Marking estimation of Petri nets with arbitrary transition labeling

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Abstract

In this paper we deal with the problem of estimating the marking of an arbitrary labeled Petri net system where two forms of nondeterminism may occur. Firstly, there may exist unobservable transitions, i.e., transitions labeled with the empty string. Secondly, there may exist undistinguishable transitions, i.e., two or more transitions sharing the same symbol taken from a given alphabet E may simultaneously be enabled.

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1 Introduction

The behavior of a discrete event system (DES) is modeled by a *language* on an alphabet E (the set of events): a sequence of events from this alphabet forms a *word* that describes a particular evolution of the system. An observer aims to provide an estimate of the system state based on the observation of the word of events.

A well-founded approach to the state estimation of DES modeled by *automata* is based on the notion of *nondeterminism*. The initial state of the automaton is usually assumed to be known but, as the system evolves, the current state may not be perfectly known if the automaton is nondeterministic. Nondeterminism originates from two different causes.

- *Silent transitions.* There may be state transitions that cause a change in the state of the DES but that are not observable by an outside observer. Transitions of this kind are labeled with the empty string ε.
- *Undistinguishable transitions.* There may be events whose occurrence from a given state yields two or more new states. Such is the case if two or more transitions labeled with the same symbol in E are enabled at a given state.

For DES modeled as finite automata, the most common way of solving the state estimation problem is that of converting the nondeterministic finite automaton (NFA) into an equivalent deterministic finite automaton (DFA) where: (i) each state of the DFA corresponds to a set of states of the NFA; (ii) the state reached on the DFA after the word w is observed, gives the set $\mathcal{C}(w)$ of *states consistent with the observed word w*. However, there are some drawbacks in the above procedure. Firstly, each set $\mathcal{C}(w)$ must be exhaustively enumerated. Then, to compute $\mathcal{C}(w)$ we first need to compute $\mathcal{C}(w')$ for all prefixes $w' \preceq w$. If the NFA has n states, the DFA can have up to 2^n states.

In previous works we have explored the possibility of using Petri nets (PN) as discrete event models and have addressed the observer design keeping the two forms of nondeterminism separated. In fact, in (Corona *et al.*, 2004), we have assumed that the only source of nondeterminism is due to silent transitions. Dually, in (Giua *et al.*, 2005), we have assumed that the only source of nondeterminism is due to undistinguishable transitions. In both cases, under some restrictions on the class of nets considered we proved that the set of markings (i.e., states) consistent with the observed behavior coincides with the set of feasible solutions of a parameterized integer constraint set: the parameters that describe this set can be recursively updated as a new event is observed.

In this paper we show that a similar approach can also be applied when the net contains both forms of nondeterminism. Combining the two approaches is not trivial, and requires identifying a new set of restrictions that the considered net must satisfy.

For some recent literature we address to (Jiroveanu and Boel, 2005; Corona *et al.*, 2004; Sundaram and Hadjicostis, 2006; Ru and Hadjicostis, 2006; Corona *et al.*, 2004; Giua *et al.*, 2005).

2 Notation and basic definitions

A *Place/Transition net* (P/T net) is a structure $N = (P, T, Pre, Post)$, where P is a set of m places; T is a set of n transitions; $Pre: P \times T \to \mathbb{N}$ and $Post: P \times T \to \mathbb{N}$ are the *pre–* and *post–* incidence functions that specify the arcs.

The incidence matrix of a net is $C = Post - Pre$. The *preset* and *postset* of a node $x \in P \cup T$ are denoted ${}^\bullet x$ and $x^\bullet$, while ${}^\bullet x^\bullet = {}^\bullet x \cup x^\bullet$.

A *marking* is a vector $M: P \to \mathbb{N}$ that assigns to each place of a P/T net a non–negative integer number of tokens, represented by black dots. We denote $M(p)$ the marking of place p. A P/T *system* or *net system* $\langle N, M_0 \rangle$ is a net N with an initial marking M_0.

A transition t is enabled at M iff $M \geq Pre(\cdot\,, t)$ and may fire yielding the marking $M' = M + C(\cdot\,, t)$. We write $M\,[\sigma\rangle$ to denote that the sequence of transitions $\sigma = t_{j_1} \cdots t_{j_k}$ is enabled at M, and we write $M\,[\sigma\rangle\,M'$ to denote that the firing of σ yields M'.

