \leq \lambda_{\min}\{P(t)\} \leq \lambda_{\max}\{P(t)\} \leq \alpha_2$$
$$\lambda_{\min}\{Q(t) - \dot{P}(t)\} \geq \alpha_3$$

3. $P(t)A(t) + A^T(t)P(t) = -Q(t) \quad (\forall t \geq 0)$

where $\lambda_{\min}$ (resp., $\lambda_{\max}$) denotes the smaller (resp., larger) eigenvalue.

Under these conditions, the linear system (24) is exponentially stable.

Now, let us consider the controller design. Let $\bar{\Delta}_\tau$ and $\bar{V}_\tau$ be the eigenvalue and eigenvector matrices for $\bar{A}_\tau^T$. Then, using the transpose of equation (10), it is possible to show that $\bar{\Delta}_t = \omega_t \bar{\Delta}_\tau$ and $\bar{V}_t = N^{-1} \bar{V}_\tau$ are eigenvalue and eigenvector matrices for $\bar{A}_t^T$. We have chosen matrix $P(t)$ in Theorem 1 as $P(t) = \bar{V}_t \bar{V}_t^H = N^{-1} \bar{V}_\tau \bar{V}_\tau^H N^{-1}$ where ^{H} denotes the complex conjugate transpose. Thus it is easy to compute analytically matrices $Q(t)$ and $\dot{P}(t)$.

Exactly the same choices can be done for the error closed loop system with matrix $\bar{E}_t$. We denoted as $P^o(t)$ and $Q^o(t)$ the corresponding matrices.

The procedure outlined above, requires the computation of the minimal eigenvalue of the symmetrical matrices $(Q - \dot{P})$ and $(Q^o - \dot{P}^o)$. This is usually done numerically and it may be the case that this number is very close to

zero. Thus one may worry that the sign of this quantity be incorrect because of numerical errors. The following proposition may be used to validate the approach.

Proposition 2. Let $M \in \mathbb{R}^{m \times m}$ be a symmetric matrix with eigenvalues λ_i and eigenvectors $\vec{v}_i$, and let $\hat{\lambda}_i$ and $\vec{\hat{v}}_i$ be the corresponding estimates.

Let us consider the intervals $\mathcal{I}_i = [\hat{\lambda}_i - \beta_i, \hat{\lambda}_i + \beta_i]$, where $\beta_i = \| M\vec{\hat{v}}_i - \hat{\lambda}_i \vec{\hat{v}}_i \|_2$. If $\mathcal{I}_i \cap \mathcal{I}_j = \emptyset$ for all $i \neq j$, then $\lambda_i \in \mathcal{I}_i$ for all i.

Proof. Follows from the fact that if M is a symmetric real matrix its eigenvalues are real, and its eigenvectors are orthogonal. Thus the relation [7]

$$\min_i |\lambda_i - \hat{\lambda}| \leq \| M\vec{w} - \hat{\lambda}\vec{w} \|_2$$

holds $\forall \hat{\lambda} \in \mathbb{R}$ and for all $\vec{w} \in \mathbb{R}^m$ with $\| \vec{w} \|_2 = 1$. $\quad \square$

6. Simulation results

The above described approach was applied to a container crane in the port of Kobe whose model is shown in Figure 1. The numerical values we assumed for simulation are: $m_T = 6000$kg, $L(t) \in [L_{\min}, L_{\max}]$, where $L_{\min} = 2$m, $L_{\max} = 10$m. In nominal operating conditions $|\dot{L}| \leq 1$m/s. These values have been taken from [5].

The p_i and q_i coefficients are derived choosing as desired eigenvalues for $\bar{A}_\tau$ $\{-0.6 \pm 0.02i, \quad -0.5 \pm 0.01i\}$, and for $\bar{E}_\tau$ $\{-1.2 \pm 0.04, \quad -1 \pm 0.02i\}$.

To show the good performance of the designed controller and observer, we present two numerical simulations in which we have been taken into account the complete nonlinear model of the crane given in [3]. We considered two different extreme situations: in the first one we assumed that the crane is lifting a heavy load; in the second one that the crane is lowering the hook wih no load. The sets of eigenvalues selected for controller and observer are valid in all the operating conditions.

