

Decentralized Supervisory Control of Petri Nets with Monitor Places

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Abstract

In this paper we deal with the problem of determining a set of decentralized controllers for P/T nets that are able to impose a given global specification on the net behaviour. More precisely, both the global specification and the decentralized specifications are given in terms of generalized mutual exclusion constraints (GMECs) thus the controllers take the form of monitor places.

In this paper we provide some preliminary results that are a first step towards a more general and systematic approach to the problem. The lines of our future research in this topic are described in details in the last section, devoted to conclusions and future works.

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1. Introduction

In the last years decentralized control has received a great attention in the discrete event system control (DES) area [6]. This fact has a lot of motivations. (a) Sometimes, real world systems are so large in scale that they require solutions that are modular such as modern automated manufacturing systems. (b) Plants may be in nature distributed across space or across a network of devices such as modern communication systems, public utilities or railway networks.

In the context of supervisory control based upon formal languages specifications a lot of decentralized DES control problems have been studied [7, 8, 9, 10]. On the contrary, decentralized supervisory control has not received a great attention in the context of Petri Nets (PNs). The compact representation of PNs may help in reducing the complexity of decentralized supervisory control problems. In the few works that can be found in the literature a state predicates formulation is adopted: in [5] global specifications are implemented by local supervisors with communication, and in [11] a central coordinator is also present; in [3] global specifications without central coordination is considered and decentralized admissibility of a state predicate formulated in terms of generalized mutual exclusion constraints (GMECs) is defined; finally, in [4], the transformation of inadmissible decentralized constraints into admissible ones is considered.

In this paper the attention is focused on global state specifications given in terms of GMECs and on a control architecture without central coordinator and without communication between local supervisors. This choice is motivated by the following considerations. (i) It is not always possible to have communication with all plant sensors or actuators because of economic reasons or bandwidth limitations. This problem is particularly relevant for plants having a wide geographic extension, or a large numbers of devices such as in modern communication systems. (ii) Even if centralized control is possible, the communication with a certain area of the plant can be lost. It could be useful to use a decentralized control without communication for this area until communication comes back.

More precisely, in this paper, that is a preliminary contribution in this context, we assume that the set of places is partitioned into a given number n_r of subsets P_i 's. Our goal is that of determining n_r decentralized GMECs in order to impose a certain specification given in terms of a global GMEC (l, k) , where the i -th decentralized GMEC (l_i, k_i) is defined over the i -th subset of places P_i . Assuming that the vectors l_i 's are taken equal to the projection of l on P_i , we investigate the possibility of determining the constants k_i 's via integer programming, so as to maximize the cardinality of the set of legal markings under the decentralized control.

Note that the problem we deal with in this paper is different from that considered by Iordache and Antsaklis [4] whose their goal was that of appropriately transform a given GMEC in order to guarantee d-admissibility.

2. Background on Petri nets

In this section we recall the formalism used in the paper. For more details on Petri nets we address to [2].

A place/transition (P/T) net is a structure $N = \langle P, T, \mathbf{Pre}, \mathbf{Post} \rangle$ where: P is a set of m places represented by circles; T is a set of n transitions represented by bars; $P \cap T = \emptyset$, $P \cup T \neq \emptyset$; \mathbf{Pre} (\mathbf{Post}) is the $m \times n$ sized, natural valued, pre-(post-)incidence matrix. For instance, $\mathbf{Pre}(p, t) = w$ ($\mathbf{Post}(p, t) = w$) means that there is an arc from p to t with weight w . The incidence matrix \mathbf{C} of the net is defined as $\mathbf{C} = \mathbf{Post} - \mathbf{Pre}$. A marking is a $m \times 1$ vector $\mathbf{m} : P \rightarrow \mathbb{N}$ that assigns to each

place of a P/T net a non-negative integer number of tokens. A P/T system or net system $\langle N, \mathbf{m}_0 \rangle$ is a P/T net N with an initial marking \mathbf{m}_0 . A transition $t \in T$ is enabled at a marking \mathbf{m} iff $\mathbf{m} \geq \mathbf{Pre}(\cdot, t)$. If t is enabled, then it may fire yielding a new marking $\mathbf{m}' = \mathbf{m} + \mathbf{Post}(\cdot, t) - \mathbf{Pre}(\cdot, t) = \mathbf{m} + \mathbf{C}(\cdot, t)$. The notation $\mathbf{m}[t > \mathbf{m}']$ means that an enabled transition t may fire at \mathbf{m} yielding \mathbf{m}' . A *firing sequence* from \mathbf{m}_0 is a (possibly empty) sequence of transitions $\sigma = t_1, \dots, t_k$ such that $\mathbf{m}_0[t_1 > \mathbf{m}_1[t_2 > \mathbf{m}_2 \dots [t_k > \mathbf{m}_k$. A marking \mathbf{m} is reachable in $\langle N, \mathbf{m}_0 \rangle$ iff there exists a firing sequence σ such that $\mathbf{m}_0[\sigma > \mathbf{m}$. Given a net system $\langle N, \mathbf{m}_0 \rangle$ the set of reachable markings is denoted $R(N, \mathbf{m}_0)$.

