Cycle time optimization for deterministic timed weighted marked graphs under infinite server semantics

Zhou He, Zhiwu Li, and Alessandro Giua

Abstract

Timed Petri nets are commonly used for modelling and analysis of automated manufacturing systems, including batch or high throughput systems. This paper consider the cycle optimization problem for a deterministic timed weighted marked graphs under infinite server semantics. The problem aims to find an initial marking to *minimize the cycle time* while the weighted sum of tokens in places is less than or equal to a desired value. We transform a timed weighted marked graph into several equivalent timed marked graphs and present a mixed integer linear programming method which can provide an optimal solution. Meanwhile, two suboptimal methods are proposed to reduce the computational cost.

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Zhou He is with the School of Electro-Mechanical Engineering, Xidian University, Xi'an 710071, China and also with with Aix-Marseille Univ., Université de Toulon, CNRS, ENSAM, LSIS, Marseille, France. zhouhe@stu.xidian.edu.cn

Zhiwu Li is with the Institute of Systems Engineering, Macau University of Science and Technology, Taipa, Macau, and also with School of Electro-Mechanical Engineering, Xidian University, Xi'an 710071, China. zhwli@xidian.edu.cn

Alessandro Giua is with Aix-Marseille Univ., Université de Toulon, CNRS, ENSAM, LSIS, Marseille, France and also with DIEE, University of Cagliari, Cagliari 09124, Italy. alessandro.giua@lsis.org, giua@diee.unica.it

I. INTRODUCTION

Performance control or performance evaluation of batch processes or high throughput manufacturing systems pose difficult issues because their representation deals with continuous and discrete models. Timed Petri nets (PNs) are well known as efficient tools for modeling discrete event systems and analysis performance of concurrent systems [1].

In this paper, we study a particular class of Petri nets called *timed weighted marked graphs* (TWMGs). The main feature of this class of nets is that each place has only one input and one output transition. Moreover, the firing delay of each transition is *deterministic*.

Several studies for this class of PNs can be found in the literature. For instance, Teruel *et al*. [2] proposed some techniques for the analysis of weighted marked graphs (WMGs). Campos *et al*. [3] discussed ergodicity and throughput bound characterization for TMGs. Munier [5] proposed a pseudo-polynomial algorithm to compute the cycle time of a TWMG under *single server* semantics hypothesis by transforming it into a TMG. Nakamura and Silva [6] discussed the cycle time computation of a TWMG with a given initial marking under *infinite server* semantics hypothesis. Gaubert [8] studied the resource optimization problem for TMGs by using *minmax algebra*.

Related problems to the one we considered are reported in the literature. For example, Sauer *et al*. [9] discussed the marking optimization problem for TWMGs and presented a heuristic algorithm to find a near optimal solution. The marking optimization problem for TWMGs consists in finding an initial marking to minimize the weighted sum of tokens in places while the cycle time is less than or equal to a given value. We [10], [11] presented a novel heuristic method to deal with the marking optimization problem which was shown to be more effective than that of Sauer [9]. We proposed a mixed integer linear programming problem (MILPP) to solve the cycle time optimization problem (also called the maximum throughput initial distributed state problem) for TWMGs under single server semantics [13].

In this paper, we consider the cycle time optimization for a TWMG under infinite server semantics (the degree of self-concurrency of each transition is infinite) which consists in finding an initial marking to maximize the throughput while the used resources are less than or equal to a given value. Under single server semantics services in a transition are provided sequentially, i.e., there is no self-concurrency, in an infinite server semantics the number of concurrent servers is equal to the enabling degree of the transition. Thus, infinite server semantics is more general than single server (or in general k server) semantics. For this reason we adopt this more general semantics in this work. Inspired by the works in [6], we prove that the transformation of TWMGs into TMGs is periodical with regard to the initial marking. Thus, we transform a TWMG into several TMGs and present an original MILPP to obtain an optimal initial marking. Besides, two suboptimal approaches are proposed in order to reduce the computational complexity. By experimental evaluation, we observe that the suboptimal approaches can obtain a good approximation of the optimal solution within a short computational time. For lack of space and make the paper more accessible, formal proofs of the results are omitted.

This paper is structured as follows. In Section II, we briefly recall some basic concepts and the main properties.

In Section III, we present the problem statement. We propose an MILPP method to solve the optimization problem in Section IV. In Section V, we provide two suboptimal solutions to reduce the computation cost and an illustrative example is given in Section VI. Conclusions and future work are finally drawn in Section VII.

II. Background

A. Generalities

We assume that the reader is familiar with the structure, firing rules, and basic properties of PNs (see [2]). In this section, we will recall the formalism used in the paper. A *place/transition* net (P/T net) is a structure $N = (P, T, \boldsymbol{Pre}, \boldsymbol{Post})$, where P is a set of n places; T is a set of m transitions; $\boldsymbol{Pre} : P \times T \rightarrow \mathbb{N}$ and $\boldsymbol{Post} : P \times T \rightarrow \mathbb{N}$ are the pre- and post-incidence functions that specify the arcs; $\boldsymbol{C} = \boldsymbol{Post} - \boldsymbol{Pre}$ is the incidence matrix, where $\mathbb{N}$ is a set of non-negative integers.

A vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_m)^T \in \mathbb{N}^{|T|}$ such that $\boldsymbol{x} \neq \boldsymbol{0}$ and $\boldsymbol{C} \cdot \boldsymbol{x} = \boldsymbol{0}$ is a *T-semiflow*. A vector $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)^T \in \mathbb{N}^{|P|}$ such that $\boldsymbol{y} \neq \boldsymbol{0}$ and $\boldsymbol{y}^T \cdot \boldsymbol{C} = \boldsymbol{0}$ is a *P-semiflow*. The supports of a T-semiflow and a P-semiflow are defined by $\|\boldsymbol{x}\|$=$\{t_i \in T | x_i > 0\}$ and $\|\boldsymbol{y}\|$=$\{p_i \in P | y_i > 0\}$, respectively. A *minimal* T-semiflow (P-semiflow) is a T-semiflow $\|\boldsymbol{x}\|$ (P-semiflow $\|\boldsymbol{y}\|$) that is not a superset of the support of any other T-semiflow (P-semiflow), and its components are mutually prime.

A marking is a vector $\boldsymbol{M} : P \rightarrow \mathbb{N}$ that assigns to each place of a P/T net a non-negative integer of tokens; we denote the marking of place p as $M(p)$. A P/T system or net system $\langle N, \boldsymbol{M_0} \rangle$ is a net N with an initial marking $\boldsymbol{M_0}$.

A P/T net is said to be *ordinary* when all of its arc weights are equal to one. A *marked graph* (also called an event graph) is an ordinary Petri net such that each place has exactly one input and one output transition. A *weighted marked graph* (also called a weighted event graph) is a net that also satisfies this structural condition but may not be ordinary, i.e., the weight associated with each arc is a non-negative integer number.

A net is *strongly connected* if there exists a directed path from any node in $P \cup T$ to every other node. Let us define an *elementary circuit* γ (or elementary cycle) of a net as a directed path that goes from one node back to the same node without passing twice on the same node.

Given a place p_i of a WMG, let $t_{in(p_i)}$ (resp., $t_{out(p_i)}$) be its unique input (resp., output) transition as shown in Fig. 1. We denote $w(p_i) = Post(p_i, t)$ the weight of its input arc and $v(p_i) = Pre(p_i, t)$ the weight of its output arc. We call gcd_{p_i} the greatest common divisor of the integers $w(p_i)$ and $v(p_i)$.

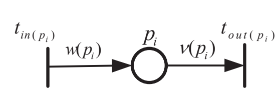

Fig. 1. A place p_i between two transitions $t_{in(p_i)}$ and $t_{out(p_i)}$.

Every elementary circuit γ of a WEG is neutral, if the following condition holds.

$$\prod_{p\in\gamma}\frac{\nu(p_i)}{w(p_i)}=1$$

A WMG is said to be neutral if all circuits are neutral. It is well known that a neutral WMG has a unique minimal T-semiflow $\boldsymbol{x}$ which contains all transitions in its support [2]. In this paper, we limit our study to strongly connected and neutral WMGs.

B. Dynamic behavior

There exists two main approaches for introducing the concept of time in PN models, namely, associating a time interpretation with either transitions (T-timed) or places (P-timed) [4]. A deterministic T-timed P/T net is a pair N^δ = (N,δ), where $N=(P,T,\boldsymbol{Pre},\boldsymbol{Post})$ is a standard P/T net, and $\delta:T\rightarrow\mathbb{N}$, called firing delay, assigns a non-negative integer fixed firing duration to each transitions. In terms of a deterministic P-timed P/T net, each place p is assigned a non-negative integer number $\delta(p)$ which represents the sojourn time that a token must spend in place p before it becomes available for its output transitions. Here, we consider deterministic T-timed WMGs.

A transition t_i is enabled at $\boldsymbol{M}_j$ if $\boldsymbol{M}_j\geq\boldsymbol{Pre}(\cdot,t_i)$ and an enabled transition t_i may fire yielding a marking $\boldsymbol{M'}$ with

$$\boldsymbol{M'}=\boldsymbol{M}_j+\boldsymbol{C}(\cdot,t_i), \tag{1}$$

where $\boldsymbol{Pre}(\cdot,t_i)$ (resp. $\boldsymbol{C}(\cdot,t_i)$) denotes the column of the matrix $\boldsymbol{Pre}$ (resp. $\boldsymbol{C}$) associated with transition t_i.