A marking M is *reachable* in $\langle N, M_0\rangle$ iff there exists a firing sequence σ such that $M_0\,[\sigma\rangle\,M$. The set of all markings reachable from M_0 defines the *reachability set* of $\langle N, M_0\rangle$, denoted $R(N, M_0)$.

Given a sequence $\sigma \in T^*$, we call $\vec{\sigma} : T \to \mathbb{N}$ the *firing vector* of σ. In particular, $\sigma(t) = k$ if the transition t is contained k times in σ.

Finally, we also define for $\tau \subseteq T$, $\sigma(\tau) = \Sigma_{t \in \tau}\sigma(t)$ the number of times transitions in τ appear in σ.

A Petri net having no directed circuits is called *acyclic*. A P/T net is *backward conflict-free* if $\forall p \in P$, $|{}^\bullet p| \leq 1$, i.e., if each place has at most one input transition.

Definition 2.1 *Given $N = (P, T, Pre,\ Post)$, and a subset $T' \subseteq T$, we define the $T'-$*induced subnet *of N as the new net $N' = (P, T', Pre', Post')$ where $Pre', Post'$ are the restriction of $Pre, Post$ to T'. The net N' is obtained from N removing all transitions in $T \setminus T'$.* ■

3 A framework for observation

3.1 Labeled Petri nets

A *labeling function* $L : T \to E \cup \{\varepsilon\}$ assigns to each transition $t \in T$ either a symbol from a given alphabet E or the empty string ε. We assume that whenever a transition t fires, only its label $L(t)$ is observed. We also denote w the word of events observed when a sequence σ fires, i.e., $w = L(\sigma)$.

Let us denote as T_e the set of transitions labeled e, i.e., $T_e = \{t \in T \mid L(t) = e\}$.

We partition the set E as $E = E_u \cup E_d$ where

- $E_u = \{e \in E \mid |T_e| > 1\}$ is the set of symbols that label two or more transitions. These symbols are called *undistinguishable* events because the observation of such a symbol e may be caused by the firing of any of the transitions in the set T_e.
- $E_d = \{e \in E \mid |T_e| = 1\}$ is the set of symbols that label just one transition. These symbols are called *deterministic* events because their observation unambiguously detects the firing of the unique transition labeled by it.

This allows us to partition the set of transitions T into three subsets: $T = T_\varepsilon \cup T_u \cup T_d$.

- $T_\varepsilon = \{t \in T \mid L(t) = \varepsilon\}$ is the set of transitions labeled ε. These transitions are called *silent* because, their occurrence generates no observable event.
- $T_u = \{t \in T \mid L(t) \in E_u\}$ is the set of *undistinguishable* transitions.
- $T_d = \{t \in T \mid L(t) \in E_d\}$ is the set of *deterministic* transitions.

In the following, without any loss of generality, we assume that $E_d = T_d$, since the restriction of the labeling function to this set is an isomorphism.

Then, we denote $T_n = T_u \cup T_\varepsilon$ the set of *nondeterministic* transitions.

The restriction of the incidence matrix C to T_e (T_ε) is denoted C_e (C_ε) and the restriction of the firing vector $\vec{\sigma}$ to T_e (T_ε) is denoted $\vec{\sigma}_e$ ($\vec{\sigma}_\varepsilon$).

Finally, to each set of undistinguishable transitions T_e we associate the set $\mathcal{T}_e$ containing all possible subsets of transitions, apart from itself and the empty set, i.e.,

$$\mathcal{T}_e = \{\tau \subseteq T_e \mid \tau \neq \emptyset,\ \tau \neq T_e\} = 2^{T_e} \setminus \{\emptyset, T_e\}.$$

3.2 Minimal explanations and minimal e-vectors

Definition 3.1 *Given a marking M and a transition $t \in T_u \cup T_d$, we define*

$$\Sigma(M,t) = \{\sigma \in {T_n}^* \ \mid M[\sigma\rangle M', \ M' \geq Pre(\cdot,t)\}$$

the set of explanations *of t at M. Then, we define*

$$Y(M,t) = \{\vec{y} \in \mathbb{N}^{n_n} \ \mid \ \exists \sigma \in \Sigma(M,t) \ : \ \vec{\sigma} = \vec{y}\},$$

where $n_n = T_n$, the set of e-vectors *(or* explanation vectors*), i.e., the firing vectors associated to the explanations.* ■

Thus $\Sigma(M,t)$ is the set of sequences in T_n whose firing at M enables t. Among the above sequences we want to select those whose firing vector is minimal. The firing vectors of these sequences are called *minimal e-vectors*.

