Finite-Time Consensus based Clock Synchronization by Discontinuous Control

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Abstract

In this paper we develop a decentralized discontinuous algorithm which guarantees the finite time synchronization of a network of clocks, which are modeled as continuous-time integrators and are subject to unknown-but-bounded time-varying disturbances. The proposed synchronization algorithm is asynchronous and consists in a randomly applied local interaction rule. After a finite transient time, the proposed algorithm provides the approximated synchronization of the network by attenuating the destabilizing effect of the disturbances. Lyapunov analysis is carried out to substantiate the suggested algorithm, and simulation results are illustrated throughout the paper.

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I. INTRODUCTION

The problem of reaching consensus, i.e. driving the state of a set of dynamical systems towards the same value, has received much attention due to its many applications in, both, the modeling of natural phenomena such as flocking (see e.g. [12], [16], [4], [14]) and in the solution of several type of control problems involving synchronization or agreement between dynamical systems (see [9], [11], [1]).

In this paper we are interested in proposing an approach to the agreement problem for a network dynamical systems modeled by first order continuous time integrators subject to unknown perturbations. The main novelty consists in proposing a local interaction rule which provides finite time convergence of the network to a condition of approximate agreement, by attenuating the effect of the disturbances.

There is a significant body of literature on practical clock synchronization algorithms for various systems and applications (see e.g. the *Network Time Protocol* in [8], the *Precision Time Protocol* in [2], the *Flooding time Synchronization Protocol* in [6], and many other examples as well, as surveyed in [13])

Our approach is based on a discontinuous local interaction rule using sliding mode control concepts and techniques (see [15]). Discontinuous local interactions rules have been already exploited in the framework of consensus or agreement algorithms, and several example of applications to flocking or synchronization problems exist (see [3]).

In [5], a finite-time consensus algorithm is proposed for the tracking problem in a multi-robot system with static topology but varying leader. In [17], [7] and [10], finite-time consensus algorithms are provided for networks of unperturbed integrators by exploiting discontinuous local interaction rules under time varying (both undirected and directed) network topologies.

The framework of sliding mode based finite-time consensus algorithms offers an interesting research direction in that the developed techniques provide superior disturbance attenuation properties with respect to unmodeled dynamics as compared to the more conventional linear methodologies.

Our approach differs from the above mentioned literature in that we address the analysis of the practical stability and disturbance attenuation properties of finite-time consensus, under switching topologies. The finite time transient to reach consensus is a design parameter that can be made arbitrarily small by varying the algorithm parameters. The disturbance rejection performance, which is quantified in this paper, depends on the time-varying network connectivity properties.

To the best of our knowledge, the above aspects were never simultaneously addressed and characterized in the existing literature.

The structure of the paper is as follows. In Section II we formulate the problem under investigation and we state the proposed local interaction rule. In Section III the convergence properties of the proposed algorithm are investigated, and, particularly, in Subsection III-A the pros and cons of the achieved result are discussed. In Section IV some simulation results are presented and, finally, in Section V conclusions are drawn and future research directions are discussed.

II. PROBLEM STATEMENT

Let us consider n clocks connected by a communication network whose topology is described by a connected undirected graph $\mathcal{G} = \{V, E\}$, where $V = \{1, \ldots, n\}$ is the set of clocks and $E \subseteq \{V \times V\}$ is the set of edges representing communication channels between clocks. Let the (i, j) elements of E be ordered such that $i < j$, and let $|E|$ denote the cardinality of E. Further assume that graph $\mathcal{G}$ does not contain self loops. Clocks are modeled by the next dynamics

$$\dot{x}_i = \alpha + \nu_i(t) + u_i(t), \quad x_i(0) = x_{i0}, \quad i = 1, \ldots, n, \tag{1}$$

where α is a known desired clock speed, the same for each clock, $\nu_i(t)$ is a bounded disturbance corrupting the clock dynamics, $u_i(t)$ is the control input used to adjust the frequency of the i-th clock, and x_{i0} is the initial clock off-set.