The state of a TWMG is defined not only by the marking, as for P/T nets, but also by the clocks associated with transitions. The enabling degree of t_i enabled at a marking $\boldsymbol{M}_j$ denoted by $\alpha_i(j)$ is the biggest integer number k such that

$$\boldsymbol{M}_j\geq k\cdot\boldsymbol{Pre}(\cdot,t_i). \tag{2}$$

Under infinite server semantics, at each time instant τ_j the number of clocks o_i associated with a transition t_i is equal to its current enabling degree, i.e., $o_i=\{o_{i,1},\ldots,o_{i,\alpha_i(j)}\}$; this number changes with the enabling degree, thus it can change each time the net evolves from one marking to another one, namely, each time that a transition fires. If transition t_i is not enabled at marking $\boldsymbol{M}_j$, it has no clock. Assuming that $o_i^*=\min\{o_{i,1},\ldots,o_{i,\alpha_i(j)}\}$ and letting $o^*=\min_{i=1,\ldots,m}\{o_i^*\}$ be the minimum among the values of the clocks o_i^*. At the time instant $\tau_{j+1}=\tau_j+o^*$, transitions whose clocks are equal to o^* fire yielding a new marking as in Eq. (1).

C. Cycle time of a TWMG

The *cycle time* $\chi(\boldsymbol{M})$ of a TWMG system $\langle N,\boldsymbol{M}\rangle$ is the average time to fire once the T-semiflow under the ASAP (as soon as possible) execution, i.e., transitions are fired as soon as possible.

In [14], the authors proved that the ASAP execution of a live and strongly connected TMG with integer delays is ultimately repetitive following an execution pattern. The period of the pattern is τ and the number of firings of every transition within a period is f (the periodicity). In terms of TWMGs, the earliest execution is also ultimately periodic. The number of firings of transition t_i within the steady period is f_i. The average cycle time of a TWMG is thus equal to $x_i \cdot \frac{\tau}{f_i}$, where $\boldsymbol{x} = (x_1, \ldots, x_m)^T$ is the minimal T-semiflow.

D. Transformation of TWMGs

One way to analytically compute the cycle time of a TWMG under infinite server semantics is to convert it into an equivalent TMG. Nakamura and Silva [6] proved that a TWMG system $\langle N, \boldsymbol{M} \rangle$ under infinite server semantics can be transformed into an *equivalent* TMG system $\langle \hat{N}, \hat{\boldsymbol{M}} \rangle$ such that the language and the cycle time[1] of the two systems are identical, i.e., $\chi(\boldsymbol{M}) = \chi(\hat{\boldsymbol{M}})$.

Note that the equivalent TMG system depends on the initial marking $\boldsymbol{M}$ and the minimal T-semiflow $\boldsymbol{x}$ of the TWMG. Since it is necessary for us to use this transformation method, we present it in Algorithm 1. All notations in the algorithm are from previous definitions and $x_{out(p_i)}$ (resp. $x_{in(p_i)}$) represents the elementary T-semiflow component corresponding to transition $t_{out(p_i)}$ (resp. $t_{in(p_i)}$).

The size of the equivalent TMG is[2] $O(|\boldsymbol{x}|_1)$. We denote by n_i the number of equivalent places corresponding to place p_i. Thus, the number of equivalent transitions is $\hat{m} = |\boldsymbol{x}|_1$ and that of places is $\hat{n} = \sum_{i=1}^{n} n_i + |\boldsymbol{x}|_1$.

Example 1: We consider the TWMG model depicted in Fig. 2 by assuming that the initial marking is $\boldsymbol{M}_0 = (4, 2)^T$.

Transformation of transitions: The minimal T-semiflow of the TWMG is $\boldsymbol{x} = (2, 3)^T$. Then the transitions t_1 and t_2 are replaced by two transition (t_1^1, t_1^2) and three transitions (t_2^1, t_2^2, t_2^3), respectively. Moreover, places $q's$ to connect these transitions are added.

Transformation of places: According to Algorithm 1, place p_1 is replaced by places $p_1^1 = (t_1^1, t_2^3)$ and $p_1^2 = (t_1^2, t_2^1)$ with initial markings $M(p_1^1) = 0$ and $M(p_1^2) = 1$. The time delays associate to the equivalent places are $\delta(p_1^1) = 4$ and $\delta(p_1^2) = 4$. For place p_2, the equivalent places are p_2^1 and p_2^2, with initial markings $M(p_2^1) = 0$ and $M(p_2^2) = 0$. The time delays are $\delta(p_2^1) = 2$ and $\delta(p_2^2) = 2$.

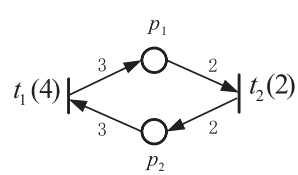

Fig. 2. TWMG of Example 1.

III. Problem statement

In this paper, the *cycle time optimization problem* for a TWMG under infinite server semantics is considered. We aim to find a marking $\boldsymbol{M}$ at which the weighted sum of tokens in places is less than or equal to a given value.

[1]In the following, we will denote by $\chi(\boldsymbol{M})$ the cycle time of a TWMG system $\langle N, \boldsymbol{M} \rangle$ and by $\chi(\hat{\boldsymbol{M}})$ the average cycle of the equivalent TMG system $\langle \hat{N}, \hat{\boldsymbol{M}} \rangle$.

[2]Here $|\boldsymbol{x}|_1$ denotes the 1-norm of T-semiflow $\boldsymbol{x}$.