Definition 3.2 Given a marking M and a transition $t \in T_u \cup T_d$, we define

$$\Sigma_{\min}(M,t) = \{\sigma \in \Sigma(M,t) \ \mid \ \nexists\, \sigma' \in \Sigma(M,t) \ : \ \vec{\sigma}' \lneqq \vec{\sigma}\}$$

the set of *minimal explanations* of t at M, and

$$Y_{\min}(M,t) = \{\vec{y} \in \mathbb{N}^{n_n} \ \mid \ \exists \sigma \in \Sigma_{\min}(M,t) \ : \ \vec{\sigma} = \vec{y}\}$$

the corresponding set of *minimal e-vectors*. ■

Similar definitions have also been given in (Giua and Seatzu, 2005; Jiroveanu and Boel, 2004). Different approaches can be used to compute $Y_{\min}(M,t)$ (Giua and Seatzu, 2005; Jiroveanu and Boel, 2005). In particular, in (Giua and Seatzu, 2005) we proposed an approach that terminates finding all vectors in $Y_{\min}(M,t)$ if applied to nets whose T_n-induced subnet is acyclic.

Theorem 3.3 ((Corona *et al.*, 2004)) *If the T_n-induced subnet is acyclic and backward conflict-free, then $|Y_{\min}(M,t)| = 1$.*

4 Problem statement

In this paper we deal with the problem of estimating the marking of a net system $\langle N, M_0\rangle$ whose marking cannot be directly observed. We assume that:

- the structure of the net N is known;
- the initial marking M_0 is known;
- the labels associated to the firing of transitions in $T \backslash T_\varepsilon$ can be observed.

Given an observation w, we define the set $\mathcal{C}(w)$ as the set of all markings in which the system may be given the observed word w.

Definition 4.1 *Given a word w, the set of w-*consistent markings *is:*

$$\mathcal{C}(w) = \{M \in \mathbb{N}^m \mid \ \exists \sigma \in T^* : M_0[\sigma\rangle M, \ L(\sigma) = w\}. \quad ■$$

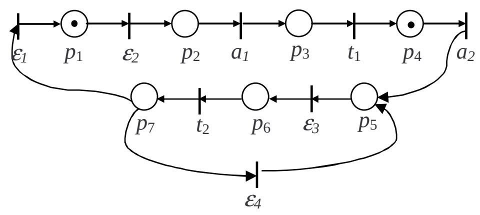

Figure 1: The net system in Example 4.2.

Example 4.2 Let us consider the Petri net system in Fig. 1 where $M_0 = [\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ ]^T$, $T_\varepsilon = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}, T_u = T_a = \{a_1, a_2\}$ and $T_d = \{t_1, t_2\}$.
Assume that no event is initially observed, i.e., $w = \varepsilon$. It holds $\mathcal{C}(\varepsilon) = \{[\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ ]^T$, $[\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ ]^T\}$. In fact, two different cases may have occurred: either no transition has fired or the silent transition ε_2 has fired. Now, let us assume that the event a is observed. The set of markings that are consistent with the observation of $w = a$ is $\mathcal{C}(a) = \{[\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ ]^T, [\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ ]^T, [\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ ]^T, [\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ ]^T, [\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ ]^T\}$. In fact, eight different sequences of transitions may have fired, namely $\sigma_1 = \varepsilon_2 a_1$, $\sigma_2 = a_2$, $\sigma_3 = a_2\varepsilon_3$, $\sigma_4 = a_2\varepsilon_2$, $\sigma_5 = \varepsilon_2 a_2$, $\sigma_6 = a_2\varepsilon_2\varepsilon_3$, $\sigma_7 = a_2\varepsilon_3\varepsilon_2$, $\sigma_8 = \varepsilon_2 a_2 \varepsilon_3$. Finally, if t_1 is observed then we are able to identify that the previously observed event a can only have been produced by the firing of a_1 which in turn is possible only if silent transition ε_2 has fired, thus $\mathcal{C}(at_1) = \{[\ 0\ 0\ 0\ 2\ 0\ 0\ 0\ ]^T\}$. ■