The bounded disturbance $\nu_i(t)$ is a general representation of all the possible modeled and unmodeled uncertainties in the clock dynamics such as time off-set, stochastic speed drift due to random electrical noise, deterministic speed drift due to temperature or other environmental effects, etc. The only assumption made on the disturbance signals $\nu_i(t)$ $(i = 1, \ldots, n)$ is uniform boundedness, in accordance with

$$|\nu_i(t)| \leq \Pi_i \leq \Pi, \qquad \Pi = \max_{i \in V} \Pi_i. \tag{2}$$

At each instant of time, only a subset of the available communication channels in $\mathcal{G}$ are active for synchronization. Let $\hat{\mathcal{G}}(t) = \{V, E(t)\}$ be a time varying graph representing at each instant of time the instantaneous topology of active links, where $E(t) \subseteq E$ for all $t \geq 0$ is the set of active edges at time t.

Our objective is to develop a clock interaction rule $u_i(t)$ compatible with the time-varying topology of graph $\hat{\mathcal{G}}(t)$ that guarantees the following **practical finite-time consensus** properties

$$\exists \rho, t_r \in \mathbb{R}^+ : \quad |x_i(t) - x_j(t)| \leq M \quad \forall t > t_r, \ \forall i, j \in E, \tag{3}$$

where $M = \rho \Pi$ and t_r are positive constants, that shall be specified later on, and Π is the upper bound to the disturbance signals.

With reference to the time varying graph $\hat{\mathcal{G}}(t)$, let $\mathcal{N}_i(t) \subseteq V$ denote the set of neighbors of node i at time t. The proposed discontinuous communication protocol takes the form

$$u_i = -\lambda \sum_{k \in V, k \neq i} r_{ik}(t) sgn(x_i(t) - x_k(t)) \tag{4}$$

where λ is the tuning constant of the algorithm and

$$r_{ik}(t) = \begin{cases} 1 & \texttt{if} \quad k \in \mathcal{N}_i(t) \\ 0 & \texttt{otherwise} \end{cases} \tag{5}$$

Since graph $\hat{\mathcal{G}}(t)$ is undirected, it follows that

$$r_{ik}(t) = r_{ki}(t), \qquad \forall i, k, \in V, \ \ i \neq k \tag{6}$$

III. Convergence analysis and main result

In this section we demonstrate the main convergence results and, particularly, we characterize under which conditions on $\hat{\mathcal{G}}(t)$ the proposed local interaction rule (4)-(6) can achieve the approximate consensus conditions (3).

Given the set of edges E, let us denote

$$E_{i,j} = \{(k, l) \in E : \quad \{k, l\} \cap \{i, j\} \neq \varnothing\} \tag{7}$$

which represents the subset of edges incident on nodes i and j only.

Let us define an **error variable** for each edge in the network

$$\delta_{ij}(t) = x_i(t) - x_j(t), \quad \forall (i, j) \in E, \tag{8}$$

The dynamics of $\delta_{ij}(t)$ are easily obtained by differentiating (8) and considering the collective clock dynamics

$$\dot{x}_i(t) = \alpha + \nu_i - \lambda \sum_{k \in V, k \neq i} r_{ik}(t) sgn(x_i(t) - x_k(t)) \tag{9}$$

which simply derives from substituting the local interaction rule (4) into (1). It yields

$$\begin{aligned} \dot{\delta}_{ij}(t) = \ & \nu_i - \nu_j - \lambda \textstyle\sum_{k \in V, k \neq i} r_{ik}(t) sgn(\delta_{ik}(t)) \\ & + \lambda \textstyle\sum_{k \in V, k \neq j} r_{jk}(t) sgn(\delta_{jk}(t)). \end{aligned} \tag{10}$$

After some manipulations, relation (10) can be rewritten as

$$\dot{\delta}_{ij}(t) = \nu_i - \nu_j + \lambda \sum_{(k,l) \in E_{i,j}} \alpha_{kl}^{ij} r_{kl}(t) sgn(\delta_{kl}(t)) \tag{11}$$

with

$$\alpha_{kl}^{ij} = \begin{cases} -2 & \texttt{if } k = i \texttt{ and } l = j, \\ -1 & \texttt{if } k = i \texttt{ xor } l = j, \\ 1 & \texttt{otherwise.} \end{cases} \tag{12}$$

In order to analyze the convergence properties of the suggested algorithm the following candidate Lyapunov function appears to be appropriate

$$V(t) = \sum_{(i,j)\in E} |\delta_{ij}(t)| \tag{13}$$

Clearly, the vanishing of $V(t)$ implies the exact consensus condition among the clocks of the network, while small values for $V(t)$ define an approximate consensus instead.