Algorithm 1 Transformation of a TWMG into a TMG

Input: A TWMG system $\langle N, \boldsymbol{M} \rangle$

Output: An equivalent TMG system $\langle \hat{N}, \hat{\boldsymbol{M}} \rangle$ such that $\chi(\boldsymbol{M}) = \chi(\hat{\boldsymbol{M}})$.

Compute the minimal T-semiflow $\boldsymbol{x} = (x_1, \ldots, x_m)^T$ of net N.

for each transition $t_i \in T$ **begin**

Replace each transition t_i by x_i transitions, t_i^1, t_i^2, …, $t_i^{x_i}$.

Places $q_i^a = (t_i^a, t_i^{a \bmod x_i + 1})$ $(a = 1, \ldots, x_i)$ is added.

$$\begin{cases} \hat{M}(q_i^a) := 1, \;\; \delta(q_i^a) := 0. \; (a = x_i) \\ \hat{M}(q_i^a) := 0, \;\; \delta(q_i^a) := 0. \; \text{(otherwise)} \end{cases} \tag{3}$$

end for;//Transformation of transitions.//

for each place $p_i \in P$ **begin**

remove place p_i and its corresponding arcs;

$a := 0$, $s := 1$;

Repeat

$$b := \left\lfloor \frac{M(p_i) + w(p_i) \cdot a}{v(p_i)} + 1 \right\rfloor \tag{4}$$

$$a := \left\lceil \frac{v(p_i) \cdot b - M(p_i)}{w(p_i)} \right\rceil \tag{5}$$

if $a \leq x_{in(p_i)}$ **then begin**

Place $p_i^s = (t_{in(p_i)}^a, t_{out(p_i)}^{(b-1) \bmod x_{out(p_i)} + 1})$ is added.

$$\hat{M}(p_i^s) := \left\lfloor \frac{b-1}{x_{out(p_i)}} \right\rfloor \tag{6}$$

$$\delta(p_i^s) := \delta(t_{in(p_i)}) \tag{7}$$

$s := s + 1$

end if

Until $a \geq x_{in(p_i)}$.

end for;//Transformation of places.//

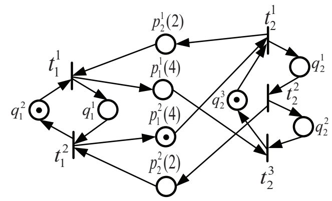

Fig. 3. The equivalent TMG of Example 1.

Among all feasible solutions, we look for those that minimize the cycle time, i.e., maximize the throughput.

In other words we look for a marking $\boldsymbol{M}$ that provides the optimal solution of the following problem:

$$\begin{aligned} & \min \ \chi(\boldsymbol{M}) \\ & \text{s.t.} \\ & \quad \boldsymbol{y}^T \cdot \boldsymbol{M} \leq R \end{aligned} \tag{8}$$

where

- $\chi(\boldsymbol{M})$ is the cycle time of the TWMG with initial marking $\boldsymbol{M}$.
- $\boldsymbol{y}^T = (y_1, \ldots, y_n)$ is a non-negative weight vector.
- R is a given positive real number, representing the upper bound on the cost of available resources.

We choose the weight vector $\boldsymbol{y}$ as a P-semiflow since the value of $\boldsymbol{y}^T \cdot \boldsymbol{M}$ for every reachable marking $\boldsymbol{M}' \in R(N, \boldsymbol{M})$ is an invariant. In particular, we choose the P-semiflow $\boldsymbol{y}$ equals to the weighted sum of all minimal P-semiflows, i.e., $\boldsymbol{y} = \sum_{\gamma \in \Gamma} \lambda_\gamma \cdot \boldsymbol{y}_\gamma$, where $\boldsymbol{y}_\gamma$ represent the minimal P-semiflow corresponding to circuit γ and λ_γ represent the cost of the resources modeled by tokens in the support of $\boldsymbol{y}_\gamma$.

IV. Optimal solution

In this section, we present a formal approach to solve the cycle time optimization problem for TWMGs. Firstly in Section IV-A, we show that the cycle time optimization problem for TMGs can be framed as an MILPP by modification of a known result to compute the cycle time of a TMG whose initial marking is given. Secondly, we discuss the conversion procedure from TWMGs to TMGs in Section IV-B. Although the exact structure of the equivalent TMG depends on the initial marking of the TWMG (which is unknown when solving an optimization problem), we show that the number of possible equivalent structures is finite and periodic with the initial making on the TWMG. This means that in fact only a finite number of equivalent TMG structures have to be considered. Finally, we propose an MILPP to solve the optimization problem (8) for all the possible equivalent TMGs in Section IV-C.