6.1 Simulation 1

In the first simulation, we considered a load mass $m_L = 42500$kg. The simulation has been performed for a lifting movement from $L_0 = 10$m to $L_f = 2$m with $\dot{L} = -1$m/s. The initial state of the crane was $x_\varphi(0) = 1.5$m, $x_C(0) = -5$m, $\dot{x}_\varphi(0) = \dot{x}_C(0) = 0$m/s. The initial state estimate was $\hat{x}_\varphi(0) = 1$m, $\hat{x}_C(0) = -4.5$m, $\dot{\hat{x}}_\varphi(0) = \dot{\hat{x}}_C(0) = 0$m/s.

In Figure 2 we show the plots of the variables of interest, where $e_\varphi(t) = x_\varphi(t) - \hat{x}_\varphi(t)$ and $e_C(t) = x_C(t) - \hat{x}_C(t)$.

6.2 Simulation 2

In the second simulation, we considered the case of no load and we assumed the presence of the only hook, so $m_L = 20$kg. The simulation has been performed for a

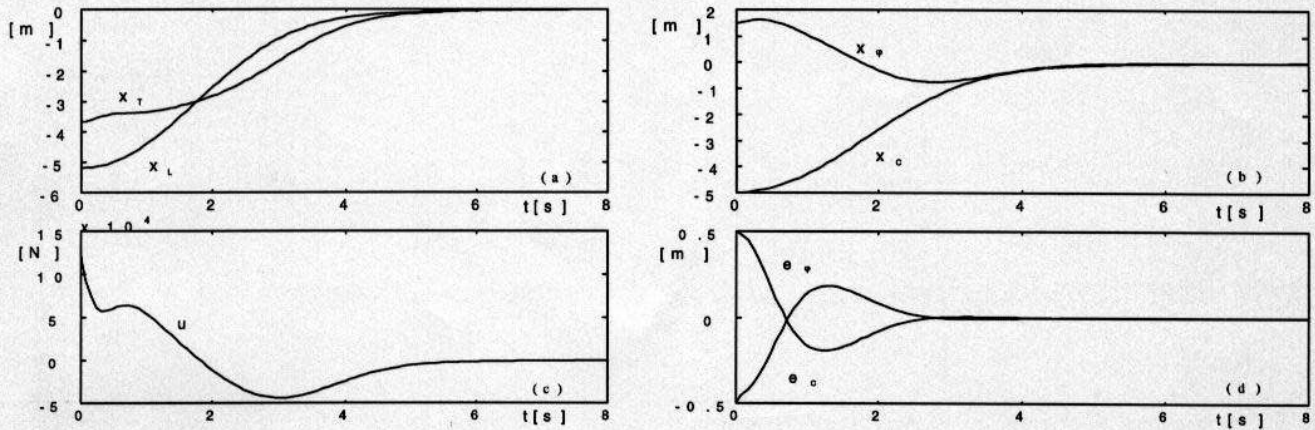Figure 2: Results of simulation 1: lifting movement with $m_L = 42500$kg. (a) Load and trolley positions $x_T(t)$ and $x_L(t)$. (b) State variables $x_1(t) = x_\varphi(t)$ and $x_2(t) = x_C(t)$. (c) Control force $u(t)$. (d) Estimate errors $e_\varphi(t)$ and $e_C(t)$.
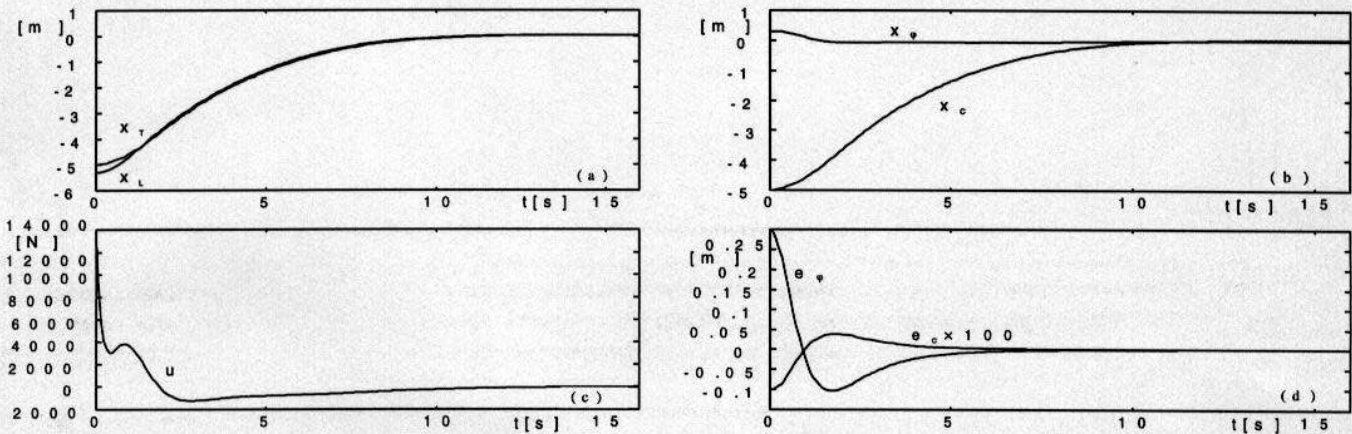