The time derivative of (13) is given by

$$\dot{V}(t) = \sum_{(i,j)\in E} \dot{\delta}_{ij}(t) sgn(\delta_{ij}(t)) \tag{14}$$

By evaluating (14) along the error variables dynamics (11) one obtains

$$\begin{aligned} \dot{V}(t) &= \sum_{(i,j)\in E} (\nu_i - \nu_j) sgn(\delta_{ij}(t)) \\ &\quad + \lambda \sum_{(i,j)\in E} \sum_{(k,l)\in E_{i,j}} \alpha_{kl}^{ij} r_{kl}(t) sgn(\delta_{kl}(t)) sgn(\delta_{ij}(t)) \end{aligned} \tag{15}$$

Lengthy but straightforward manipulations yield the next expression

$$\begin{aligned} &\dot{V}(t) = \sum_{(i,j)\in E} (\nu_i - \nu_j) sgn(\delta_{ij}) - 2\lambda \sum_{(i,j)\in E} r_{ij} \left[sgn(\delta_{ij})\right]^2 \\ &+\lambda \sum_{(i,j)\in E} \sum_{k\in V, k\neq i, k\neq j} r_{ij} sgn(\delta_{ij}) [sgn(\delta_{jk}) - sgn(\delta_{ik})]. \end{aligned} \tag{16}$$

Now we introduce the next Lemma.

Lemma 3.1: The next relations holds true for all $k \in V$, with $k \neq i$, $k \neq j$

$$\begin{aligned} &sgn(\delta_{ij})[sgn(\delta_{jk}) - sgn(\delta_{ik})] = \\ &-\tfrac{[sgn(\delta_{ij})]^2}{2} \left[2 - |sgn(\delta_{ik})|\right] \times \\ &\left[|sgn(\delta_{ik})| - sgn(\delta_{ij}\delta_{jk})\right] \left[1 + sgn(\delta_{ij}\delta_{ik})\right] \end{aligned} \tag{17}$$

Proof: The Theorem's statement (17) is separately demonstrated to hold in the three cases $\delta_{ij}(t)\delta_{ik}(t) \lesseqgtr 0$ which cover all possible situations.

Case 1 - $\delta_{ij}(t)\delta_{ik}(t) < 0$.

Under the condition $\delta_{ij}(t)\delta_{ik}(t) < 0$ one has that

$$sgn(\delta_{ij}\delta_{ik}) = -1, \tag{18}$$

and, therefore, the right hand side of (17) is zero. Relation (18) readily implies that

$$sgn(\delta_{ij}) = -sgn(\delta_{ik}) \tag{19}$$

Since, by construction

$$\delta_{jk}(t) = \delta_{ik}(t) - \delta_{ij}(t) \tag{20}$$

then it follows from (19) and (20) that

$$sgn(\delta_{jk}) = sgn(\delta_{ik}(t)) \tag{21}$$

which means that the left hand side of (17) will be zero, too. Hence condition (17) holds when $\delta_{ij}(t)\delta_{ik}(t) < 0$.

Case 2 - $\delta_{ij}(t)\delta_{ik}(t) > 0$.

Now assume that

$$\delta_{ij}(t)\delta_{ik}(t) > 0, \tag{22}$$

which implies that

$$sgn(\delta_{ij}) = sgn(\delta_{ik}), \tag{23}$$

and that

$$sgn(\delta_{ij}\delta_{ik}) = 1, \tag{24}$$

which also means that both $sgn(\delta_{ij})$ and $sgn(\delta_{ik})$ are nonzero. This means that $[sgn(\delta_{ij})]^2 = 1$ and $|sgn(\delta_{ik})| = 1$, which allow us to simplify (17) as

$$\begin{aligned} & sgn(\delta_{ij})[sgn(\delta_{jk}) - sgn(\delta_{ik})] = \\ & -\tfrac{1}{2}\left[1 - sgn(\delta_{ij}\delta_{jk})\right]\left[1 + sgn(\delta_{ij}\delta_{ik})\right] \end{aligned} \tag{25}$$

Considering (24) into the right hand side of (25) yields the further simplification

$$sgn(\delta_{ij})[sgn(\delta_{jk}) - sgn(\delta_{ik})] = sgn(\delta_{ij}\delta_{jk}) - 1 \tag{26}$$

Now substituting (23) into the left hand side of (26) one obtains

$$sgn(\delta_{ij})[sgn(\delta_{jk}) - sgn(\delta_{ij})] = sgn(\delta_{ij})sgn(\delta_{jk}) - 1 \tag{27}$$

which is clearly an identity. Hence condition (17) holds when $\delta_{ij}(t)\delta_{ik}(t) > 0$.

