Petri Net Controllers for Disjunctive Generalized Mutual Exclusion Constraints

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Abstract

In this paper a type of specifications called OR-GMEC for place/transition nets is defined. Such a specification consists of a set of disjunctive Generalized Mutual Exclusion Constraint, i.e. the requirement is that, at any given time, the controlled system should satisfy at least one of them. We show that a bounded OR-GMEC can be enforced by a special control structure composed by a set of monitor places (one for each constraint) plus a switcher that determines the current active constraint. We also show that such a simple control structure is not maximally permissive, and characterize this problem identifying a special subset of transitions that may be over-restricted. A modified controller that ensures maximal permissiveness is also presented. Finally, we discuss a particular control problem, that consists in preventing the firing of a given set of transitions and show that it can be reduced to an OR-GMEC problem.

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1 Introduction

Generalized Mutual Exclusion Constraints [1] (GMECs) are a class of state specifications that can be efficiently enforced in a Petri net by a simple control structure, called monitor place, that is maximally permissive. Since the monitor design does not require to enumerate the reachability set but can be solved working on the net structure, the state explosion problem is avoided and the controller design process is quite efficient.

Although this approach has many advantages and has been used in a wide range of contexts [2–6], its modeling power is rather restricted. In fact a single GMEC only considers a very special class of legal markings that belong to an n-dimensional half-space, where n is the number of places in the net. The legal marking set defined by a set of GMECs is given by the intersection of half-spaces and thus is always convex.

Furthermore, when a net contains uncontrollable transitions the set of legal markings under control must be further restricted to avoid reaching not only forbidden marking, i.e., markings that are not legal, but also weakly forbidden markings, i.e., legal markings from which a forbidden marking is reached by firing a sequence of of uncontrollable transitions. In such a case we say that the GMEC is not controllable. A solution to such a problem has been provided in [6]: in these works an efficient technique that given an uncontrollable GMEC $(\mathbf{w}, k)$ determines a more restrictive controllable GMEC $(\mathbf{w}', k')$ was presented. However, in [1] it was shown that in general the set of legal markings under control for an uncontrollable GMEC is not convex. In such a case the solution provided by the approach of [6] is suboptimal, because the set of legal markings of the controllable GMEC $(\mathbf{w}', k')$ is a strict subset of the set of legal markings under control of the uncontrollable GMEC (w, k). An optimal solution would require an approach capable of handling non convex sets of legal markings.

A similar problem arises when Petri nets models are used in *Supervisory Control Theory* [7] for arbitrary language specifications [8, 9] or for nonblocking and deadlock prevention issues [10]: the set of legal markings in this type of problems is not usually convex and the classical GMEC approach is not directly applicable. The approach followed in this paper is to extend the concept of GMEC to a more general class of constraints capable of describing non convex legal sets. We consider a constraint composed by a set of disjunctive GMECs which we call *OR-GMEC* to distinguish it from a set of (classical) conjunctive GMECs that we call *AND-GMEC*. While in an AND-GMEC a marking is legal if it satisfies all constraints, in an OR-GMEC a marking is legal if it satisfies at least one constraint.

Disjunctive constraints of this type have also been studied by Iordache and Antsaklis [5,11]. These authors designed a closed-loop supervisor to precisely keep track of the violation information, i.e., which constraints are currently violated, to ensure that the plant would not violate the control law. However, in [5, 11] an additional external decision agent is needed to determine if a firing of a transition in the plant net changes the true value of each constraint, otherwise non-determinism may arise in the supervisor. However this external decision agent is not compiled into a Petri net structure. Furthermore a very restrictive assumption in their approach is that the firing a each transition should modify the token count of each GMEC by one at most.

Here we propose a different approach to convert a given set of disjunctive constraints into a compiled place/transition supervisor that does not require the restrictive assumption of [5, 11] on the class of GMEC considered. The fundamental advantage of a compiled supervisor is the possibility of constructing a model of a closedloop system as a place/transition net that can be validated using existing techniques such as structural analysis.

We present two different design procedures for a controller capable of enforcing an OR-GMEC under the assumption that the OR-GMEC is bounded (we refer to Section 3 for a formal definition of bounded OR-GMEC and for a discussion of its limitations). The controller obtained using the first procedure, called monitor switcher, can be easily implemented but in some cases may not be maximally permissive. We characterize the conditions under which the monitor switcher is not maximally permissive identifying a special subset of transitions that may be overrestricted. We also show how the monitor switcher may be modified, if necessary, to always obtain a maximally permissive controller. The design procedures we propose work on the net structure and thus the control problem can be efficiently solved without enumerating the state space.

Finally, in the last section of the paper we show that a particular control problem, that we call transition disabling problem, i.e., ensuring that a given set of transitions is never enabled, can always be reduced to an OR-GMEC problem.

The paper is organized in six sections. Section 2 presents the Petri net formalism used in the paper. Section 3 presents the monitor switcher design. Section 4 discusses the conditions under which the switcher is maximally permissive and shows how it can be modified, if necessary, to obtain a maximally permissive controller. Section 5 deals with the transition disabling problem. Conclusions are presented in Section 6.

2 Preliminaries

2.1 Petri Net

A Petri net is a four-tuple $N = (P, T, Pre, Post)$, where P is a set of m *places* represented by circles; n *transitions* represented by bars; $Pre : P \times T \to \mathbb{N}$ and $Post : P \times T \to \mathbb{N}$ are the *pre-* and *post-incidence functions* that specify the arcs in the net and are represented as matrices in $\mathbb{N}^{m \times n}$ (here $\mathbb{N} = \{0, 1, 2, \ldots\}$).

The *incidence matrix* of a net is defined by $C = Post - Pre \in \mathbb{Z}^{m \times n}$ (here $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$). A net is said to be *self-loop free* if for all places $p \in P$ and for all transitions $t \in T$ holds $Pre(p,t) \cdot Post(p,t) = 0$. For a self-loop free net, from the incident matrix one may univocally determine the *Pre* and *Post* functions.