5 Particular cases

5.1 Petri nets with silent transitions

In (Corona *et al.*, 2004) we assume that a different label is associated to transitions in $T \setminus T_\varepsilon$. In such a case $T_n = T_\varepsilon$.
Under the assumptions that

(A1) the $T_\varepsilon-$induced subnet of N is *acyclic*,

(A2) the $T_\varepsilon-$induced subnet is backward conflict-free, i.e., all silent transitions have no common output place,

we showed that the set of consistent markings can be written as the solution of a linear system with a fixed structure that depends on the value of a vector $M_{b,w} \in \mathbb{N}^m$, called the *basis marking*. It represents the marking that is reached from M_0 by firing all the observed deterministic transitions and all those silent transitions whose firing is strictly necessary to enable the observed sequence. The basis marking can be recursively computed, and depends on the actual observation w.
Thus, the set of markings consistent with w can be written as: $\mathcal{C}(w) = \{M \in \mathbb{N}^m \mid M = M_{b,w} + C_\varepsilon\ \vec{\sigma}_\varepsilon, \vec{\sigma}_\varepsilon \in \mathbb{N}^{n_\varepsilon}\}$,where n_ε is the number of silent transitions. This means that the set of consistent markings can be characterized as the set of markings reachable from the basis marking by firing any sequence of silent transitions.

5.2 λ-free Petri nets

In (Giua *et al.*, 2005) we assume that the label function is λ-free, i.e., there exists no silent transition. In such a case $T_n = T_u$. Under the following assumption

(B1) undistinguishable transitions are contact-free, i.e., for any two undistinguishable transitions t_i and t_j, it holds that ${}^\bullet t_i^\bullet \cap {}^\bullet t_j^\bullet = \emptyset$ and ${}^\bullet t_i \cap t_i^\bullet = \emptyset$, we proved that, for all words $w \in E^*$, the set of w consistent markings $\mathcal{C}(w)$ is equal to

$$\begin{aligned} \mathcal{C}(w) = \{M \in \mathbb{N}^m \mid \quad & M = M_{b,w} + \sum_{e \in E_u} C_e \vec{\sigma}_e; \\ & \vec{\sigma}_e \in \mathcal{S}_e(w)\} \end{aligned} \tag{1}$$

where

$$\begin{aligned} \mathcal{S}_e(w) \stackrel{\text{def}}{=} \{\vec{\sigma} \in \mathbb{N}^{n_e} \mid (\forall \tau \in \mathcal{T}_e) \quad & \sigma(\tau) \leq u_w(\tau), \\ & \sigma(T_e) = u_w(T_e)\} \end{aligned}$$

is the set of w-consistent undistinguishable firing vectors and the upper bounds $u_w(\tau)$ and $u_w(T_e)$, as well as the basis marking $M_{b,w}$, are computed using an appropriate recursive algorithm (Giua *et al.*, 2005). Therefore, the number of constraints used to describe the set $\mathcal{S}_e(w)$ is equal to $2^{n_e} - 1$, regardless of the length of the observed word w.

6 Arbitrary labeled Petri nets

In this section we consider the general problem formulation given in Section 4.

Let us first define a *silent path* of N from transition t to t' as a sequence $t_0 p_1 t_1 \ldots t_{k-1} p_k t_k$ where $t = t_0$, $t' = t_k$, $t_i \in T_\varepsilon$ for $i = 1, \ldots, k-1$, and for $i = 1, \ldots, k$ it holds $t_{i-1} \in {}^\bullet p_i$ and $p_i \in {}^\bullet t_i$. In plain words, a silent path is a directed path that contains only silent transitions (apart possibly for the initial and final transition).