Case 3 - $\delta_{ij}(t)\delta_{ik}(t) = 0$.

When $sgn(\delta_{ij}) = 0$ condition (17) is trivially verified regardless of the value of $sgn(\delta_{ik})$.

When $sgn(\delta_{ik}) = 0$ and $sgn(\delta_{ij}) \neq 0$ both sides of (17) are equal to $sgn(\delta_{ij})sgn(\delta_{jk})$, hence condition (17) is fulfilled in the considered Case 3 as well.

Lemma 3.1 is proven.

■

We are now in position to state the main result of this paper.

***Theorem** 3.2:* Consider the network of clocks (1), which satisfies (2), along with the discontinuous local interaction rule (4)-(6). Assume that

(i) there are $T, \epsilon > 0$ with $T \geq \epsilon$, such that during the receding horizon time interval $\mathcal{I}(t) = (t, t+T)$, $\hat{\mathcal{G}}(t)$ contains a spanning tree for at least ϵ units of time.

Let parameter λ be chosen according to

$$\lambda \geq \tfrac{|E|\ \Pi\ T + \mu^2}{\epsilon}, \quad \mu \neq 0. \tag{28}$$

Then, the collective dynamics (9) reaches the approximate consensus condition

$$V(t) \leq M, \quad t \geq t_r \tag{29}$$

where $V(t)$ is defined in (13), with the constant ρ specified as

$$\rho \leq 2|E|(T - \epsilon) \tag{30}$$

and the finite transient time t_r

$$t_r \leq \frac{V(0)}{2\mu^2}T = \frac{T}{2\mu^2}\sum_{(i,j)\in E}|x_{i0} - x_{j0}| \tag{31}$$

Proof: To analyze the convergence to the approximate consensus condition (3) we consider the error variables (8) along with the associated dynamic equations (11)-(12) and the Lyapunov function (13). The time derivative of the Lyapunov function is

$$\begin{aligned} & \dot{V}(t) = \textstyle\sum_{(i,j)\in E}(\nu_i - \nu_j)sgn(\delta_{ij}) - 2\lambda \sum_{(i,j)\in E} r_{ij}\left[sgn(\delta_{ij})\right]^2 \\ & +\lambda \textstyle\sum_{(i,j)\in E} r_{ij} sgn(\delta_{ij}) \sum_{k\in V, k\neq i, k\neq j}[sgn(\delta_{jk}) - sgn(\delta_{ik})] \end{aligned} \tag{32}$$

According to Lemma 3.1, the next relations hold true for all $k \in V$, with $k \neq i$, $k \neq j$

$$sgn(\delta_{ij})[sgn(\delta_{jk}) - sgn(\delta_{ik})] \leq 0 \tag{33}$$

since the right hand side of (17) turns out to be nonpositive. Hence the last term in the right hand side of (32) is nonpositive, too, and the next estimation is in force

$$\dot{V}(t) \quad \leq \sum_{(i,j)\in E}(\nu_i - \nu_j)sgn(\delta_{ij}) - 2\lambda \sum_{(i,j)\in E} r_{ij}\left[sgn(\delta_{ij})\right]^2 \tag{34}$$

The first term in the right hand side of (34) can be bounded as follows

$$\left|\sum_{(i,j)\in E}(\nu_i - \nu_j)sgn(\delta_{ij})\right| \leq 2|E|\Pi \tag{35}$$

Thus we get

$$\dot{V}(t) \quad \leq 2|E|\Pi - 2\lambda \sum_{(i,j)\in E} r_{ij}\left[sgn(\delta_{ij})\right]^2 \tag{36}$$

The rightmost stabilizing term in the right hand side of (36) is nonzero at time t if there is at least an active edge in the graph, say (i,j), connecting two non-synchronized clocks of the network (i.e. such that $\delta_{ij} \neq 0$).

The hypothesis (i) given in the statement of the Theorem guarantees that during each time interval $\mathcal{I}(t)$ there is a subinterval $\mathcal{S}(t) \subseteq \mathcal{I}(t)$, not necessarily connected (e.g., possibly composed by the union of disjoint intervals, see Figure 1), of duration not less than ε, during which (at least) an arc connecting non synchronized nodes is active.

