Robust Common Reference Estimation and Formation Control for Multi-Agent Systems

Daniele Rosa, Mauro Franceschelli and Alessandro Giua
Dept. of Electrical and Electronic Engineering, Univ. of Cagliari, Italy
(e-mail: {daniele.rosa,mauro.franceschelli,giua}@diee.unica.it.)

Abstract

In this paper we propose a decentralized coordination strategy that allows a dynamic multi-agent system to estimate a common reference point and achieve arbitrary spatial formations with respect to the estimated point. The method is robust against measurement noise of odometry or inertial navigation. We assume that the agents are mobile point-mass vehicles that do not have access to absolute positions (GPS).

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I. Introduction

Multi-agent systems consisting in a network of autonomous vehicles benefit greatly from the global positioning system (GPS) in that it allows to close feedback control loops on estimated positions in a global reference frame common to every vehicle, enabling several control tasks such as surveillance, patrolling, formation control or search and rescue missions to be performed. Unfortunately such a powerful tool may not always be exploited for several reasons: for instance the GPS signal is unreliable for indoor/underwater environments, during adverse atmospheric conditions or in locations close to transmission power lines and is vulnerable to jamming attacks. Furthermore, if the desired scale of relative distances between the vehicles is of the order of meters, the accuracy provided by the GPS system might not be enough. The problem of how to coordinate a network of agents in absence of absolute position information has thus received great attention from the control theory community [1], [2], [3]. Furthermore, it is usually assumed that the full network topology is not known by the agents and that only local point-to-point communication or sensing are available to model sensors with limited capabilities. In [4] a theoretical framework and a method to achieve flocking in a multi-agent system is proposed based on the famous three rules of flocking by Reynolds [5] and on local interaction rules based on virtual potentials that allow the achievement of flocking as global emergent behavior. In [6], [7], [8], [9] the consensus problem, i.e., the problem of how to make the state of a set of agents converge toward a common value, was presented regarding also the application of multi-agent coordination. In particular control strategies based on consensus algorithms were described in these papers as a fundamental tool to achieve synchronization of velocities, directions or the attainment of constant relative distances between the agents.

In our approach we assume that each agent estimates relative positions with its neighbors in its own local reference frame centered on it. A similar assumption was made in [10], where a Nyquist criterion to determine the effect of the topology of a multi-agent system performing formation control was proposed; in this case the agents were assumed to have a common coordinate system but not a common origin. Furthermore we assume that each agent has an onboard compass, which allows all the local frames to have the same orientation.

Leader-based approaches [11], [12] require the network of vehicles to properly follow one or more leaders, possibly controlled by a pilot, satisfying eventually some constraints. Also some formation control strategies in the literature take advantage from the presence of leaders exploiting network properties such as graph rigidity [13]. In this paper we design a coordination strategy for point-mass agents in which leaders are not required, and the desired formation is expressed with coordinates centered at the estimated common reference point. We also show that the proposed strategy, based on an *overcompensation* of the agents' displacement, is robust against measurement noise. The concept of overcompensation is presented in the following sections.

In [14] a decentralized algorithm to make a network of agents agree on the location of the network centroid in absence of common reference frames was presented; the algorithm is based on gossip (only random asynchronous pairwise communications) and assumes static agents displaced in a 3-d space. In [15] a decentralized algorithm based on gossip to make a network of agents agree on a common reference point and frame was proposed, assuming static agents in a 2-d plane. This paper differs from [14], [15] in that we consider dynamic agents that move while the the estimation process is executed, we assume that all the agents local reference frames are oriented in the same direction and that noise is affecting the relative position measurements. Furthermore, the proposed approach is used to implement formation control.

This paper is organized as follow. In Section III we present the considered system and the set of assumptions adopted. In Section IV we propose a formation control strategy which is characterize by a parallel application of two different decentralized algorithms: a local displacement control rule which move each agent toward a target point and a consensus algorithm which allows agents to reach an agreement on a common reference frame. The concept of overcompensation is here presented. In Section V the robustness of the proposed strategy is investigated and an optimal choice of the algorithm parameters is discussed.