For a transition $t \in T$ we define its *set of input places* as ${}^\bullet t = \{p \in P \mid Pre(p,t) > 0\}$ and its *set of output places* as $t^\bullet = \{p \in P \mid Post(p,t) > 0\}$. This notation can also be applied to sets of transitions $\bar{T} \subseteq T$, defining ${}^\bullet\bar{T} = \cup^\bullet_{t\in\bar{T}} t$ and $\bar{T}^\bullet = \cup_{t\in\bar{T}} t^\bullet$.

A *marking* is a vector $M : P \to \mathbb{N}$ that assigns to each place of a Petri net a non-negative integer number of tokens, represented by black dots. We denote by $M(p)$ the marking of place p. A *marked net* $\langle N, M_0\rangle$ is a net N with an initial marking M_0.

A transition t is *enabled* at M if $M \geq Pre(\cdot,t)$ and may fire reaching a new marking $M' = M_0 + C(\cdot,t)$. We write $M[\sigma\rangle$ to denote that the sequence of transitions σ is enabled at M, and we write $M[\sigma\rangle M'$ to denote that the firing of σ yields M'.

A marking M is *reachable* in $\langle N, M_0\rangle$ iff there exists a firing sequence σ such that $M_0[\sigma\rangle M$. The set of all markings reachable from M_0 defines the reachability set of $\langle N, M_0\rangle$ and is denoted by $R(N, M_0)$. We denote by $PR(N, M_0)$ the *potentially reachable set*, i.e., the set of all markings $M \in \mathbb{N}^m$ for which there exists a vector $\mathbf{y} \in \mathbb{N}^n$ that satisfies the *state equation* $M = M_0 + C\cdot\mathbf{y}$, i.e., $PR(N, M_0) = \{M \in \mathbb{N}^m \mid \exists \mathbf{y} \in \mathbb{N}^n : M = M_0 + C\cdot\mathbf{y}\}$. We have that $R(N, M_0) \subseteq PR(N, M_0)$.

A place $p \in P$ of a marked net $\langle N, M_0\rangle$ is said to be *bounded* if there exists a nonnegative integer K such that for all $M \in R(N, M_0)$ it holds $M(p) \leq K$. A marked net is bounded if all its places are bounded.

Definition 1 *A Generalized Mutual Exclusion Constraint (GMEC) is a pair $(\boldsymbol{w}, k)$ where $\boldsymbol{w} \in \mathbb{Z}^m$ and $k \in \mathbb{N}$. A GMEC defines a set of legal markings:*

$$\mathscr{M}(\boldsymbol{w}, k) = \{M \in \mathbb{N}^m \mid \boldsymbol{w}^T \cdot M \leq k\}.$$

An AND-GMEC is a set of r GMECs denoted by a pair $(\boldsymbol{W}, \boldsymbol{k})_{AND}$ where $\boldsymbol{W} = [\boldsymbol{w}_1 \cdots \boldsymbol{w}_r] \in \mathbb{Z}^{m\times r}$ and $\boldsymbol{k} = [k_1 \cdots k_r]^T \in \mathbb{N}^r$. An AND-GMEC defines a set of legal markings

$$\mathscr{M}_{AND}(\boldsymbol{W}, \boldsymbol{k}) = \{M \in \mathbb{N}^m \mid \forall i \in \{1, \ldots, r\}, \boldsymbol{w}_i^T \cdot M \leq k_i\}.$$

A GMEC $(\mathbf{w}, k)$ on a net system $\langle N, M_0\rangle$ with $N = (P, T, Pre, Post)$ (that we call *plant net*) can be easily enforced by a control structure by adding to the net a loop-free place q called *monitor* with incidence matrix $C(q,\cdot) = -\mathbf{w}^T \cdot C(\cdot,t)$ and initial marking $M(q) = k - \mathbf{w}^T \cdot M_0$. The resulting *closed-loop* net $\langle N', M_0'\rangle$ with $N' = (P \cup \{q\}, T, Pre', Post')$ is such that the projection of its reachability set on the set of places P on N satisfies $R(N', M_0')_{\uparrow P} \subseteq \mathscr{M}(\mathbf{w}, k)$. An AND-GMEC can be enforced by a set of monitor places using the previous technique.

3 Monitor Switcher Design for OR-GMEC

In the classical GMEC condition, the logical relationship between constraints is **AND**, i.e., each legal marking must satisfies all the constraints. We consider the cases, however, in which the system is not required to satisfy all constraints but is only required to satisfy at least one of them. This type of constraints is called *OR-GMEC*.

Definition 2 *An OR-GMEC is a pair* $(\boldsymbol{W},\boldsymbol{k})_{OR}$ *where* $\boldsymbol{W} = [\boldsymbol{w}_1 \cdots \boldsymbol{w}_r] \in \mathbb{Z}^{m\times r}$ *and* $\boldsymbol{k} = [k_1 \cdots k_r]^T \in \mathbb{N}^r$*. An OR-GMEC defines a set of legal markings*

$$\mathscr{M}_{OR}(\boldsymbol{W},\boldsymbol{k}) = \{M \in \mathbb{N}^m \mid \exists i \in \{1,\ldots,r\}, \boldsymbol{w}_i^T \cdot M \leq k_i\}.$$

Unlike an AND-GMEC, in an OR-GMEC the logical condition among constraints is **OR**.

In the classical AND-GMEC controller design approaches, the controller consists in a set of monitor places P_S that are added to the plant net to determine a closed-loop net N' with $P' = P \cup P_S$ and $T' = T$. For an OR-GMEC, however, it is not in general possible to build a controller which only consists of control places. We look for a control structure that contains both additional control places P_S and control transitions T_S, i.e., the closed-loop net shall have set of places $P' = P \cup P_S$ and set of transitions $T' = T \cup T_S$. Considering that the firing of a transition $t \in T_S$ should not change the marking of the plant net, we assume that T_S is such that ${}^\bullet T_S \cup T_S^\bullet \subseteq P_S$. This motivates the following problem.