Figure 3: Results of simulation 2: lowering movement with $m_L = 20$kg. (a) Load and trolley positions $x_T(t)$ and $x_L(t)$. (b) State variables $x_1(t) = x_\varphi(t)$ and $x_2(t) = x_C(t)$. (c) Control force $u(t)$. (d) Estimate errors $e_\varphi(t)$ and $e_C(t)$.
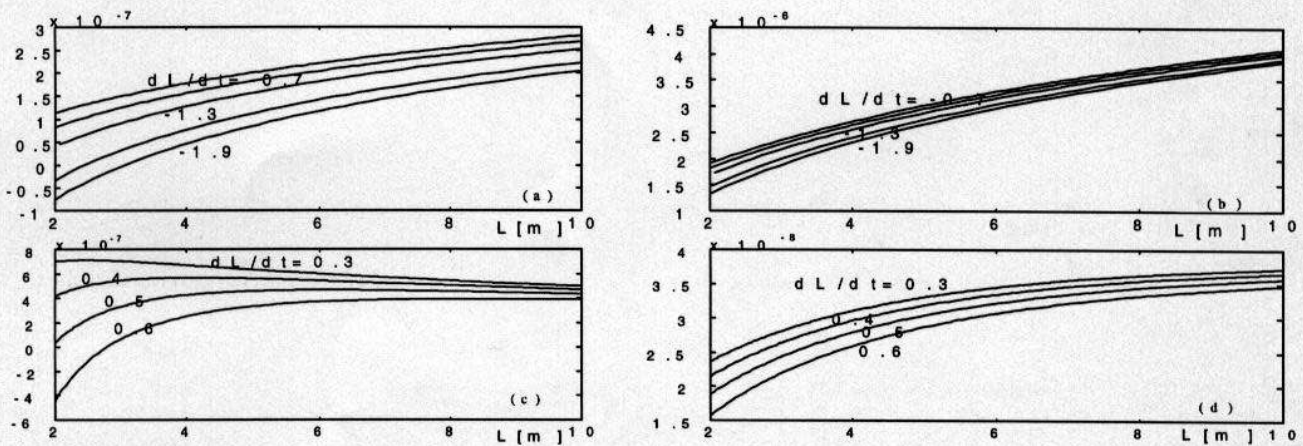

Figure 4: Stability analysis. (a) Plot of $\lambda_{\min}\{Q - \dot{P}\}$ with $m_L = 42500$kg. (b) Plot of $\lambda_{\min}\{Q^o - \dot{P}^o\}$ with $m_L = 42500$kg. (c) Plot of $\lambda_{\min}\{Q - \dot{P}\}$ with $m_L = 20$kg. (d) Plot of $\lambda_{\min}\{Q^o - \dot{P}^o\}$ with $m_L = 20$kg.

lowering movement from $L_0 = 2$m to $L_f = 10$m with $\dot{L} = 0.5$m/s. The initial state of the crane was $x_\varphi(0) = 0.3$m, $x_C(0) = -5$m, $\dot{x}_\varphi(0) = \dot{x}_C(0) = 0$m/s; while the initial state estimation was $\hat{x}_\varphi(0) = 0$m, $\hat{x}_C(0) = x_T(0) = -4.9990$m, $\dot{\hat{x}}_\varphi(0) = \dot{\hat{x}}_C(0) = 0$m/s. In such a case (no load) it is not difficult to have a good initial estimation of the position of the centre of gravity: we can suppose that it is coincident with the trolley position that is a measurable variable. In Figure 3 we show the plots of the variables of interest.