II. Background

Let a network of agents be described by a time-varying undirected graph $\mathcal{G}(t) = \{V, \mathcal{E}(t)\}$, where $V = \{1, \dots, n\}$ is the set of nodes (agents), $E \subseteq \{V \times V\}$ is the set of edges e_{ij} representing point-to-point bidirectional communication channels available to the agents, $\mathcal{E}(t) : \mathbb{R}^+ \to E$ is the set of edges being active at time t. Given a time interval T, the joint graph $\mathcal{G}([t, t+T))$ is the union of graphs $\mathcal{G}(t)$ in the time interval $[t, t+T)$ defined as $\mathcal{G}([t, t+T)) = \{V, \mathcal{E}([t, t+T)))\}$, where

$$\mathcal{E}([t, t+T)) = \mathcal{E}(t) \bigcup \mathcal{E}(t+1) \bigcup \dots \bigcup \mathcal{E}(t+T)$$

A node $u \in V$ is said to be reachable from $v \in V$ if there exists a path in the graph from v to u. Node $u \in V$ is said to be a *center node* if it is reachable from any node in V. In a connected undirected graph all the nodes are center nodes. A node $u \in V$ is said to be *aperiodic* if the greatest common divisor of all the possible path length from u to u is 1.

The state of each agent i is characterized by its absolute position x_i and an estimation of the origin of the common reference frame $s_i \in \mathbb{R}^2$. Agents are assumed to be modeled by discrete time integrator dynamics

$$x_i(t+1) = x_i(t) + u_i(t),$$

where $u_i \in \mathbb{R}^2$ is the control input.

Let $\mathcal{N}_i(t) = \{j : e_{ij}(t) \in \mathcal{E}(t)\}$ be the set of neighbors of agent i at time t. We define the *degree* of node i as $\delta_i(t) = |\mathcal{N}_i(t)|$. Graph $\mathcal{G}(t)$ is encoded by the Laplacian matrix $\mathcal{L}_{\mathcal{G}}$, whose entries l_{ij} are

$$l_{ij} = \begin{cases} -1, & if\ (i,j) \in \mathcal{E}(t) \\ \delta_i(t). & if\ i = j \\ 0 & \text{otherwise} \end{cases}$$

Given a generic square matrix $M_{n \times n}$, the *associated graph* $\mathcal{G}_M = \{V_M, E_M\}$ is composed as follow:

- $\mathcal{G}_M$ has n nodes, with index $i \in [1, n]$, so $V_M = \{1, \dots, n\}$;
- $\mathcal{G}_M$ has an edge e_{ij} if the entry $m_{ij} \in M$ is nonzero, so $E_M = \{(i,j) | m_{i,j} \neq 0\}$

If M has non zero diagonal entry m_{ii}, than node $i \in \mathcal{G}_M$ has a self loop. If M is symmetric than $\mathcal{G}_M$ is an undirected graph. For a time-varying square matrix $M(t)$ the associated graph is $\mathcal{G}_M(t) = \{V_M, E_M(t)\}$.

A square matrix A is *stochastic* if its elements are non-negative and the row sums equals one. A stochastic matrix said to be *ergodic* if

$$rank\left(\lim_{k \to \infty} A^k\right) = 1,$$

An ergodic matrix A is *SIA* (stochastic, indecomposable and aperiodic) if

$$\lim_{k \to \infty} A^k = \mathbf{1}_n \pi^T,$$

where π is the left eigenvector of A corresponding to the unitary eigenvalue and $\mathbf{1}_n$ is the n-element vector of ones.

Given two generic matrices $A_{(m \times n)}$ and $B_{(p \times q)}$, the *Kronecker product* $A \otimes B_{(mp \times nq)}$ is defined as follow:

$$A \otimes B = \begin{bmatrix} a_{11}B & \dots & a_{1,n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \dots & a_{mn}B \end{bmatrix}$$

where a_{ij} is the (i,j) entry of A.

III. System description

In this section we describe in detail the model of the multi-agent systems and the set of fundamental working assumptions.

A. Assumptions

- Agents are modeled by point mass dynamics;
- agent exchange informations through bidirectional communication channels;
- agents sense relative distances and directions;
- communications channels are available only between subsets of agents and could be both synchronous or asynchronous;
- a local coordinate system is fixed on each agent, it translate rigidly with it but does not rotate;
- odometry or an inertial navigation system is available to each agent;
- agents have a common orientation of their local coordinate systems, achieved for instance by using compasses.