Problem 1 *Given a Petri net system* $\langle N, M_0 \rangle$ *with* $N = (P,T,Pre,Post)$ *and an OR-GMEC* $(\boldsymbol{W},\boldsymbol{k})_{OR}$*, determine a closed-loop net* $\langle N', M_0' \rangle$ *where* $N = (P \cup P_S, T \cup T_S, Pre', Post')$ *which satisfies:* ${}^\bullet T_S \cup T_S^\bullet \subseteq P_S$ *and such that the projection of the reachability set of the net* N' *on the set of places* P *of* N *satisfies* $R(N', M_0')_{\uparrow P} \subseteq \mathscr{M}_{OR}(\boldsymbol{W},\boldsymbol{k})$*. To ensure the problem has a solution we assume* $M_0 \in \mathscr{M}_{OR}(\boldsymbol{W},\boldsymbol{k})$*.*

Let us first give the definition of boundedness for an OR-GMEC.

Definition 3 *An OR-GMEC* $(\boldsymbol{W},\boldsymbol{k})_{OR}$ *is said to be bounded (with respect to* $\langle N, M_0 \rangle$*) if there exist an integer* K *such that for all constraints* $(\boldsymbol{w}_i, k_i)$ *and for all markings* $M \in R(N, M_0)$ *it holds* $\boldsymbol{w}_i^T \cdot M \leq K$*.*

Proposition 1 *An OR-GMEC* $(\boldsymbol{W},\boldsymbol{k})_{OR}$ *is bounded with respect to a plant net* $\langle N, M_0 \rangle$ *if for all constraints* $(\boldsymbol{w}_i, k_i)$ *holds:* $w_i(p_j) > 0 \Rightarrow$ *place* p_j *is bounded.*

Proof: Without loss of generality, for an arbitrarily chosen i let us assume $w_i(p_j) > 0$ for $j \leq r'$ and $w_i(p_j) \leq 0$ for $j > r'$. Assume that all place p_j are bounded for $j \leq r'$; then for all $M \in R(N, M_0)$ and for all

$j \leq r'$ holds $M(p_j)c_j$, where $c_j \in \mathbb{N}$ is an upper bound. Define a vector $\mathbf{w}'_i \in \mathbb{Z}^m$ where $w'_i(p_j = w_i(p_j)$ for $j \leq r'$, $w'_i(p_j) = 0$ for $j > r'$. Therefore for all $M \in R(N, M_0)$ we have:

$$\mathbf{w}_i^T \cdot M \leq \mathbf{w}_i'^T \cdot M \leq \sum_{j=1}^{r} w_i(p_j) \cdot c_j \leq K_i \in \mathbb{N} \tag{1}$$

According to Definition 3, define $K \in \mathbb{N}$ such that $K = \max\{K_i\}$. Since all constraints are K-bounded then $(\mathbf{W}, \mathbf{k})_{OR}$ is bounded. ■

An immediate result of the previous proposition is the following one.

Corollary 1 *If $\langle N, M_0 \rangle$ is bounded, any OR-GMEC defined on its reachability set is bounded with respect to $\langle N, M_0 \rangle$.*

Note that Proposition 1 provide only sufficient (but not necessary) conditions for the boundedness of an OR-GMEC. We believe that this property is not too restrictive since it applies not only to the class of bounded nets but to meaningful classes of unbounded nets as well.

We found that for a given Petri net and a bounded OR-GMEC $(\mathbf{W}, \mathbf{k})_{OR}$, the OR-GMEC can always be implemented by a net structure. If the OR-GMEC is unbounded, however, even in a very simple system a maximally permissive closed-loop net may not exist (see discussion in Section 6). As a result, in this paper we only focus on the bounded OR-GMECs.

The next algorithm transforms a bounded OR-GMEC to a closed-loop net with a control structure called *monitor switcher*.

Algorithm 1 Monitor Switcher Design

Input: A plant net $\langle N, M_0 \rangle$, a constraint $(\mathbf{W}, \mathbf{k})_{OR}$
Output: A closed-loop net $\langle N', M'_0 \rangle$ such that $R(N', M'_0)_{\uparrow P} \subseteq \mathscr{M}_{OR}(\mathbf{W}, \mathbf{k})$

1: Select a large constant K:

$$K \geq K' = \max_{i \in \{1,\ldots,r\}} \max_{M \in R(N, M_0)} \{\mathbf{w}_i^T \cdot M - k_i\} \tag{2}$$

2: For each single constraint $(\mathbf{w}_i, k_i)$ $(i \in \{1, \ldots, r\})$, add a loop-free place q_i to the plant net $\langle N, M_0 \rangle$ with $C(q_i, t) = -\mathbf{w}_i^T \cdot C(\cdot, t), M_0(q_i) = k_i - \mathbf{w}_i^T \cdot M_0 + K$.
3: Add r places $q'_1, \ldots, q'_r$. Add $r \times (r-1)$ transition $t_{i,j}$ where $i \neq j$. Let $Pre(q'_i, t_{i,j}) = 1$ and $Post(q'_j, t_{i,j}) = 1$. Let $Post(q_i, t_{i,j}) = K$ and $Pre(q_j, t_{i,j}) = K$.
4: Pick an $l : M(q_l) \geq K$. Let $M(q'_l) = 1$ and $M(q'_i) = 0$ for all $i \neq l$. Let $M(q_l) = M(q_l) - K$.
5: Output the closed-loop net $\langle N', M'_0 \rangle$.

We will illustrate the algorithm by showing an example.