6.3 Stability analysis

The stability analysis presented in Section 5 requires the computation of $\lambda_{\min}\{Q - \dot{P}\}$ (as a function of L) for different values of $\dot{L}$.

Figure 4.a shows the plot of $\lambda_{\min}\{Q - \dot{P}\}$ versus L for different values of $\dot{L}$ and for $m_L = 42500$kg. According to Theorem 1, the upper bound on $|\dot{L}|$ is the value corresponding to the first curve that, as $|\dot{L}|$ is increased, goes to negative values. As can be seen from Figure 4.a, relative to lifting operations, this happens for $|\dot{L}| > 1.5$m/s. The same conclusion can be derived in the case of a lowering movement (the corresponding figures are not reported). Hence it can be concluded that the time-varying system with system matrix $\bar{A}_t$ is stable if $|\dot{L}| \leq 1.5$m/s, that is to say it is always stable in nominal conditions.

The same discussion has been done for the error closed loop system with matrix $\bar{E}_t$. In Figure 4.b we reported the curves corresponding to those in Figure 4.a where $Q^o(t)$ and $P^o(t)$ are determined in the same manner as $P(t)$ and $Q(t)$. The same conclusion can be derived in the case of a lowering movement (the corresponding figures are not reported). Note that stability of the observer is guaranteed for any velocity of practical interest. This is due to our choice for the set of observer eigenvalues, that are much more stable than those of the controller.

The analogous curves with $m_L = 20$kg and relative to a lowering operation are shown in Figure 4.c and Figure 4.d. Similar curves can be drawn in the case of a lifting movement. Here it is evident that stability is guaranteed for smaller values of the rope velocity. We can repeat the same discussion for every intermediate value of m_L and it is possible to observe that stability is guaranteed for values of $|\dot{L}|$ that decrease with the load. This is not a surprising fact.

In reality what is plotted in the previous figures is not $\lambda_{\min}$ but its estimate $\hat{\lambda}_{\min}$ computed with a numerical procedure. Even if all computed values $\hat{\lambda}_{\min}$ are close to zero, Proposition 2 can be used to ensure that all $\lambda_{\min}$ are positive. In fact, in all cases the estimated eigenvector $\vec{\hat{w}}$ associated to the estimated eigenvalue $\hat{\lambda}$ was such that the values of the norms $\| \{Q - \dot{P}\}\vec{\hat{w}} - \hat{\lambda}\vec{\hat{w}} \|_2$ and $\| \{Q_o - \dot{P}_o\}\vec{\hat{w}} - \hat{\lambda}\vec{\hat{w}} \|_2$ are $\simeq 10^{-14}$, while all estimated eigenvalues are spaced much further apart.

7. Conclusions

In this paper we presented a general metodology for controlling mechanical cranes. It is a generalization of the procedure applied to the same model in [3] in which a time scaling has been used to reduce the original time-varying system to a stationary one. In that case we supposed a measurable state. This is not always the case, so we extended the procedure to construct also a state observer. Furthermore in this paper we derived the time-varying laws for controller and observer design, by imposing the desired eigenvalues to the stationary closed loop system and to the error estimation system respectively. We also studied the stability of the time-varying systems with gain scheduling. Using a Lyapunov-like theorem it was possible to find upper bounds for the rate of change of the varying parameter (the lenght of the suspending rope) that ensure the stability of a given crane in all its possible operating conditions: maximum load and no load, lifting and lowering movements. The gain matrices for controller and observer are given in a parametrized form. The chosen sets of eigenvalues are valid in every condition with caution to use a quite smaller rope velocity when the load is light or even null.

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