B. Coordinate systems

A 2-d reference frame $\Sigma' = (o', \theta')$ is an orthogonal coordinate system characterized by an origin $o' \in \mathbb{R}^2$ and orientation of the x-axis $\theta' \in [0, 2\pi)$ respect to a global coordinate system Σ defined by $o = (0, 0)$ and $\theta = 0$. We deal with three kinds of coordinate systems, which are showed in Fig. 1.

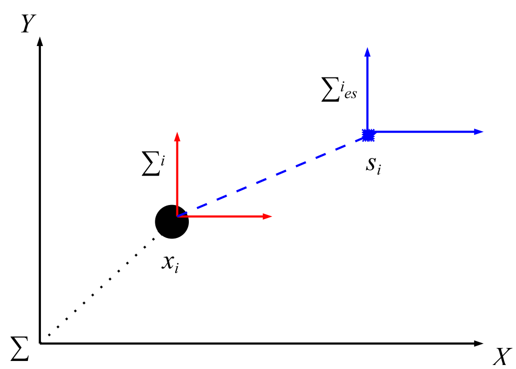

Fig. 1. The used coordinate systems.

- *Global coordinate system:* is the reference frame used to describe the system from the point of view of an external observer. We denote it with Σ, and the current position of agent i specified in Σ is $x_i \in \mathbb{R}^2$.
- *Local coordinate system:* each agent owns a local reference frame centered on it. Since we assume that each agent has a compass on board, all the local frames have the same orientation. The local coordinate system of agent i is denoted with $\Sigma^i = (x_i, 0)$, where x_i is the position of agent i in Σ. We denote the position of a generic point j with respect to Σ^i as x^i_j and $x_j = x^i_j + x_i$ with respect of Σ.
- *Estimated coordinate system:* each agent estimates a point s_i which is supposed to be the center of a common coordinate system. With respect to Σ the estimation by agent i of the common reference frame is denoted with $\Sigma^{i,es} = (s_i, 0)$. We denote the position of a generic point j with respect to $\Sigma^{i,es}$ as $x^{i,es}_j$. With respect of Σ^i, the position of j is:

$$x^i_j = x^{i,es}_j + s^i_i$$

IV. Formation control strategy without a common reference center

In this section we present a decentralized control strategy which allows a network of mobile agents in a 2-d space to reach an agreement on a common center and simultaneously converge to a desired formation. We assume that agents do not have a common positioning system: each agent refers to a local coordinate system. However we assume that agents have a compass on board which allow them to have a common direction, so that all the local frames have the same orientation. If they are able to establish a common reference point in the 2-d space, then a common reference frame could be achieved for the system. The state of i-th agent is characterized by a position $x_i \in \mathbb{R}^2$ and another variable $s_i \in \mathbb{R}^2$, which represents the estimated center of the common reference frame. When referring to the state of the agent in its own reference frame Σ^i we denote its current estimation as $s_i^i \in \mathbb{R}^2$.

Our strategy involves two local state update rules:

- a local interaction rule to update the position of the agents;
- a local interaction rule to make the agents agree on a common reference center.

A. *Position update rule*

We suppose that each agent i at time t has a target position $d_i^i(t) = s_i^i(t) + D_i$ with respect to its local reference frame Σ^i, where $D_i \in \mathbb{R}^2$ is a constant vector which represent the desired final position of agent i in the common reference frame. We can express the target position of agent i with respect to Σ as

$$d_i(t) = x_i(t) + s_i^i(t) + D_i = s_i(t) + D_i. \tag{1}$$

Since the objective is to make each agent asymptotically reach its target position, at each time t agent i moves toward his target position $d_i^i(t)$ making a step of

$$x_i(t+1) - x_i(t) = q[d_i(t) - x_i(t)] \tag{2}$$

with respect to Σ. The parameter $0 < q < 1$ specifies the travelled distance at each time step. By replacing equation (1) in (2) we find the following position update rule:

$$x_i(t+1) = (1-q)x_i(t) + q(s_i(t) + D_i) \tag{3}$$

Due to the movement of the agent it is necessary at each step to *compensate* the estimated reference center according to the displacement. In other words, because the agents' local frame is centred on x_i and moves rigidly with it, each agent i needs to update s_i^i, and consequently d_i^i.