Example 1 *In Figure 1 the Petri net on the left is the plant, and the related OR-GMEC are: $M(p_2) + M(p_3) \leq 2$ **OR** $M(p_3) + M(p_4) \leq 2$. By applying Algorithm 1 we obtain the closed-loop net on the right. The subnet in red is the monitor switcher. In the closed-loop net, q_i is the monitor associated to the i-th constraint. If the*

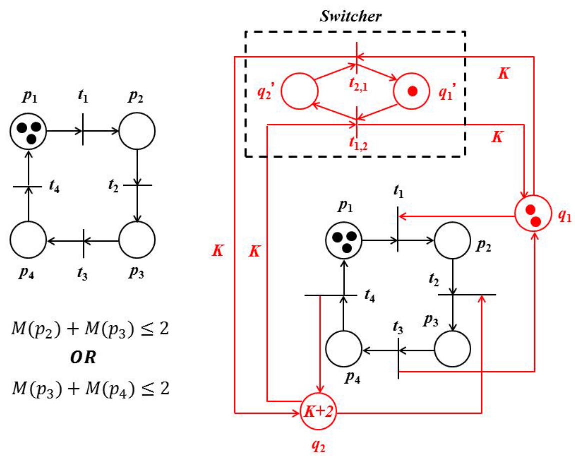

Figure 1: The net in Example 1. The subnet in red is the monitor switcher.

unique token in the switcher (the subnet in the dashed box) is in q_i', all monitors q_j (for $j \neq i$) have a sufficient number of tokens to make the corresponding constraint inactive. Therefore the unique token in q_i' indicates that the i-th constraint is active, i.e., the current legal marking satisfies (at least) the i-th constraint.

Note that in the first step the value of K should be larger than but not necessarily identical to K'. Therefore one can efficiently find a proper K by solving the potential reachable markings by linear programming. This algorithm cannot be applied for an unbounded OR-GMEC because a finite K that makes any constraint inactive does not exist.

From the structure of the resulting net we have the following theorem that ensure that only legal markings are reached under control. The following theorem also ensures that a solution disjunctive GMEC based on monitor places and switcher always exists if the OR-GMEC is bounded.

Theorem 1 *A net $\langle N', M_0' \rangle$ constructed by Algorithm 1 satisfies $R(N', M_0')_{\uparrow P} \subseteq \mathscr{M}_{OR}(\mathbf{W}, \boldsymbol{k})$.*

Proof: In Step 2 of the algorithm, the resulting net has among its invariants the following ones:

$$\mathbf{w}_i^T \cdot M + M(q_i) = k_i + K. \tag{3}$$

After the monitor switcher is added, the resulting net has among its invariants the following ones:

$$\begin{cases} \mathbf{w}_i^T \cdot M + M(q_i) = k_i + K - M(q_i') \cdot K \\ \sum_i M(q_i') = 1 \end{cases} \tag{4}$$

The last invariant ensures that for any marking $M' \in R(N', M_0')$ there exists a constraint $\bar{i}$ such that $M(q_{\bar{i}}') = 1$ and by (4) M' also satisfies $\mathbf{w}_{\bar{i}}^T \cdot M + M(q_{\bar{i}}) = k_{\bar{i}}$. So $M = M'_{\uparrow P}$ satisfies at least the $\bar{i}$-th constraint $(\mathbf{w}_{\bar{i}}, k_{\bar{i}})$ in

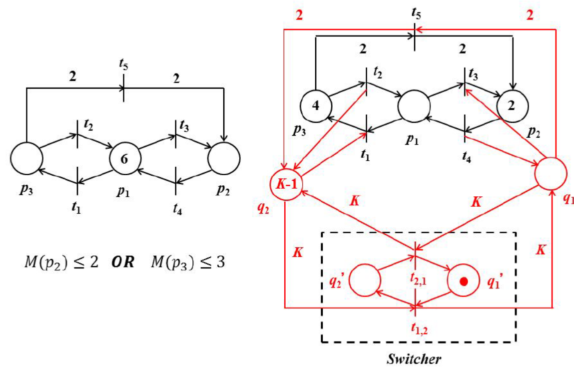

Figure 2: An example in which $\langle N', M'_0 \rangle$ is not maximally permissive.

$(\mathbf{W}, \mathbf{k})_{OR}$. ■

A desirable property of the obtained $\langle N', M'_0 \rangle$ is *maximal permissiveness*, that is, if the firing of a transition t in the plant net is legal, it should also be firable in the closed-loop net. In the closed-loop net, however, the firing of t may depend on the previous firing of a control transition in the switcher. This motivates the following weaker definition of permissiveness.

Definition 4 *A closed-loop net $\langle N', M'_0 \rangle$ (with respect to $\langle N, M_0 \rangle$) is said to be* maximally permissive *if for all $M' \in (N', M'_0)$ the following condition*[1] *holds:*

$$(M'_{\uparrow P}[t\rangle_N M \in \mathscr{L}) \Longrightarrow (\exists \sigma \in T_S^* : M'[\sigma t\rangle_{N'}) \tag{5}$$

In the classical supervisory control theory [7], for each marking M of the plant the supervisor generates a control pattern $\xi(M)$ that contains all admissible transitions that can fire without violating the specification. In our approach, since the monitor switcher has its own transitions, when the closed-loop net by the firing of a plant transition reaches a marking M'_1 corresponding to the plant marking $M = M'_{1\uparrow P}$, the closed-loop nets may evolve to marking $M'_2, M'_3, \ldots$ by only firing transitions in T_S, while $M'_{i\uparrow P} = M$ for all i. In such a case, we assume that the equivalent control pattern for M is $\xi_{tot}(M)$ which contains all plant transitions enabled by any of the marking M'_i.

The following theorem shows that Algorithm 1 does not determine a maximally permissive net.