The approach we will present in the following applies to nets that satisfy these four assumptions:

(C1) the T_n-induced subnet of N is *acyclic*;

(C2) the T_n-induced subnet is backward conflict-free, i.e., all silent and undistinguishable transitions have no common output place;

(C3) for any two undistinguishable transitions $t, t' \in T_u$ there exists no silent path from t to t';

(C4) if silent path $t_0 p_1 t_1 \ldots t_{i-1} p_k t_k$ leads from a silent transition $t_0 \in T_\varepsilon$ to an undistinguishable transition $t_k \in T_u$, then for all p_i it holds ${}^\bullet p_i = \{t_{i-1}\}$, i.e., each place p_i on the path has a single input transition, namely t_{i-1}.

6.1 An algebraic characterization of the set of consistent markings

Theorem 6.1 *Let us consider an arbitrary labeled Petri net system $\langle N, M_0 \rangle$ and let $L : T \to E \cup \{\varepsilon\}$ be its labeling function. Let assumptions C1 to C4 be verified. Then, for all words $w \in E^*$ the set of w-consistent markings $\mathcal{C}(w)$ is equal to*

$$\begin{aligned} \mathcal{C}(w) = \{ \quad M \quad & \in \mathbb{N}^m \mid \\ & M = M_{b,w} + C_\varepsilon \vec{\sigma}_\varepsilon + \textstyle\sum_{e \in E_u} C_e \vec{\sigma}_e \\ & \vec{\sigma}_\varepsilon \in \mathbb{N}^{n_\varepsilon} \\ & \vec{\sigma}_e \in \mathcal{S}_e(w)\} \end{aligned} \tag{2}$$

where

$$\begin{aligned} \mathcal{S}_e(w) \stackrel{\text{def}}{=} \{\vec{\sigma} \in \mathbb{N}^{n_e} \mid (\forall \tau \in \mathcal{T}_e) \quad & \sigma(\tau) \leq u_w(\tau), \\ & \sigma(T_e) = u_w(T_e)\} \end{aligned}$$

*is the set of w-*consistent undistinguishable firing vectors*; the upper bounds $u_w(\tau)$ and $u_w(T_e)$, as well as the marking $M_{b,w}$, are computed using the recursive Algorithm 6.7 in Fig. 3*[1]*.*

Proof: A formal proof of this statement would require to repeat several arguments and intermediate results that have already been proved in (Giua et al.*, 2005), and would not fit in the allotted number of pages. Thus, here we present sketch of the main arguments that enable us to generalize the results in (Corona* et al.*, 2004; Giua* et al.*, 2005) under assumptions C1 to C4.*

We first observe that the physical meaning of the basis marking $M_{b,w}$ is the same as in the previous particular cases. In fact, $M_{b,w}$ is the marking that is reached from the initial one after the firing of the deterministic events in w plus the nondeterministic transitions (either undistinguishable or silent) that are strictly necessary to enable them.

Moreover, the linear algebraic characterization in (2) only differs from (1) because of the additional term $C_\varepsilon \vec{\sigma}_\varepsilon$ that keeps into account that a certain number of silent transitions may have fired without being identified.

Now, let us discuss the importance of Assumptions C1 to C4. Assumptions C1 and C2 are similar to A1 and A2 in Subsection 5.1. The only difference is that now it is required that the T_n-induced net be acyclic and backward conflict-free. These assumptions ensure that, whenever a deterministic transition t fires, if the current basis marking M_b does not enable it, then there exists a unique minimal e-vector $\vec{y}$ that allows this firing and it is possible to update the new basis marking to $M'_b = M_b + C_n\vec{y} + C(\cdot, t)$. The proof is analogous to that of Theorem 3.3.

Assumption C3 is inspired by assumptions B1 in Subsection 5.2. In fact, Assumption B1 ensures that any two undistinguishable transitions are contact-free, i.e., they do not share: (a) a common input place; (b) a common output place; (c) a place outputting the first and inputting the second. In the present approach these conditions should be generalized to all silent paths leading to or starting from these transitions. However in the present approach condition (a) is not necessary because, as explained later on in Remark 6.5, we are using an IPP (integer programming problem) to compute the enabling degree of sets of transitions. Furthermore, (b) is prevented by assumption C2. Thus we only have to consider the generalization of (c) and this is done in C3.

The last assumption is rather technical. It ensures that the dependency of the firing of an undistinguishable transition from the silent transitions in its minimal explanation is not influenced by the firing of other transitions (Example 6.6 well clarifies this)[2]*.* □

6.2 A detailed explanation of Algorithm 3

Let us first introduce two definitions.