If the agent knows exactly how much it moved then,

$$\begin{cases} d_i^i(t+1) = d_i^i(t) - q d_i^i(t) \\ s_i^i(t+1) = s_i^i(t) - q[s_i^i(t) + D_i] \end{cases}$$

which can be written in global coordinates as:

$$\begin{cases} d_i(t+1) = d_i(t) \\ s_i(t+1) = s_i(t). \end{cases}$$

These updates, however are not robust as the movement of the agent may have been different from the nominal one due to disturbances, actuator malfunctioning, etc. Thus we propose a novel updating rule for the target position of agent i, based on *overcompensation*, as follows:

$$d_i(t+1) = h x_i(t) + (1-h) d_i(t) \tag{4}$$

Equation (4) represents a *compensation* of agent displacement based on a new introduced parameter h, which controls how much the agents compensate their displacement. Note that:

- if $h = -q$ the distance vector $d_i(t) - x_i(t)$ is constant, thus there is no compensation;

- if $-q < h < 0$, $d_i(t)$ translate in the same direction of $x_i(t)$ and $|d_i(t+1) - x_i(t+1)| < |d_i(t) - x_i(t)|$, thus there is only a partial compensation;
- if $h = 0$ the target position $d_i(t)$ is constant, thus the compensation is perfect;
- if $h > 0$, $d_i(t)$ moves toward $x_i(t)$, thus an *overcompensation* is made.

Using equation (3) and rewriting equation (4) in terms of $s_i(t)$, we can express the general update rule as follow:

$$\begin{cases} x_i(t+1) = (1-q)x_i(t) + q(s_i(t) + D_i) \\ s_i(t+1) = hx_i(t) + (1-h)(s_i(t) + D_i) - D_i \end{cases} \tag{5}$$

B. Achieving consensus on a common reference center

Each agent has a local estimate $s_i^i(t)$ which considers as the center of a common estimated frame. By exchanging this local information with neighbours, agents are able to reach an agreement on a common reference center, which means that:

$$\forall i, j \in V, \quad \lim_{t \to \infty} \|s_i(t) - s_j(t)\| = 0$$

At each time step agent i receive the value s_j^j from each agent $j \in \mathcal{N}_i(t)$. In Figure 2 it is shown how agent i is able to determine the correct value s_j^i of agent j with respect to Σ^i by only knowing x_j^i and the received value s_j^j.

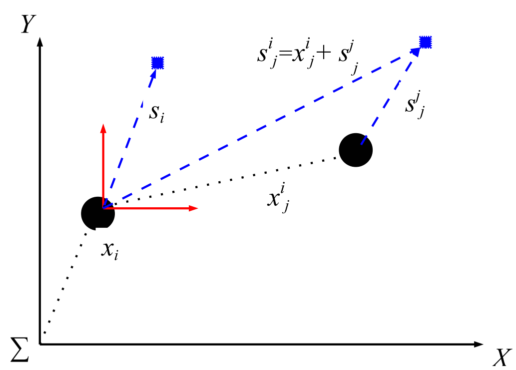

Fig. 2. Neighbour's estimate detection for agent i.

The update rule for the local estimate is:

$$s_i^i(t+1) = s_i^i(t) + \varepsilon \sum_{j \in \mathcal{N}_i(t)} (s_j^j(t) + x_j^i(t) - s_i^i(t)) \tag{6}$$

with $0 < \varepsilon \leq |\mathcal{N}_i(t)|$. The same rule could be written with respect to Σ:

$$s_i(t+1) = s_i(t) + \varepsilon \sum_{j \in \mathcal{N}_i(t)} (s_j(t) - s_i(t)) \tag{7}$$

With respect to Σ the overall estimate update rule could be expressed as follow:

$$s(t+1) = (P(t) \otimes I_{2\times 2})s(t) \tag{8}$$

where $P(t) \in \mathcal{P}$ is a time-varying matrix which depends on network topology at time t and ε, and $\mathcal{P}$ is the set of all possible matrices representing the system update defined in (7). Due to the update rule definition all matrices $P(t) \in \mathcal{P}$ are stochastic. Note that equation (8) can represent both deterministic synchronous consensus algorithms and randomized gossip algorithms. At each t, algorithm (8) can be represented by the associated graph $\mathcal{G}_P(t)$. If $\forall t > 0$ there exists a $T > 0$ such that $\mathcal{G}_P([t, t+T))$ is connected, than $\lim_{t\to\infty} s_1(t) = \ldots = \lim_{t\to\infty} s_n(t)$, where $\mathcal{G}_P([t, t+T))$ is the union of graphs $\mathcal{G}_P(t)$ in the time interval $[t, t+T)$ [7][8].