Theorem 2 *The closed-loop net $\langle N', M'_0 \rangle$ obtained from Algorithm 1 is not always maximally permissive with respect to $\langle N, M_0 \rangle$.*

Proof: We prove this result by providing a counterexample. Consider the plant net $\langle N, M_0 \rangle$ on the left of Figure 2 and the bounded OR-GMEC $M(p_2) \le 2$ **OR** $M(p_3) \le 3$. In $\langle N, M_0 \rangle$, both [0 2 4] and [0 4 2]

[1] Here we distinguish the enabling in the nets N and N' using the notation $[\cdot\rangle_N$ and $[\cdot\rangle_{N'}$, respectively.

are reachable legal markings and [0 2 4][$t_5\rangle$[0 4 2]. If we apply Algorithm 1 on this plant, we obtain the closed-loop net on the right in the figure where the marking shown is reached by firing $\sigma = t_1t_1t_1t_1t_3t_3$, which is now corresponding to the plant marking [0 2 4]. Transition t_5 in $\langle N', M'_0\rangle$ is now blocked by q_1, and the monitor switcher cannot shift from q'_1 to q'_2 since $M(q_2) = K - 1 < K$. Therefore the marking [0 4 2] cannot be reached from [0 2 4] by firing a sequence (possibly empty) of transitions in T_S followed by t_5. ■

One may have noticed that by applying Algorithm 1 the resulting Petri net in Figure 2 is not maximally permissive according to Definition 4, but all the legal markings are still reachable, e.g., the marking [0 4 2] is still reachable. However, in some cases of OR-GMECs there may exist a legal marking which is only reachable by firing a transition unnecessarily blocked by the monitor switcher, and thus the maximal permissiveness in terms of markings is not guaranteed neither. One can easily find an example which is not maximally permissive in terms of markings.

The reason why the closed-loop net $\langle N', M'_0\rangle$ constructed by Algorithm 1 is not always maximally permissive can be explained as follows: when the switcher marking is in a state under which a constraint i is active, if it wants to shift to a state under which another constraint j is activated, both constraints must be satisfied by the plant marking at the same time. However, in general a legal firing may also occur from a plant net marking that satisfies only constraint i yielding a new plant marking that satisfies only constraint j. The undesired restriction of the switcher reduces the permissiveness of the closed-loop net. Since we desire a maximally permissive closed-loop net, the closed-loop net $\langle N', M'_0\rangle$ has to be suitably modified: this is possible as shown in the next section.

4 Modifying a Monitor Switcher to Ensure Maximal Permissiveness

Definition 5 *A transition $t \in T$ is said to be maximally permissive in a closed-loop net $\langle N', M'_0\rangle$ if for all $M' \in R(N', M'_0)$ the following condition holds:*

$$M'_{\uparrow P}[t\rangle_N \bar{M} \in \mathscr{L} \quad \Rightarrow \quad \exists \sigma_S \in T_S^* : M'[\sigma_S t\rangle_{N'}$$

In the closed-loop net in Figure 2 we can see that t_1, t_2, t_3, and t_4 are all maximally permissive while t_5 is not. This is because the current marking [0 2 4] violates the constraint $M(p_3) \leq 3$ and the firing of t_5 will lead to a marking [0 4 2] that violates the constraint $M(p_2) \leq 2$. However, the monitor switcher does not consider that the firing of t_5 will lead to a marking that violates $M(p_2) \leq 2$ while concurrently satisfies $M(p_3) \leq 3$.

The following definition aims to characterize this situation.

Definition 6 *The* influence vector *of a transition t is a vector $\boldsymbol{z}_t \in \mathbb{Z}^r$ defined as $\boldsymbol{z}_t = \boldsymbol{W}^T \cdot C(\cdot, t)$.*

Definition 7 *A transition t is called a* migrating transition *if* $(z_t \ngeq \mathbf{0}) \wedge (z_t \nleq \mathbf{0})$.

For such a transition we define its positive support as $\mathscr{Z}_t^+ = \{i \mid z_t(i) > 0\}$ *and its negative support as* $\mathscr{Z}_t^- = \{i \mid z_t(i) < 0\}$.

We now show that the non-maximal permissiveness may be only due to migrating transitions.

Theorem 3 *If a transition t is not maximally permissive, then t is a migrating transition.*

Proof: For an arbitrarily chosen t which is non-maximally permissive, according to Definition 5, $\exists M_1' : M'_{1\uparrow P} = M_1 \in \mathscr{L}, M_1[t\rangle_N M_2 \in L, \forall\sigma \in T_S^*, \neg M[\sigma t\rangle_{N'}$. Since $M_1, M_2 \in \mathscr{L}$, let $S_1 \neq \emptyset$ and $S_2 \neq \emptyset$ be the constraints M_1 and M_2 satisfies, respectively.

We first prove, by contradiction, that $S_1 \cap S_2$ is empty. If $S_1 \cap S_2 \neq \emptyset$, pick any i: $(\mathbf{w}_i, k_i) \in S_1 \cap S_2$, if the unique token in the monitor switcher is now in q_i', t is not blocked (σ equals to the empty string) since all the constraints are deactivated. If the unique token in the monitor switcher is in $q_j' \neq q_i'$, by firing $t_{j,i}$ it can be moved into q_i' so that t may fire after $\sigma = t_{j,i}$.

Since $S_1 \cap S_2$ is the empty set, therefore, M_2 must violate all the constraints in S_1 while M_1 must violate all the constraints in S_2. As a result, it must hold for all $M_1[t\rangle M_2, \mathbf{w}_i^T \cdot M_2 < \mathbf{w}_i^T \cdot M_1$ for $(\mathbf{w}_i, k_i) \in S_1$ and $\mathbf{w}_i^T \cdot M_2 > \mathbf{w}_i^T \cdot M_1$ for $(\mathbf{w}_i, k_i) \in S_2$. This implies $z_t(i) < 0, i \in \{1, \ldots, r'\}$ and $z_t(i) > 0, i \in \{r'+1, \ldots, r''\}$. Therefore t is a migrating transition. ■

Proposition 2 *The firing of a migrating transition t does not lead to a violation of the OR-GMEC if the current plant net marking M satisfies the condition:* $\exists i, \mathbf{w}_i^T \cdot M \leq k_i - z_t(i)$.