The next relation is in force for all $t \geq 0$

$$V(t+T) - V(t) = \int_{\mathcal{S}(t)} \dot{V}(\tau)d\tau + \int_{\mathcal{I}(t)\setminus\mathcal{S}(t)} \dot{V}(\tau)d\tau \tag{37}$$

Whenever $t \in \mathcal{S}(t)$ and $V(t) \neq 0$ the next estimation can be made

$$\dot{V}(t) \quad \leq 2|E|\Pi - 2\lambda, \qquad t \in \mathcal{S}(t) \tag{38}$$

while for $t \in \mathcal{I}(t) \setminus \mathcal{S}(t)$ we have that

$$\dot{V}(t) \quad \leq 2|E|\Pi, \qquad t \in \mathcal{I}(t) \setminus \mathcal{S}(t) \tag{39}$$

By noticing that the length of the subinterval $\mathcal{S}(t)$ is at least ε by assumption, it follows that the length of the interval $\mathcal{I}(t) \setminus \mathcal{S}(t)$ will not exceed the value of $T - \varepsilon$.

Thus, in light of (38) and (39), one can manipulate (37) as

$$\begin{array}{rcl} V(t+T) - V(t) & \leq & \epsilon\left[2|E|\Pi - 2\lambda\right] + (T-\varepsilon)2|E|\Pi \\ & = & -[2\varepsilon\lambda - 2T|E|\Pi]. \end{array} \tag{40}$$

By considering the expression (28) of λ into (40) one obtains the next contraction condition

$$V(t+T) - V(t) \leq -2\mu^2 \tag{41}$$

Notice that the above condition (41) will be satisfied as long as $V(\tau) \neq 0$ for all $\tau \in (t, t+T)$. Therefore, it can be found a finite t_r such that $V(t_r) = 0$. The value of t_r can be readily overestimated as in (31) by taking into account condition (41). Starting from the time instant t_r we now compute the maximal steady state deviation from zero of $V(t)$, i.e. the quantity

$$V_M = sup_{t\geq t_r} V(t) \tag{42}$$

which sets the ultimate precision of the achieved consensus. If, at any time t' one has that $V(t') = 0$ then along the time interval $t \in (t', t'+T)$ the Lyapunov function $V(t)$ may grow, at most, by a quantity $2(T-\varepsilon)|E|\Pi$, which is obtained by integrating (39) for a time $T - \varepsilon$ (the maximal consecutive time interval without a spanning tree, according to hypothesis (i)) starting from the zero initial condition.

Thereby, the domain

$$V(t) \leq 2(T-\varepsilon)|E|\Pi. \tag{43}$$

is positively invariant at any $t \geq t_r$. Theorem 3.2 is proven. ■

Theorem 3.2 allows to define the majorant curve $\bar{V}(t)$ such that $V(t) \leq \bar{V}(t)$ as follows

$$\bar{V}(t) = \max\{V(0) - 2\mu^2 + M, M\}, \tag{44}$$

see Figure 2.

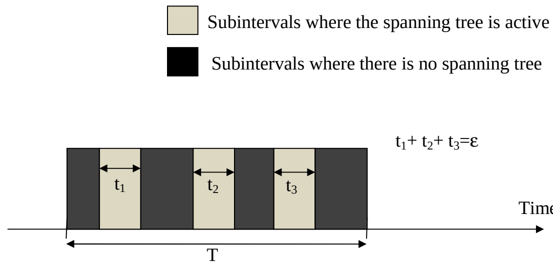

Fig. 1. Changes in network topology and communication constraints.

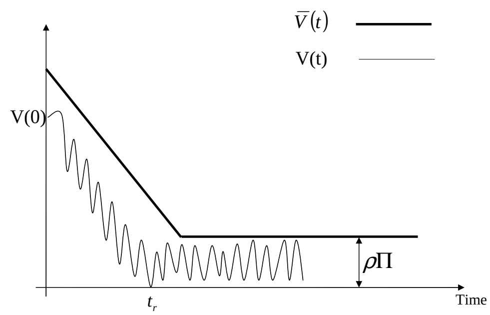

Fig. 2. Actual and majorant curves of V(t)

A. *Comments*

Synchronization property (29)-(31) clearly implies the approximate finite-time consensus condition (3). Some comments about the achieved conditions and results are given throughout this subsection.