C. Formation control strategy

Let us define column vectors $x(t) = \{x_1(t), \dots, x_n(t)\}$, $s(t) = \{s_1(t), \dots, s_n(t)\}$ and $D = \{D_1, \dots, D_n\}$. Note that D represents the desired formation respect to a common center. By summing the contributions of equations (5) and (8) the overall formation control strategy could be expressed as follow:

$$\begin{bmatrix} x(t+1) \\ s(t+1) \end{bmatrix} = (M(t) \otimes I_{2\times 2}) \begin{bmatrix} x(t) \\ s(t) \end{bmatrix} + \begin{bmatrix} qD \\ -hD \end{bmatrix} \tag{9}$$

where

$$M(t) = \left[\begin{array}{c|c} (1-q)I_{n\times n} & qI_{n\times n} \\ \hline hI_{n\times n} & (P(t) - hI_{n\times n}) \end{array} \right] \tag{10}$$

For all t, $M(t) \in \mathcal{M}$, where $\mathcal{M}$ is the set of all possible matrices of type (10) corresponding to different $P(t) \in \mathcal{P}$. A given formation is considered to be achieved if

- $x(t) = s(t) + D$;
- $\forall i, j \in V, \quad \|s_i(t) - s_j(t)\| = 0$

Lemma IV.1 *Consider system* (9)*. If*

$$\lim_{t\to\infty} (M(1)M(2)\dots M(t) \otimes I_{2\times 2}) \begin{bmatrix} x(0) \\ s(0) \end{bmatrix} = c\mathbf{1}_{2n} \tag{11}$$

then

- $\lim_{t\to\infty} x(t) = s(t) + D$,
- $\forall i, j \in V, \quad \lim_{t\to\infty} \|s_i(t) - s_j(t)\| = 0$.

Thus the desired formation is asymptotically achieved.

Proof. *Condition* (11) *implies that system* (9) *is stable. At the equilibrium $x(t+1) = x(t)$ and $s(t+1) = s(t)$. From the first equation of* (9) *we find:*

$$(1-q)Ix(t) + qIs(t) + qID = x(t)$$
$$x(t) = s(t) + D$$

By substituting in the second equation:

$$Ps(t) = (I - \varepsilon L)s(t) = s(t)$$

which implies $s(t) = c\mathbf{1}$, where $c \in \mathbb{R}$ is a constant. $square$

Convergence of the proposed strategy toward the desired formation can thus be addressed by studying the stability of the following linear time-varying system

$$\begin{bmatrix} x(t+1) \\ s(t+1) \end{bmatrix} = (M(t) \otimes I_{2\times 2}) \begin{bmatrix} x(t) \\ s(t) \end{bmatrix} \tag{12}$$

1) Case I: static topology:
If the network topology is static and connected, than $M(t) = M$, $\forall t$.

Lemma IV.2 *(Lin,[16]) A stochastic matrix M is SIA if and only if the associated graph $\mathcal{G}_M$ has a centre node which is aperiodic.* ■

Now we are able to prove the following result.

Theorem IV.1 *Consider a network of agents with a static connected topology. Given system* (12) *with $M(t) = M$, if*

$$0 \le h \le 1 - \varepsilon\delta_{max} \tag{13}$$

where $\delta_{max} = max\{\delta_1, \cdots, \delta_n\}$ represents the maximum degree for the network, then

$$\lim_{t\to\infty} \begin{bmatrix} x(t) \\ s(t) \end{bmatrix} = c\mathbf{1}_{2n},$$

where $c \in \mathbb{R}$ is a constant.