Definition 6.2 *For all $e \in E_n$ and all $t \in T_e$, we denote as $\Sigma_{t_e} = \{t_e\} \cup \{t \in T_\varepsilon \mid \exists$ a silent path from to t to t_e $\}$.* ■

Definition 6.3 *Given a marking M and a subset of transitions $\tau \in \mathcal{T}_e$, we define $z(M, \tau)$ as the optimal value of the objective function of the following IPP*

$$\begin{cases} \max & \sum_{t \in \tau} \sigma_n(\tau) \\ s.t. & M_{b,we} + C_n \vec{\sigma}_n \geq \vec{0}, \qquad \vec{\sigma}_n \in \mathbb{N}^{n_n}. \end{cases} \quad ■$$

[1]A detailed explanation of Algorithm 6.7 in given in Subsection 6.2.

[2]Note that assumption C4 is different from C2. In fact, C2 specifies that any place must be backward conflict-free in the *nondeterministic* net, i.e., it can have in input at most one nondeterministic transition, but it may also have in input one or more deterministic transitions. On the contrary, assumption C4 specifies that some places must be backward conflict-free in the *complete* net, i.e., they can only have in input the transition of the silent path they belong to.

In simple words $z(M, \tau)$ denotes how many times transitions in τ may have fired at M, taking into account that their firing may also occur after the firing of an appropriate number of silent transitions.

Now, let us discuss in detail all cases of Algorithm 3. For clearness of explanation we refer to the labeled Petri net in Fig. 2 that represents the generic substructure of a more complex Petri net that satisfies assumptions C1 to C4. Note that without loss of generality we assume that in this subnet the only undistinguishable transitions are those labeled with a.

Moreover, for simplicity of notation in Fig. 2 we denote as a_1, a_2 and a_3 the undistinguishable transitions labeled a; analogously, we denote silent transitions as ε_1, ε_2, …, ε_8.

Let w be the actual observed word of events and let $M_{b,w}$ be the marking shown in Fig. 2.

- Case A: *a deterministic transition t fires, whose minimal explanation does not include undistinguishable transitions.* If certain silent transitions belong to the minimal explanation of t, we know for sure that such transitions have fired and we can also evaluate how many times they have fired. We consequently update the basis marking taking into account the firing of t and of its minimal explanation.

As an example, assume that transition t_4 in Fig. 2 fires. Its minimal explanation is $\sigma = \varepsilon_1\varepsilon_1$. Thus, we update the basis marking taking into account the firing of $\varepsilon_1\varepsilon_1$ and of t_4.

- Case A': *t is such that some undistinguishable transition (or a transition on the silent path that leads to it) shares an input place with t or with its minimal explanation.* This is a subcase of Case A. If this happens, then it may occur that the upper bounds associated to the subsets of undistinguishable transitions may decrease. In particular, if $z(M_{b,we}, \tau)$ denotes the maximum number of times transitions in τ may have fired at the basis marking $M_{b,we}$ (provided that an appropriate number of silent transitions have fired), then the value of the upper bound of τ is equal to the minimum among $z(M_{b,we}, \tau)$ and the previous value of the upper bound, i.e., $u_w(\tau)$.

As an example, assume that transition t_6 has fired. We know for sure that silent transitions ε_2 and ε_3 should have fired two times and one time, respectively. But the firing of ε_3 removes one token from p_3, thus limiting to one the firings of ε_8 that belongs to the minimal explanation of a_2 at the current basis marking. Thus, if the previous bound of a_2 was 2, we have to reduce it to 1 because we can be sure that neither ε_8 nor a_2 have fired twice.

- Case B: *a deterministic transition t fires, whose minimal explanation includes undistinguishable transitions.* The same reasoning of Case A applies, with the difference that now the minimal explanation of t also contains undistinguishable transitions. Therefore, we update the basis marking taking into account the firing of t and of its minimal explanation, and we also update the upper bounds relative to all subsets of undistinguishable transitions containing transitions in the minimal explanation of t.

As an example, assume that the firing of t_7 is observed. The firing of t_7 is only possible if ε_6, $\varepsilon_5 a_1 \varepsilon_7$ and $\varepsilon_8 a_2$ have fired. Thus, we conclude that one of the previous observations of a was due to a_1 and another one to a_2.