Proof: Suppose the current marking M satisfies: $\exists i, \mathbf{w}_i^T \cdot M \leq k_i - z_t(i)$. After the firing of t the system reach M_1. From the definition of $z_t(i)$ it must hold $\mathbf{w}_i^T \cdot M_1 \leq k_i$. Therefore M_1 is a legal marking which satisfies at least the i-th single GMEC. ■

From Theorem 3 we can clearly see that only migrating transitions should be treated to enhance permissiveness. We propose to add a set of mirror transitions to a migrating transition thus constructing from the closed-loop net $\langle N', M_0' \rangle$ obtained from Algorithm 1 a modified closed-loop net $\langle N'', M_0'' \rangle$. Such a set of mirror transitions is described in the following definition (where Pre'' and $Post''$ denote the pre and post matrices of the modified net).

Definition 8 *For a transition $t \in T$, its mirror set $\mathscr{T}(t)$ is defined as:*

$$\begin{cases} \mathscr{T}(t) = \{t' \mid \exists t_S \in T_S, \\ \quad \forall p \in P \cup P_S \setminus \{q_l, l = 1, \ldots, r\}, \\ \qquad Pre''(p,t') = Pre'(p,t) + Pre'(p,t_S), \\ \qquad Post''(p,t') = Post'(p,t) + Post'(p,t_S); \\ \quad \forall p \in \{q_l, l = 1, \ldots, r\}, \\ \qquad Pre''(p,t') = Pre'(p,t_S) - Post'(p,t), \\ \qquad Post''(p,t') = Post'(p,t_S) - Pre'(p,t)\} \end{cases} \tag{6}$$

Theorem 4 *If under M a migrating transition t is enabled in N while its firing is legal, but t is blocked in N', there must exist a $t' \in \mathscr{T}(t)$ which is enabled under M. And the firing of a transition $t' \in \mathscr{T}(t)$ has the same impact on state evolution as the firing of sequence $\sigma = tt_S$ in N'.*

Proof: Suppose under M the current active constraint is i and the firing of t will deactivate i and activate j. For the places $p \in \{q_l\}$ from Definition 8 $Pre''(q_j,t') = Pre'(q_j,t_S) - Post'(q_j,t) = K + z_t(j)$. Since j-th constraint is not activated, $M(q_j) = k_j + K - \mathbf{w}_j^T \cdot M$. According to Proposition 2 it holds $\mathbf{w}_j^T \cdot M \leq k_j + z_t(j)$. Therefore we have $M(q_j) \geq K + z_t(j) = Pre''(q_j,t)$, t' is not blocked by q_l. For the places $p \in P \cup P_S \setminus \{q_l\}$ which are the input places of t' it is obvious that $M(p) \geq Pre(p,t')$. Therefore t' is enabled under M.

The fact that the firing of a transition $t' \in \mathscr{T}(t)$ has the same impact on state evolution as the firing of sequence $\sigma = tt_S$ in N' is straight forward from its definition. ■

Theorem 4 ensures if in N' neither t nor t_S is enabled but $M + C_{N'}\sigma \geq \mathbf{0}$ where $\sigma = tt_S$, we can fire t' instead of firing t and t_S sequentially.

We present an algorithm to construct the modified net.

Algorithm 2 Modified closed-loop net design

Input: A closed-loop net $\langle N', M_0' \rangle$ for $(\mathbf{W}, \mathbf{k})_{OR}$ obtained by Algorithm 1
Output: A modified closed-loop net $\langle N'', M_0'' \rangle$ that is maximally permissive
1: Let $N'' = N'$, $M_0'' = M_0'$.
2: Let T_{mig} be the set of migrating transitions.
3: Pick a transition t from T_{mig}.
4: For all $(i,j) \in \mathscr{Z}_t^+ \times \mathscr{Z}_t^-$, add to T'' a transition $t_{dup,ij}$ with: $\forall p \in P \cup P_S \setminus \{q_l\}$, $Pre''(p,t_{dup,ij}) = Pre'(p,t) + Pre'(p,t_{i,j}), Post''(p,t_{dup,ij}) = Post'(p,t) + Post'(p,t_{i,j})$; $\forall p \in \{q_l\}, Pre''(p,t_{dup,ij}) = Pre'(p,t_{i,j}) - Post'(p,t), Post''(p,t_{dup,ij}) = Post'(p,t_{i,j}) - Pre'(p,t)$.
5: Remove t from T_{mig}.
6: If $T_{mig} \neq \emptyset$ goto Step 2.
7: Output the closed-loop net $\langle N'', M_0'' \rangle$.

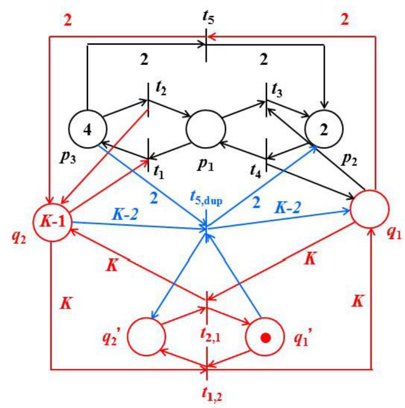

Figure 3: A modified closed-loop net that is maximally permissive constructed by Algorithm 2 for the net in Figure 2.

The Algorithm 2 can be illustrated in the following way. First we compute all migrating transitions which should be deal with. For each migrating transition t, according to Theorem 3, the only condition under which t is abnormally blocked under a marking $M'_{\uparrow P} = M$ is when some satisfied constraints $i \in \mathscr{Z}_t^+$ are going to be violated by the firing of t while some violated constraints $j \in \mathscr{Z}_t^-$ are going to be satisfied. At this moment the unique token in the monitor switcher must be in an q'_i. Since the firing of t is legal, it must hold $M' + C_{N'} \cdot \mathbf{y} \geq \mathbf{0}$ where $\mathbf{y}$ is the count vector which represents $\sigma = tt_{i,j}$. So a transition $t_{dup,ij}$ is added. The firing of $t_{dup,ij}$ indicates the firing of t and, simultaneously, deactivate the i-th constraints while activate the j-th constraint.