Proof: Assume condition (13) holds, then matrix $\mathbf{M}$ in eq. (10) is stochastic since all the diagonal entries of matrix $P(t)$ satisfy by construction $p_{ii} \leq 1 - \varepsilon\delta_{max}$ for all $i \in V$, thus $1 \geq p_{ii} - h \geq 0$, thus as all entries are non negative and row sums are equal to 1. Now we have to prove that $\mathbf{M}$ is SIA. We can represent system (12) using a undirected graph $\mathcal{G}_{\mathbf{M}}$ associated to matrix $\mathbf{M}$. In this graph each agent i is represented by two nodes:

- one associated to the agent position x_i, that we call position node;
- one associated to the agent estimate s_i, that we call estimation node.

For each agent the two associated nodes are connected together by a bidirectional edge, as the position update depends on the position estimate and vice versa. The connections between agents depend on matrix $P - hI$. In particular, given a couple of agents (i, j) there exists an edge between their estimation nodes if the p_{ij} entry of P is non zero. As the network is connected and undirected by assumption, the graph $\mathcal{G}_{\mathbf{M}}$ is connected as well and each node is a center node. More, as all diagonal entries in $(1 - q)I$ are nonzero, each position node in the associated graph has a self loop, so $\mathcal{G}_{\mathbf{M}}$ is aperiodic. It follows from Lemma IV.2 that matrix $\mathbf{M}$ is SIA, so

$$\lim_{t\to\infty} \mathbf{M}^t \begin{bmatrix} x(0) \\ s(0) \end{bmatrix} = c\mathbf{1}_{2n}$$

where c is a constant. □

2) Case II: time-varying topology.: In order to prove the robustness of (12) we need first to present some preliminary notions.

Lemma IV.3 *(Jadbabaie et al.,[8]) Let $\{M_1, M_2, \ldots, M_m\}$ be a set of stochastic matrices of the same order such that the joint graph $\{\mathcal{G}(M_1) \bigcup \mathcal{G}(M_2) \bigcup \ldots \bigcup \mathcal{G}(M_m)\}$ is connected. Then the matrix product $M_1 M_2 \ldots M_m$ is ergodic.* ■

Lemma IV.4 *(Wolfowitz,[17]) Let $\{M_1, M_2, \ldots, M_m\}$ be a set of ergodic matrices with the property that for each sequence $M_{i_1}, M_{i_2}, \ldots, M_{i_j}$ of positive length j the matrix product $M_{i_1} M_{i_2} \ldots M_{i_j}$ is ergodic. Then for each infinite sequence $M_{i_1}, M_{i_2}, \ldots$ there exists a row vector $\mathbf{c}$ such that*

$$\lim_{j\to\infty} M_{i_1} M_{i_2} \ldots M_{i_j} = \mathbf{1c}.$$

■

Now we can state the following theorem

Theorem IV.2 *Consider a network of agents with time-varying topology described by (12). Let us assume that $\forall t > 0$ there exists a $T > 0$ such that $\mathcal{G}_P([t, t+T))$ is connected. The following condition is sufficient for the system to converge to the desired formation:*

$$0 \leq h \leq 1 - \varepsilon\delta_{max} \tag{14}$$

Proof: Let $\mathcal{M}_c$ be the set of all possible product matrices in $\mathcal{M}$ of length T such that the joint graph $\mathcal{G}_P([t, t+T))$ is connected. In the theorem we assume that for each time interval $[t, t+T)$ the matrix

$$M(t)M(t+1)\ldots M(t+T) \in \mathcal{M}_c$$

Thus we can represent the evolution of the system as a product of matrices $M_c(t) \in \mathcal{M}_c$. If condition (14) holds, then all matrices $M(t) \in \mathcal{M}$ are stochastic as showed in the proof of Theorem IV.1, and it follows from Lemma IV.3 that all matrices $M_c(t) \in \mathcal{M}_c$ are ergodic as well as all products in $\mathcal{M}_c$. Finally it follows from Lemma IV.4 that:

$$\lim_{t\to\infty} (M_c(1)M_c(2)\ldots M_c(t) \otimes I_{2\times 2}) \begin{bmatrix} x(0) \\ s(0) \end{bmatrix} = c\mathbf{1}_{2n}$$

□

V. Robustness of the control strategy

The proposed coordination strategy described in section IV can be affected by errors due to the odometry or inertial navigation system. In particular the desired displacement that the generic agent $x_i(t)$ should achieve within one sample of time is as follows

$$x_i(t+1) = x_i(t) - q_i(t)(x_i(t) - s_i(t)). \tag{15}$$

where the time-varying parameter $q_i(t) = q + \Delta_i(t)$ models a random error in the position update at time t. We point out that parameter $\Delta_i(t)$ is unknown to the agents.