First of all let us observe that the "synchronization error" M tends to zero when the disturbances ν_i are vanishing, thereby proving some "consistency" of the suggested scheme in the sense that the guaranteed accuracy depends continuously on the disturbance magnitude.

The synchronization error also increases with the complexity of the network, i.e., with the value of $|E|$, which is reasonable.

A parameter having a major effect on the steady state accuracy is the difference $T - \varepsilon$, that sets the relative amount of time during which a spanning tree is active (and, therefore, information can profitably propagate through the network). See Fig. 1 for a graphical representation of the scenario in which $\varepsilon < T$

Remarkably, when $\varepsilon = T$, i.e. when a spanning tree is always active in the time-varying communication graph, the **complete rejection** of the disturbances is guaranteed by the proposed methodology, which would provide, in this case, the ideal consensus condition after a finite time.

Finally, all parameters in eq. (28) or upper or lower bounds to them are assumed to be known and $\mu > 0$ is a design parameter that constrains the upper bound to the convergence time.

Some important aspects still need additional investigations since we are considering clocks that are synchronized in continuous time and we are neglecting possible delays in the communication channels, which are both limiting restrictions in applications. We expect, however, that the effect of communication delay can be considered equivalent to a further "artificial" reduction of ε. This conjecture will be explored in next research.

IV. NUMERICAL SIMULATIONS

In the considered network of 20 clocks, $\mathcal{G}$ is a random connected graph. For numerical simulation purposes, fixed-step Euler integration method is used with sampling step of size 0.0001 seconds. The

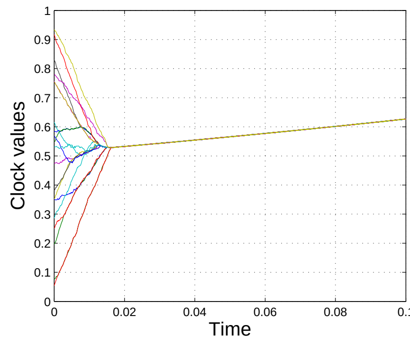

Fig. 3. Test 1. Transient evolution of the clock variables.

additive disturbances are selected of the form $\nu_i(t) = \frac{N}{2}(n_i(t) + sin(3|n_i(t)|t))$ with $n_i(t)$ being a uniformly distributed random variable in the range $n_i(t) \in [-1, 1]$ for all t. With the given parameters, the upper bound to the disturbance signals is $\Pi = N = 5$. The value $\lambda = 30$ is chosen and the desired clock speed is $\alpha = 1$. Initial clocks values are chosen uniformly at random in the range $[0, 1]$.

The outline of the performed simulations is as follows. In the first simulation, Test 1, the graph $\hat{\mathcal{G}}(t)$ is randomly chosen so that at each time instant it contains a spanning tree (i.e., $\varepsilon = T$). In the second simulation, Test 2, the switching policy is modified in such a way that $\varepsilon = 0.3T$, i.e., the spanning tree in active on average for the 30% of time. In the final test, Test 3, the same switching policy of Test 1 is used, and the disturbance signals are removed.

In Figure 3 the evolution of the clock variables relative to Test 1 are shown. It clearly emerges that after a finite time transient the clocks will be exactly synchronized, in accordance with (3). Figure 4 shows the time evolution of the Lyapunov function $V(t)$. After a finite-time transient $V(t)$ is actually steered to zero. In Figure 5, the evolution of the clock variables relative to Test 2 are shown, with the same network parameters used in Test 1 except the value $\epsilon = 0.3T$. The Figure 6, which reports a zoom on the transient evolution of $V(t)$, puts into evidence the effect of the bounded disturbances on the convergence properties of the network, and the robustness properties of the proposed local interaction rule. Finally, in Figures 7 the clock variables time evolutions relative to Test 3, having removed any additive disturbance, are displayed.

In both Test 1 (where $\varepsilon = T$) and Test 3 (where $\Pi = 0$), the achievement of the consensus with $V(t) = 0$ condition is expected, according to the demonstrated convergence properties, while in Test 2 the approximate attainment of the synchronization ("practical consensus") is guaranteed. The obtained simulation results confirm the theoretical analysis."