- Case C: *a nondeterministic event is observed.* In such a case we cannot establish which transition has fired. Thus we do not update the basis marking and we take $u_{we}(T_a) = u_w(T_e)+1$. Moreover, for any undistinguishable event e and any subset $\tau \in \mathcal{T}_e$ we update the corresponding upper bound as the minimum among $z(M_{b,w}, \tau)$ and $u_w(\tau)$. This implies that the number of firings of transitions in τ is maximum given the actual basis marking $M_{b,w}$, with the constraint that such a number is consistent with the actual observation of events labeled e. Note that this last requirement is verified because the algorithm is iterative and at each iteration we allow that each upper bound is at most increased of one unity.

As an example, assume that the nondeterministic event a is observed. We know that either transition a_1 or a_2 have fired, while transition a_3 is not enabled at the basis marking and there exists no sequence of silent transitions that enables it.

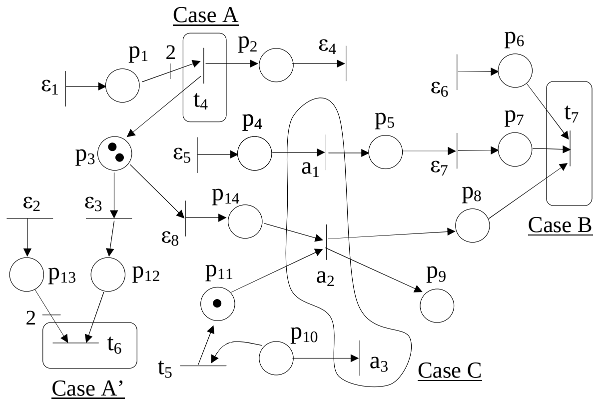

Figure 2: The generic substructure of a Petri net that satisfies assumptions C1 to C4.

w	$M_{b,w}$	$u_w(\tau_1)$	$u_w(\tau_2)$	$u_w(\tau_3)$	$u_w(\tau_{12})$	$u_w(\tau_{13})$	$u_w(\tau_{23})$	$u_w(T_a)$
ε	$[1\ 0\ 0\ 2\ 0\ 0\ 0]^T$	0	0	0	0	0	0	0
a	$[1\ 0\ 0\ 2\ 0\ 0\ 0]^T$	1	1	0	1	1	1	1
at_1	$[1\ 1\ 0\ 2\ 0\ 0\ 0]^T$	1	1	0	1	1	1	1
at_1t_2	$[1\ 1\ 0\ 2\ 0\ 1\ 0]^T$	1	1	0	1	1	1	1
at_1t_2a	$[1\ 1\ 0\ 2\ 0\ 1\ 0]^T$	2	2	1	2	2	2	2

Table 1: The results of Example 6.6.

Example 6.4 Consider the net of the Example 4.2. Assume the observed word is $w = at_1$. Using Algorithm 6.7 we update the basis marking and the upper bounds of subsets τ's as summarized in the following table. Here, $\tau_1 = \{a_1\}$ and $\tau_2 = \{a_2\}$. One can readily verify that the solutions of (2) with the parameters given in the table coincides with the sets of consistent markings explicitly enumerated in Example 4.2. ■

w	$M_{b,w}$	$u_w(\tau_1)$	$u_w(\tau_2)$	$u_w(T_a)$
ε	$[1\ 0\ 0\ 1\ 0\ 0\ 0]^T$	0	0	0
a	$[1\ 0\ 0\ 1\ 0\ 0\ 0]^T$	1	1	1
at_1	$[0\ 0\ 0\ 2\ 0\ 0\ 0]^T$	0	1	1

Remark 6.5 We observe that in this paper we are slightly extending the approach of (Giua *et al.*, 2005) even if we consider nets without silent transitions. The generalization consists in removing the assumption that two undistinguishable transitions may not have a common input place. This assumption was used in (Giua *et al.*, 2005) to simplify the evaluation of the enabling degrees of sets of transitions τ as the sum of the enabling degrees of each transition in the set. Since in this paper we resort to an IPP to compute the enabling degree of the sets τ's, the assumption is not necessary any more. ■

We conclude with an example that shows the necessity of Assumption C4.