Thus, the proposed local interaction rule becomes

$$\begin{cases} x_i(t+1) = & (1 - q_i(t))x_i(t) + q_i(t)s_i(t) \\ s_i(t+1) = & h(t)(x_i(t) - s_i(t)) \\ & +(s_i(t) + \varepsilon \sum_{j \in \mathcal{N}_i} l_{ij}(s_j(t) - s_i(t))) \end{cases} \tag{16}$$

where $h_i(t) = h - \Delta_i(t)$.

Let $Q(t)$ and $H(t)$ be $n \times n$ diagonal matrices where $Q_{ii} = q_i(t)$ and $H_{ii}(t) = h_i(t)$. The global system dynamics are thus described by

$$\begin{bmatrix} x(t+1) \\ s(t+1) \end{bmatrix} = (M_\Delta(t) \otimes I_{2\times 2}) \begin{bmatrix} x(t) \\ s(t) \end{bmatrix} \tag{17}$$

$$M_\Delta(t) = \left[\begin{array}{c|c} I - Q(t) & Q(t) \\ \hline H(t) & P(t) - H(t) \end{array}\right] \tag{18}$$

For all t, $M_\Delta(t) \in \mathcal{M}_\Delta$, where $\mathcal{M}_\Delta$ is a infinite set of matrices $M_\Delta(t)$ characterized by different values of $q(t)$, $h(t)$ and $P(t)$. Now we characterize the robustness of the proposed strategy with respect to measurement noise.

Theorem V.1 *Consider a system as in eq.* (17)*. Let us assume that* $\forall t > 0$ *there exists a* $T > 0$ *such that* $\mathcal{G}_P([t, t+T))$ *is connected . If the measurement noise* $\Delta_i(t)$ *is bounded by*

$$h + \varepsilon\delta_{max} - 1 \leq \Delta_i(t) \leq min\{h, (1-q)\}, \quad \forall i, t \tag{19}$$

then

$$\lim_{t\to\infty} \begin{bmatrix} x(t) \\ s(t) \end{bmatrix} = c\mathbf{1}_{2n}$$

where c *is a constant.*

Proof: The diagonal entries of the matrices $I - Q(t)$ and $P(t) - H(t)$ are

$$[I - Q(t)]_{ii} = 1 - q - \Delta_i(t)$$

$$[P(t) - H(t)]_{ii} = 1 - \varepsilon\delta_i - h + \Delta_i(t)$$

We can assume that $q > \Delta_i(t)$. If condition (19) hold, then all matrices in $\mathcal{M}_\Delta$ are stochastic, because all entries are non negative and row sums equal to one. Thus, the proof follows as in theorem IV.2. □

Note that $\Delta(t)$ could be positive or negative.

Best parameter choice for maximum robustness: Given a fixed value of q, the optimum value of h is the one which maximizes the following objective function:

$$\max_h\{\min\{h,(1-q),|h+\varepsilon\delta_{max}-1|\}\}$$

By substitution we find that:

- if $\dfrac{1-\varepsilon\delta_{max}}{2}\leq(1-q)$ the optimum value of h is
$$h=\frac{1-\delta_{max}}{2}$$
thus the bound (19) become symmetric:
$$-\frac{1-\varepsilon\delta_{max}}{2}\leq\Delta_i(t)\leq\frac{1-\varepsilon\delta_{max}}{2},\quad\forall i,t$$
- if $\dfrac{1-\varepsilon\delta_{max}}{2}>(1-q)$ the optimum value of h is $h=(1-q)$. The bound (19) become:
$$\varepsilon\delta_{max}-q\leq\Delta_i(t)\leq1-q,\quad\forall i,t$$

VI. Conclusions

In this paper we have proposed a novel coordination strategy, based on an overcompensation of agent displacement, to achieve an arbitrary formation in a multi-agent system. We have proved that our strategy is robust with respect to measurement noise of odometry or inertial navigation. Our strategy is characterized by a decentralized algorithm to achieve agreement on a common reference point and a consensus based strategy to provide cohesion in the network. The system achieves arbitrary formations by specifying positions in the estimated common reference frame on which the agents agree upon.

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