Due to the approximation of the continuous-time evolution by the Euler method, we may still observe some small numerical residual errors in the steady state even in presence of theoretically exact synchronization. These errors, however, tend to vanish when the sampling time is progressively reduced as shown in Figure 8 which displays the steady state evolutions of the Lyapunov function relative to Test 3 using two different sampling times in the Euler integration method.

V. Conclusions and future work

In this paper we proposed a distributed algorithm to solve the finite time consensus problem in a network of first order dynamical systems with additive disturbance signals. The proposed system models a network of clocks that synchronize themselves with a time-varying network topology. It has been proven that the proposed local interaction rule is robust against bounded disturbance signals and the system converges in finite time to a synchronized state in which each clock evolves at the same speed with a bounded error can be made arbitrarily small if the control parameter λ and the relative amount of time along which the

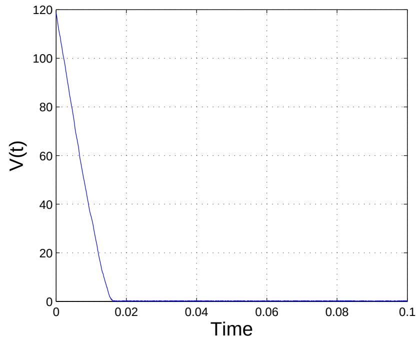

Fig. 4. Test 1. Transient evolution of the Lyapunov function $V(t)$.

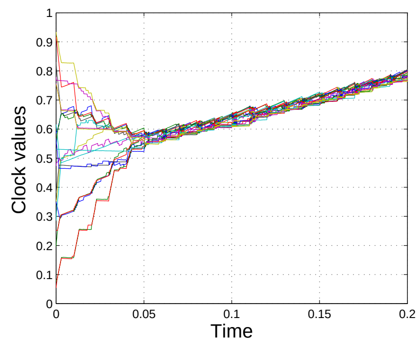

Fig. 5. Test 2. Transient evolution of the clock variables.

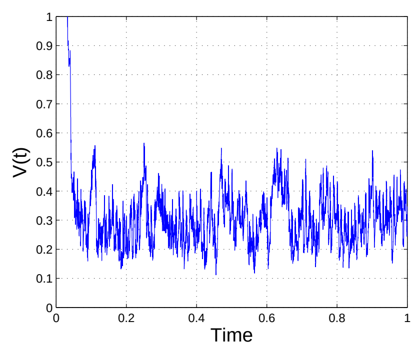

Fig. 6. Test 2. Zoomed transient evolution of the Lyapunov function $V(t)$.

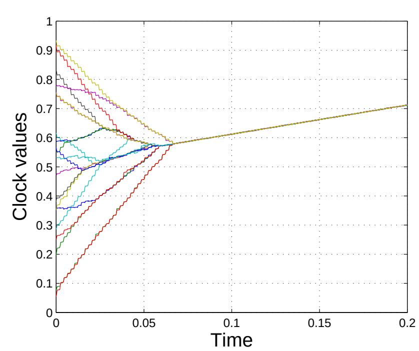

Fig. 7. Test 3. Transient evolution of the clock variables.

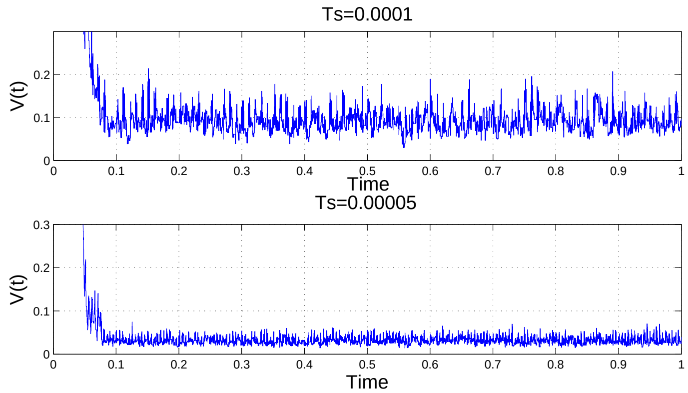

Fig. 8. Test 3. Zoomed transient evolution of the Lyapunov function $V(t)$.

network possesses a spanning tree is increased. Numerical simulations have been provided to corroborate the analytical results.

In future work we will extend the sufficient condition on the network topology to the case in which the graph is only jointly connected over a finite time window.

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