Since the firing of a transition $t' \in \mathscr{T}(t)$ has the same impact on state evolution as the firing of sequence $\sigma = tt_S$ in N', the definition of maximal permissiveness given in Definition 4 should be changed for the net $\langle N'', M''_0 \rangle$ as follows.

Definition 9 *A closed-loop net $\langle N'', M''_0 \rangle$ (with respect to $\langle N, M_0 \rangle$) is said to be maximally permissive if $\forall M'' \in R(N'', M''_0)$ the following condition holds:*

$$(M''_{\uparrow P}[t\rangle_N \bar{M} \in \mathscr{L}) \Longrightarrow (\exists \sigma \in T_S^*, \exists t' \in \mathscr{T}(t) \cup \{t\} : M''[\sigma t'\rangle_{N''}) \tag{7}$$

According to Definition 9 we can state the following result.

Theorem 5 *The closed loop-net $\langle N'', M''_0 \rangle$ obtained from Algorithm 2 is maximally permissive with respect to $\langle N, M_0 \rangle$.*

Proof: The result follows from the previous discussion, since the only conditions that violate maximal permissiveness are removed in the the net $\langle N'', M''_0 \rangle$ by the addition of the mirror transitions. ■

Example 2 *Figure 3 shows the resulting net obtained from the net in Figure 2 by Algorithm 2. In the net in Figure 2, t_5 is the only migrating transition. $z_{t_5} = [2 - 2]$ indicates the firing of t_5 may activate the second constraint $M(p_3) \leq 2$ while concurrently deactivate the first constraint $M(p_2) \leq 2$. The fact that $|\mathscr{Z}_{t_5}^{+}| = |\mathscr{Z}_{t_5}^{-}| = 1$ indicates $|\mathscr{T}(t_5)| = 1$. Therefore one transition $t_{5,dup}$ is added with its incident matrix $Pre''(\cdot, t_{5,dup})$ and $Post''(\cdot, t_{5,dup})$ are computed according to Step 4 so that the firing of $t_{5,dup}$ is equivalent to fire t_5 and $t_{1,2}$ concurrently (although neither of them is enabled under [0 2 4]). The resulting net $\langle N'', M_0'' \rangle$ is shown in Figure 3 in which the subnet in blue is newly added. The net is maximally permissive.*

Let us now discuss the complexity of the algorithms presented so far. To convert an OR-GMEC with r single constraints to a monitor switcher Algorithm 1 adds $2r$ places and $r \times (r-1)$ transitions. Applying Algorithm 2 for each migrating transition t, $|\mathscr{Z}_t^{+}| \times |\mathscr{Z}_t^{-}|$ mirror transitions are added, which in the worst case means $(r/2) \times (r/2)$ transitions will be added. Assuming all transitions are migrating, the total number of places and transitions added in the modified monitor switcher net is of order $O(r)$ and $O(nr^2)$, respectively, where n is the number of transitions in the net. Considering that the complexity is linear in the size of the net (number of transitions) and quadratic in the number of constraints, while the enumeration of the reachability set is not required, we believe this approach is efficient.

We have some final comments concerning Algorithm 2. From our observation after the first stage of controller design, some of the migrating transitions may be maximally permissive so that in the monitor switcher controlled closed-loop net it is not necessary to add the corresponding mirror transitions. However, since the characterization of a maximally permissive transition relies on the Petri net structure, initial marking distribution, and the OR-GMEC itself, one cannot determine if a migrating transition is maximally permissive or not unless the whole reachability graph is checked. Thus it is preferable to add the mirror transitions (albeit redundant) for all migrating transitions rather than to compute the reachability graph.

5 Transition Disabling Problem

In this section we consider a particular control problem and show that it can be reduced to an OR-GMEC problem.

Problem 2 *(Transition Disabling). Given a Petri net $\langle N, M_0 \rangle$, let T_d be a subset of it transitions. We want to restrict the behavior of this net so that no marking M that enables a transition in T_d is reachable under control, i.e., the set of legal markings is $\mathscr{L} = \{M \in R(N, M_0) \mid (\forall t \in T_d) \neg M[t\rangle\}$. To ensure the problem has a solution we assume that no transition in T_d is enabled at the initial marking M_0 and that for all $t \in T_d$ holds ${}^{\bullet}t \neq \emptyset$.*

In plain words the problem consists in preventing the firing of all transitions in T_d without directly disabling them (e.g., because they are uncontrollable in the supervisory control framework).

Definition 10 *For all transitions $t \in T_d$, we define the* minimal enabling marking *of t as $M_{en,t} = Pre(\cdot, t)$.*

We now show that Problem 2 can be formulated as an OR-GMEC problem.

Theorem 6 *Given a Petri net $\langle N, M_0 \rangle$ and a transition t such that ${}^{\bullet}t \neq \emptyset$, there exist an OR-GMEC $(\boldsymbol{W}, \boldsymbol{k})_{OR}$ such that*

$$\{M \in R(N, M_0) \mid \neg M[t\rangle = R(N, M_0) \cap \mathscr{M}_{OR}(\boldsymbol{W}, \boldsymbol{k})$$

Furthermore the OR-GMEC is bounded if all places $p \in^{\bullet} t$ are bounded.