A. Solving the cycle time optimization problem for TMGs

The cycle time optimization problem of a TMG net $\hat{N}$ can be formulated as follows:

$$\begin{aligned} &\min\ \chi(\hat{\boldsymbol{M}}) \\ &\text{s.t.} \\ &\quad \hat{\boldsymbol{y}}^T \cdot \hat{\boldsymbol{M}} \leq R \end{aligned} \tag{9}$$

Proposition 1: Let $(\hat{\boldsymbol{M}}^*, \beta^*, \boldsymbol{\alpha}^*)$ be the optimal solution of the MILPP:

$$\begin{aligned} &\max\ \beta \\ &\text{s.t.} \\ &\begin{cases} \hat{\boldsymbol{C}} \cdot \boldsymbol{\alpha} + \hat{\boldsymbol{M}} \geq \boldsymbol{D}_p \cdot \hat{\boldsymbol{Post}} \cdot \boldsymbol{v} \cdot \beta, \\ \hat{\boldsymbol{y}}^T \cdot \hat{\boldsymbol{M}} \leq R \end{cases} \end{aligned} \tag{10}$$

with variables $\hat{\boldsymbol{M}} \in \mathbb{N}^{\hat{n}}$, $\boldsymbol{\alpha} \in \mathbb{R}^{\hat{m}}$, $\beta \in \mathbb{R}^+$. $\boldsymbol{v}$ is the visit ratio vector that equals to $\vec{\boldsymbol{1}}_{\hat{n}\times 1}$ and $\boldsymbol{D}_p$ is a $\hat{n} \times \hat{n}$ matrix such that $D_p(i,j) = \delta(p_i)$, when $i = j$ and otherwise $D_p(i,j) = 0$.

Then the optimal solutions for problem (9) are $\hat{\boldsymbol{M}} = \hat{\boldsymbol{M}}^*$ and the corresponding optimal cycle time $\chi(\hat{\boldsymbol{M}}) = 1/\beta^*$.

B. Transformation the cycle time optimization problem for TWMGs into TMGs

According to Algorithm 1, the structure of the equivalent TMG (i.e., the input and output arcs of equivalent places) depends on the initial marking $\boldsymbol{M}_0$ of the TWMG. However, we find that this dependence is periodic as shown in the following proposition.

Proposition 2: Consider a TWMG N with minimal T-semiflow $\boldsymbol{x} = (x_1, x_2, \ldots, x_m)^T$ and two possible initial markings $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$. Let $\langle \hat{N}_1, \hat{\boldsymbol{M}}_1 \rangle$ (resp., $\langle \hat{N}_2, \hat{\boldsymbol{M}}_2 \rangle$) be the equivalent TMG obtained by Algorithm 1 with input $\langle N, \boldsymbol{M}_1 \rangle$ (resp., $\langle N, \boldsymbol{M}_2 \rangle$).

If for a place $p_i \in P$

$$M_2(p_i) = M_1(p_i) + \xi \cdot v(p_i) \cdot x_{out(p_i)} \quad with\ \xi \in \mathbb{N},$$

then the structure corresponding to p_i in $\hat{N}_1$ and $\hat{N}_2$ is the same and the markings of the transformed places p_i^s corresponding to p_i in Eq. (6) satisfy

$$\hat{M}_2(p_i^s) = \hat{M}_1(p_i^s) + \xi. \tag{11}$$

The previous result implies that the structure corresponding to place p_i in the equivalent TMG is periodic with regard to $M(p_i)$ and the period ϕ_i is equal to $v(p_i) \cdot x_{out(p_i)}$. However, the number of possible equivalent TMG structure is very big. Following the technique in [13], [11] for single server semantics, we restrict our analysis to the markings that belong to a restricted number of partitions which are guaranteed to find an optimal solution. In fact, to rule out the presence of useless tokens that do not contribute to the cycle time, we can assume that token content of each place p_i is a multiple of gcd_{p_i}. Thus the set of possible markings of place p_i can be partitioned into $\frac{\phi_i}{gcd_{p_i}}$ subsets such that

$$\bar{\mathcal{M}}_{p_i}^{k_i} = \{k_i \cdot gcd_{p_i} + \xi \cdot \phi_i | \xi \in \mathbb{N},\ k_i = 1, \ldots, \frac{\phi_i}{gcd_{p_i}} - 1\} \tag{12}$$

and all makings of p_i in the same partition $\bar{\mathcal{M}}_{p_i}^{k_i}$ correspond to the same equivalent structure.

Thus the set of possible markings of a TWMG are divided into Φ partitions:

$$\mathcal{M}_j = \bar{\mathcal{M}}_{p_1}^{k_{j,1}} \times \bar{\mathcal{M}}_{p_2}^{k_{j,2}} \times \ldots \times \bar{\mathcal{M}}_{p_n}^{k_{j,n}} \tag{13}$$

where

$$\Phi = \prod_{p_i \in P} \frac{\phi_i}{gcd_{p_i}} \tag{14}$$

C. *Optimal solution: an MILPP*

According to Proposition 2, the equivalent structure of each place p_i is finite. For a TWMG, one way to find the optimal solution of the optimization problem (8) is to enumerate all possible equivalent TMGs and adopt MILPP (10) for each of them to find a marking which has the maximal throughput. However, there are two main problems that should be emphasized:

- The constraint on the cost of resources for a TWMG should be transformed into a new constraint for each equivalent TMG.
- We have to add in Eq. (10) a series of constraints to make sure the marking $\hat{\boldsymbol{M}}$ that we find for a given net structure $\hat{N}$ is consistent with the marking $\boldsymbol{M}$ of N that produces the structure $\hat{N}$.