Example 6.6 Consider the generic substructure of a more complex Petri net in Fig. 4, where $M_0 = [\ 1\ 0\ 0\ 2\ 0\ 0\ 0\ ]^T$, $T_\varepsilon = \{\varepsilon_1\}, T_u = T_a = \{a_1, a_2, a_3\}$ and $T_d = \{t_1, t_2\}$.
Assume the observed word is $w = at_1t_2a$. Using Algorithm 6.7 we update the basis marking and the upper bounds of subsets τ's as summarized in Table 1. Here, $\tau_j = \{a_j\}$ and $\tau_{ij} = \{a_i, a_j\}$ for $i, j = 1, 2, 3$ and $i \neq j$. As a result the marking $M = [\ 1\ 0\ 1\ 2\ 0\ 0\ 1\ ]^T$, obtained firing the sequence $\sigma = a_1t_1t_2a_3$ is considered as consistent with the observed word w. This is clearly not correct because $\sigma = a_1t_1t_2a_3$ is not enabled at M_0. ■

Algorithm 6.7 **1.** Let $w = \varepsilon$ and $M_{b,w} = M_0$.
2. Let $u_w(\tau) = 0$ for all $e \in E^u$ and for all $\tau \in \mathcal{T}_e$.
3. Let $u_w(T_e) = 0$ for all $e \in E^u$.
4. Wait until an event e is observed.
5. If $e \in E^d$, then
  let $t = L^{-1}(e)$,
  let $\vec{y} = Y_{\min}(M_{b,w}, t)$,
  let $\Sigma = \{t \in T_n \mid y(t) \neq 0\} \cup \{t\}$,
  if $\Sigma \cap T_u = \emptyset$, then (Case A)
    $M_{b,we} = M_{b,w} + C_n\vec{y} + C(\cdot, t)$
    if ${}^\bullet\Sigma \cap^\bullet \bigcup_{t_e \in T_u} \Sigma_{t_e} \neq \emptyset$,
      then (Case A')
      let $\mathcal{T}_r(t) = \{\hat{t} \in T_u \mid {}^\bullet\Sigma \cap^\bullet \Sigma_{\hat{t}} \neq \emptyset\}$
      for all $\tau \in \mathcal{T}_{L(\hat{t})} \cup \{T_{L(\hat{t})}\} : \hat{t} \in \tau$,
        then $u_{we}(\tau) =$
          $\min\{u_w(\tau), z(M_{b,we}, \tau)\}$
      endfor
    endif
  endif
  if $\Sigma \cap T_u \neq \emptyset$, then (Case B)
    for all $\tau \in \bigcup_{e \in E^u} 2^{T_e} \setminus \emptyset \ : t \in \Sigma$, then
      $$u_{we}(\tau) = u_w(\tau) - \sum_{t \in \tau} y(t)$$
    endfor
    $M_{b,we} = M_{b,w} + C(\cdot, t) + C_n\vec{y}$
  endif
 else (Case C)
  for all $\tau \in \mathcal{T}_e$, then
    $u_{we}(\tau) = \min\{u_w(\tau) + 1, z(M_{b,w}, \tau)\}$
  endfor
  $u_{we}(T_e) = u_w(T_e) + 1$
  $M_{b,we} = M_{b,w}$
 endif
7. $w = we$
8. Goto 4. ■

Figure 3: The algorithm for the upper bounds and the basis marking computation.

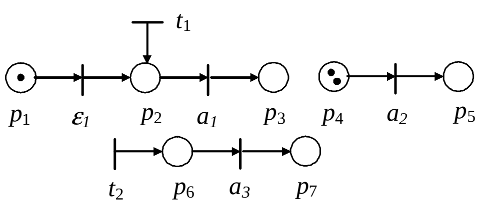

Figure 4: The net in Example 6.6.

7 Conclusions

In this paper we considered Petri nets with arbitrary transition labeling, and assumed that only labels associated to transitions may be observed. The main contribution consists in providing a linear algebraic characterization of the set of markings that are consistent with an observation w, whose structure does not depend on the length of the observed word, but only on a certain number of parameters that may be computed using an appropriate recursive algorithm. The proposed result holds under certain assumptions on the nondeterministic subnet.

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