Proof: Consider a transition t which needs to be prevented from firing and let ${}^{\bullet}t = \{p_{j1}, \ldots, p_{jq}\}$. From the definition of minimal enabling marking, the legal marking set is:

$$\mathscr{L} = \{M \in \mathbb{N}^m \mid M \ngeq M_{en,t}\}$$

This condition can be rewritten as the following OR-GMEC condition:

$$\bigvee \begin{cases} \mathbf{e}_{j1}^T \cdot M < M_{en,t}(p_{j1}) \\ \mathbf{e}_{j2}^T \cdot M < M_{en,t}(p_{j2}) \\ \cdots \\ \mathbf{e}_{jq}^T \cdot M < M_{en,t}(p_{jq}) \end{cases} \tag{8}$$

where $\mathbf{e}_i \in \mathbb{N}^m$ denotes i-th canonical basis vector. Obviously (8) is an OR-GMEC problem with:

$$\mathbf{W} = [\mathbf{e}_{j1}, \mathbf{e}_{j2}, \ldots, \mathbf{e}_{jq}] \quad \text{and} \quad \mathbf{k} = \begin{bmatrix} M_{en,t}(p_{j1}) - 1 \\ M_{en,t}(p_{j2}) - 1 \\ \vdots \\ M_{en,t}(p_{jq}) - 1 \end{bmatrix}.$$

The boundedness of the constraint follows from Proposition 1. ■

From Theorem 6 we see that a transition disabling problem can be reduced to an OR-GMEC problem, and if all places in ${}^{\bullet}T_d$ are bounded we can use the technique proposed in Section 4 to convert the OR-GMEC condition to a net controller as described in the following algorithm.

Example 3 *An example of transition disabling problem is given in Figure 4. The origin marked Petri net is*

Algorithm 3 Transition disabling

Input: A marked net $\langle N', M'_0 \rangle$, a subset of transitions T_d

Output: A closed-loop net $\langle N', M'_0 \rangle$ where no transition in T_d is enabled at $M \in R(N, M_0)$

1: Let $TEMP = T_d$.
2: Pick a t in $TEMP$, compute its $M_{en,t}$.
3: If $|^\bullet t| \geq 2$, transform the constraint $M \ngeq M_{en,t}$ to an OR-GMEC problem. Convert the OR-GMEC to a Petri net controller and add it to the net. Goto Step 5.
4: If $|^\bullet t| = 1$, transform the constraint $M \ngeq M_{en,t}$ to a classical single GMEC. Convert the single GMEC to a Petri net controller and add it to the net.
5: Remove t from $TEMP$.
6: If $TEMP \neq \emptyset$ goto Step 2.
7: Output the closed-loop net $\langle N', M'_0 \rangle$.

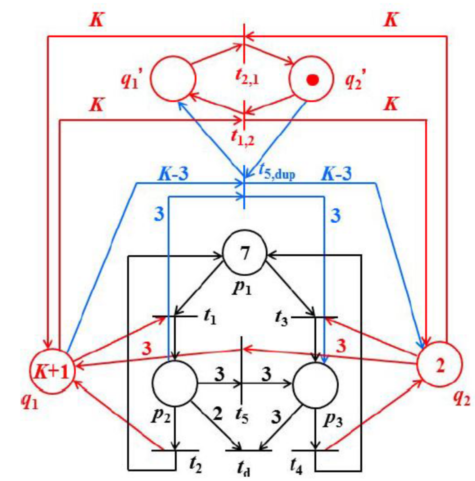

Figure 4: An example and its solution of the transition disabling problem.

*the subnet in black. The transition t_d is the transition we want to disable. Because $^\bullet t = \{p_2, p_3\}$ and $M_{en,t_d} = [023]$, by Theorem 6 this can be converted to the OR-GMEC: $M(p_2) \leq 1$ **OR** $M(p_3) \leq 2$. By converting the OR-GMEC to a Petri net controller, the closed-loop net is the entire net in Figure 4 and it is maximally permissive. In this example the arcs from t_d to q_1 and q_2 are not drawn since t_d would never be enabled.*

The previous algorithm adds to the net to be controlled a monitor switcher for each transition in T_d. If some transition in T_d is dead (i.e., may never fire) the corresponding monitor switcher is redundant and does not constrain the net evolution.

6 Conclusions

In this paper we are considering the OR-GMEC, i.e., a constraint composed by a set of disjunctive GMECs. Since an OR-GMEC can be transformed to a Petri net control structure with maximal permissiveness, it would be a supplement and extension to the classical GMEC approach. The algorithms can be widely used

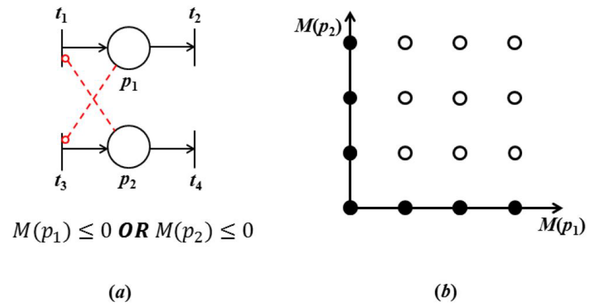

Figure 5: An OR-GMEC with unbounded places:(a) the original net and the inhibitor solution; (b) part of its reachability graph.

in different type of systems where, according to the control demand, the legal marking set $\mathscr{L}$ is not a convex set.

In this paper we restricted our attention to bounded OR-GMECs. An expected extension of this approach is to convert an arbitrarily OR-GMEC (possibly not bounded) into a maximally permissive control structure. However we believe that if the OR-GMEC is unbounded, it may be impossible to construct such a controller as a place/transition net. For instance, consider the Petri net in Figure 5(a) without the red dash arc. Here the OR-GMEC $M(p_1) \leq 0$ **OR** $M(p_2) \leq 0$ is unbounded. The legal markings are partially presented as solid circles in Figure 5(b). Such OR-GMEC can be enforced by inhibitor arcs as in Figure 5(a) considering red dashed arcs. We conjecture that for an arbitrary OR-GMEC there exists a maximally permissive closed-loop net with inhibitor arcs. However, it is well known that a Petri net with inhibitor arcs is equivalent to a Turing Machine and may not always be converted to a place/transition net. This will be the the object of our future work.

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