For each place p_i with an initial marking

$$M(p_i) = k_{j,i} \cdot gcd_{p_i},\ k_{j,i} = 0, \ldots, \frac{\phi_i}{gcd_{p_i}} - 1, \tag{15}$$

we compute

- the equivalent structure of place p_i, i.e., places $p_i^1, \ldots, p_i^{n_i}$,
- the initial markings corresponding to Eq. (15), i.e., $\hat{M}(p_i^1) = \mu_j(p_i^1), \ldots, \hat{M}(p_i^{n_i}) = \mu_j(p_i^{n_i})$.

Thus for each partition $\mathcal{M}_j$ ($j = 1, \ldots, \Phi$), we can compute the equivalent TMG system $\langle \hat{N}_j, \hat{\boldsymbol{M}}_j \rangle$.

Proposition 3: For each partition $\mathcal{M}_j$ ($j = 1, \ldots, \Phi$) in Eq. (13), we consider the following MILPP

$$
\begin{aligned}
&\max \beta_j \\
&\text{s.t.} \\
&\begin{cases}
\hat{\boldsymbol{C}}_j \cdot \boldsymbol{\alpha}_j - \boldsymbol{D}_p \cdot \hat{\boldsymbol{Post}}_j \cdot \boldsymbol{v} \cdot \beta_j + \hat{\boldsymbol{M}}_j \geq 0, & (a) \\
\boldsymbol{y}^T \cdot \boldsymbol{M}_j \leq R, & (b) \\
M_j(p_i) = k_{j,i} \cdot gcd_{p_i} + \xi_{j,i} \cdot \phi_i, \ \forall p_i \in P, & (c) \\
\hat{M}_j(p_i^s) = \mu_j(p_i^s) + \xi_{j,i}, s = 1, \ldots, n_i, & (d) \\
\hat{M}_j(q_i^a) = 0, \ i = 1, \ldots, m, \ a = 1, \ldots, x_i - 1, & (e) \\
\hat{M}_j(q_i^{x_i}) = 1, \ i = 1, \ldots, m, & (f) \\
\xi_{j,i} \in \mathbb{N}, & (g)
\end{cases}
\end{aligned}
\tag{16}
$$

with variables $\beta_j \in \mathbb{R}_{\geq 0}$, $\boldsymbol{M}_j \in \mathbb{N}^n$, $\hat{\boldsymbol{M}}_j \in \mathbb{N}^{\hat{n}}$, $\hat{\boldsymbol{\alpha}}_j \in \mathbb{R}^{\hat{m}}$, and $\xi_{j,i} \in \mathbb{N}$. Let $(\beta_j^*, \boldsymbol{M}_j^*, \hat{\boldsymbol{M}}_j^*, \hat{\boldsymbol{\alpha}}_j^*, \boldsymbol{\xi}_j^*)$ be an optimal solution of Eq. (16). Thus $\boldsymbol{M}_j^*$ is also an optimal solution of Eq. (8) restricted to partition $\mathcal{M}_j$.

Remark 1: ([12]) Among all the Φ optimal solutions associate to each partition, we can obtain the maximal throughput and its corresponding marking, i.e., optimal solutions of the cycle time optimization problem (8).

V. SUBOPTIMAL SOLUTIONS

We find that the number of partitions of equivalent structure increases exponentially as the number of places gets bigger. When this number becomes large, the efficiency of the optimal approach will be low and sometime it may not be possible to obtain an optimal solution because of the high computational cost. In this Section, we aim to reduce this cost while obtaining a near optimal solution.

Place Subset Allocation (PSA): In the manufacturing domain, the cycle time optimization problem corresponds to allocating a given number of resources to the manufacturing system (which is empty at the beginning) to maximize its throughput (productivity). From a practical point of view, it may be interesting to put resources such as pallets and machines in some specific places instead of taking all places into consideration, and we believe that in many cases this initial assignment can also lead to an optimal solution. This means that tokens are allocate to a subset of places and other places are initialized to have zero token. As a consequence, the number of partitions can be significantly reduced.

Throughput Upper Bound (TUB): It was shown in [3] that an upper bound of the throughput (lower bound of the cycle time) of a TWMG system $\langle N, \boldsymbol{M} \rangle$ under infinite server semantics can be computed by solving the following LPP:

$$
\begin{aligned}
&\max \ \beta' \\
&\text{s.t.} \\
&\boldsymbol{C} \cdot \boldsymbol{z} + \boldsymbol{M} - \boldsymbol{Pre} \cdot \boldsymbol{\theta} \cdot \beta' \geq \boldsymbol{0}
\end{aligned}
\tag{17}
$$

where $\boldsymbol{\theta} = (x_1 \cdot \delta(t_1), x_2 \cdot \delta(t_2), \ldots, x_m \cdot \delta(t_m))^T$ (recall $\boldsymbol{x}$ is the minimal T-semiflow of the TWMG). The decision variables are $\beta' \in \mathbb{R}^+$ and $\boldsymbol{z} \in \mathbb{R}^m$, and the optimal value of β' provides a upper bound of the throughput, i.e.,

$$\beta' \geq \beta. \tag{18}$$

As we discussed in remark following Proposition 2, we can with loss further refine the admissible domain considering only markings whose number of tokens in any place p_i is a multiple of gcd_{p_i}.

Additionally, it may also happen that the marking obtained by Eq. (17) is a dead marking. To ensure the liveness for the TWMG system, we propose to use a sufficient condition from [2].

Proposition 4: (Teruel *et al.* [2]) If a weighted elementary circuit satisfies $W(\boldsymbol{M}) > W(\boldsymbol{M}_D)$, then the circuit is live,

where $\boldsymbol{M}_D = (v(p_1) - 1, v(p_2) - 1, \ldots, v(p_n) - 1)^T$ and $W(\boldsymbol{M}) = \boldsymbol{y}^T \cdot \boldsymbol{M}$.

Combining these results we present the following proposition.

Proposition 5: Let $(\boldsymbol{M}, \beta')$ be the optimal solution of the MILPP

$$\begin{aligned} &\max\ \beta' \\ &\text{s.t.} \\ &\begin{cases} \boldsymbol{y}_\gamma^T \cdot \boldsymbol{M} > W(\boldsymbol{M}_D^\gamma), \forall \gamma \in \Gamma, \\ \boldsymbol{C} \cdot \boldsymbol{z} + \boldsymbol{M} - \boldsymbol{Pre} \cdot \boldsymbol{\theta} \cdot \beta' \geq \boldsymbol{0}, \\ M(p_i) \text{ mod } \text{gcd}_{p_i} = 0, i = 1, \ldots, n, \\ \boldsymbol{y} \cdot \boldsymbol{M} \leq R. \end{cases} \end{aligned} \tag{19}$$

where y_γ^T represents the P-semiflow of a weighted elementary circuit γ and the decision variables are $\beta' \in \mathbb{R}_{\geq 0}$, $\boldsymbol{M} \in \mathbb{N}^n$, and $\boldsymbol{z} \in \mathbb{R}^m$. Then $\boldsymbol{M}$ is a suboptimal live solution for problem (8) and β' is an upper bound of the throughput it produces.

VI. Test case

In this Section, we present an example taken from the literature [7] to compare the optimal solution with the suboptimal solutions.

Example 2: The TWMG model is depicted in Fig. 4. It combines cyclic assembly process, buffers, WIP, and batch operations. Two parallel machines (machine one and machine two) are working on items. Machine three loads two parts produced by machine one and three parts produced by machine two and assembles them to get one product. The assembly process is finished by machine four. The batching transportation device removes one finished products from the workshop and brings six items (resp. nine items) to machine one (resp. machine two). The TWMG model of the assembly process is depicted by Fig. 4. Transitions t_1, t_2, t_3, t_4, and t_5 represent machine one, machine two, machine three, machine four, and the transportation device, respectively.

The minimal T-semiflow of the TWMG is $\boldsymbol{x}_1 = (2, 3, 1, 1, 1)$ and the minimal P-semiflows are $\boldsymbol{y}_1 = (3, 0, 0, 6, 0, 6, 1, 0)^T$, $\boldsymbol{y}_2 = (0, 3, 0, 9, 0, 9, 0, 1)^T$, $\boldsymbol{y}_3 = (1, 0, 1, 2, 0, 0, 0, 0)^T$, and $\boldsymbol{y}_4 = (0, 1, 0, 3, 1, 0, 0, 0)^T$. Thus, the weighted

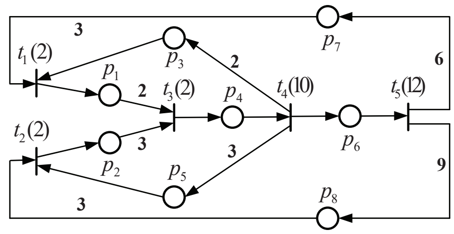

Fig. 4. The TWMG model of an assembly line.

elementary circuits corresponding to all minimal P-semiflows are $\gamma_1 = p_1t_3p_4t_4p_6t_5p_7t_1$, $\gamma_2 = p_2t_3p_4t_4p_6t_5p_8t_2$, $\gamma_3 = p_1t_3p_4t_4p_3t_1$, and $\gamma_4 = p_2t_3p_4t_4p_5t_2$. The number of tokens in γ_1 (resp. γ_2) represents the number of items processed (i.e., WIP) by machines one and two, respectively. Thus, we initialize the cost of the tokens in these circuits to ten, and that of circuits three and four to five. Thus, the P-semiflow we used in the criteria is $\boldsymbol{y} = (35, 35, 5, 175, 5, 150, 10, 10)^T$. The markings of the TWMG are partitioned into $\Phi = 216$ subsets (Case 1). For the considered net in Fig 4, we also study another case that we slightly change the weight of arcs connecting transition t_5 by multiplying two, i.e., $Pre(p_6, t_5) = 2$, $Post(p_7, t_5) = 12$, $Post(p_8, t_5) = 18$ and the number of partitions will be $\Phi = 55296$ (Case 2). For both two cases, we initialize the cost of resource R to 1000.

As for the PSA approach, we assume that the number of tokens in p_7 (resp. p_8) represents the number of items proceed by machine one (resp. machine two), and that in p_3 (resp. p_5) represents the number of servers that can be used for machine one (resp. machine two). Tokens are initially assigned to these four places.

For the application of the proposed approaches, MATLAB has been used with the MILPP toolbox YALMIP [15]. Table I summarizes the results obtained by the three approaches for the net in Fig. 4 and considers both Case 1 and Case 2. For each experiment we show the obtained optimal throughput β and the computation time. In addition, for the optimal and PSA approaches we show the number of partitions Φ that must be considered.

The results show the tradeoff between computational cost and quality of the solution. For both Case1 and Case2, we observe that the PAS approach can obtain an optimal solution while the computation time is much faster than the one required by the optimal approach. However, this result may not hold in general. Furthermore, so far we do not have an algorithm to optimally select a subset of places to which initially allocate tokens. The solutions obtained by TUB are good candidates. For Case2, the number of partitions of optimal approach is very big and the computation time is enormous. In practice, reduction of the computational cost are very important and needed.

VII. Conclusion

This paper deals with the cycle time optimization problem for deterministic TWMGs under infinite server semantics, which generalizes the case of single server semantics we considered in [13]. We propose an optimal approach and two suboptimal approaches to solve the optimization problem. Future work will explore the possibility of reducing the number of partitions of a TWMG and the computational burden of solving the MILPP proposed in

TABLE I

SIMULATION RESULTS OF THE THREE APPROACHES PROPOSED.

		Φ	β	Computation time [s]
Case1	Optimal	216	0.23	70
	PSA	36	0.23	12
	TUB	–	0.21	3
Case2	Optimal	55296	0.12	45703
	PSA	576	0.12	162
	TUB	–	0.1	3

this paper.

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REFERENCES

[1] J. Magott. "Performance evaluation of concurrent systems using Petri nets," *Information Processing Letters*, vol. 18, no. 1, pp. 7-13, 1984.

[2] E. Teruel, P. Chrzastowski-Wachtel, J.M. Colom and M. Silva. "On Weighted T-Systems," *Application and Theory of Petri Nets*, LNCS, vol. 616, pp. 348–367, 1992.

[3] J. Campos, G. Chiola, and M. Silva. "Ergodicity and throughput bounds of Petri nets with unique consistent firing count vector," *IEEE Trans. on Software Engineering*, vol. 17, no. 2, pp. 117–125, 1991.

[4] J.H. van Schuppen, M. Silva, and C. Seatzu. Control of discrete-event systems-Automata and Petri Net perspectives, *Lect. Notes Control Inf. Sci.*, Springer, Vol. 433, pp. 319-340, 2012.

[5] A. Munier. "Régime asymptotique optimal d'un graphe d'événements temporisé généralisé : Application à un problème d'assemblage," *RAIPO-APII*, vol. 27, pp. 487–513, 1992.

[6] M. Nakamura and M. Silva. "Cycle time computation in deterministically timed weighted marked graphs," *in Proc. 7th IEEE Int. Conf. Emerg. Technol. Factory Autom.*, vol. 2, pp. 1037–1046, 1999.

[7] A. Benabid-Najjar, C. Hanen, O. Marchetti, and A. Munier. "Periodic schedules for bounded timed weighted event graphs," *IEEE Trans. on Autom. Control*, vol. 57, no. 5, pp. 1222–1232, 2012.

[8] S. Gaubert. "An algebraic method for optimizing resources in timed event graphs," *Analysis and Optimization of Systes*, Springer, pp 957–966, 1990.

[9] N. Sauer. "Marking optimization of weighted marked graphs," *Discrete Event Dyn. Syst.*, vol. 13, no. 3, pp. 245–262, 2003.

[10] Z. He, Z.W. Li, and A. Giua. "Marking optimization of deterministic timed weighted marked graphs," *in Proc. the 10th IEEE Int. Conf. on Autom. Sci. Eng.*, Taipei, Taiwan, pp. 413–418, August 2014.

[11] Z. He, Z.W. Li, and A. Giua. "Optimization of deterministic timed weighted marked graphs," *IEEE Trans. Autom. Sci. Eng.*, 2016 (To appear). doi:10.1109/TASE.2015.2490538.

[12] Z. He, Z.W. Li, and A. Giua. "Cycle time optimization of deterministic timed weighted marked graphs by transformation," *IEEE Trans. Control Systems Tech.*, (under review).

[13] Z. He, Z.W. Li, and A. Giua. "Cycle time optimization of deterministic timed weighted marked graphs," *in Proc. 11th IEEE Int. Conf. Autom. Sci. Eng.*, pp. 274–279, Sweden, August, 2015.

[14] J.V. Millo and R. De Simone. “Periodic scheduling of marked graphs using balanced binary words,” *Theor. Comput. Sci.*, vol. 458, pp. 113–130, 2012.

[15] J. Löfberg. “A toolbox for modeling and optimization in MATLAB,” *in Proc. IEEE Int. symp. Comput.-Aided Control Syst. Des.*, pp. 284–289, Taipei, Taiwan, 2004.