3. Background on monitor approach

Assume we are given a set of legal markings $\mathcal{L} \subseteq \mathbb{N}^m$, and consider the basic control problem of designing a supervisor that restricts the reachability set of the plant in closed loop to $\mathcal{L} \cap R(N, \mathbf{m}_0)$. Of particular interest are those PN state-based control problems where the set of legal markings \mathcal{L} is expressed by a set of n_c linear inequality constraints called Generalized Mutual Exclusion Constraints. A single GMEC is a couple (\mathbf{l}, k) where $\mathbf{l} : P \rightarrow \mathbb{Z}$ is a $1 \times m$ weight vector and $k \in \mathbb{Z}$. Given the net system $\langle N, \mathbf{m}_0 \rangle$, a GMEC defines a set of markings that will be called *legal markings*: $\mathcal{M}(\mathbf{l}, k) = \{\mathbf{m} \in \mathbb{N}^m \mid \mathbf{l}\mathbf{m} \leq k\}$. The markings that are not legal are called *forbidden markings*. A controlling agent, called supervisor, must ensure the forbidden markings will be not reached. So the set of legal markings under control is $\mathcal{M}_c(\mathbf{l}, k) = \mathcal{M}(\mathbf{l}, k) \cap R(N, \mathbf{m}_0)$. Without loss of generality, from now on we confine our attention to the case of $n_c = 1$.

It has been shown [1] that the Petri net controller that enforces (\mathbf{l}, k) is a place p_c called *monitor* with incidence matrix $\mathbf{c}_c \in \mathbb{Z}^{1 \times n}$ given by $\mathbf{c}_c = -\mathbf{l}\mathbf{C}_p$ where \mathbf{C}_p is the incidence matrix of the plant. The initial marking of the monitor, denoted as $m_{c0} \in \mathbb{N}$, is given by $m_{c0} = k - \mathbf{l}\mathbf{m}_{p0}$ where $\mathbf{m}_{p0} \in \mathbb{N}^{m \times 1}$ is the initial marking of the plant. The controller exists iff the initial marking is a legal marking, i.e. $k - \mathbf{l}\mathbf{m}_{p0} \geq 0$. By definition a monitor is loop-free¹, thus its incidence matrix \mathbf{c}_c uniquely defines the post- and pre- incidence matrices \mathbf{c}_c^+ and \mathbf{c}_c^- , i.e., $\mathbf{c}_c^+ = \max\{\mathbf{c}_c, \mathbf{0}\}$ and $\mathbf{c}_c^- = \max\{-\mathbf{c}_c, \mathbf{0}\}$. The monitor so constructed is maximally permissive, i.e. it prevents only transitions firings that yield forbidden markings.

4. Problem statement

Let $\langle N, \mathbf{m}_{p0} \rangle$ be the P/T system to be controlled, where $N = \langle P, T, \mathbf{Pre}, \mathbf{Post} \rangle$.

Assume that the set of places P is partitioned into n_r subsets P_1, \dots, P_{n_r} , i.e., $P_i \cap P_j = \emptyset$ if $i \neq j$, and $\cup_{i=1}^{n_r} P_i = P$.

Assume that a global specification is given in terms of a GMEC (\mathbf{l}, k) with nonnegative weights, i.e., $l(p) \geq 0$ for all $p \in P$, and $k > 0$.

We want to determine a set of *decentralized* GMECs (\mathbf{l}_i, k_i) , $i = 1, \dots, n_r$, whose support of places are P_i , $i = 1, \dots, n_r$, respectively, such that

$$\cap_{i=1}^{n_r} \mathcal{M}(\mathbf{l}_i, k_i) \subseteq \mathcal{M}(\mathbf{l}, k).$$

We call *decentralized monitors* the set of monitor places forcing the GMECs (\mathbf{l}_i, k_i) , $i = 1, \dots, n_r$.

¹A transition t cannot be at same time input and output transition of a monitor.

Note that here we are assuming that all transitions are controllable and observable. The relaxation of this hypothesis will be one of the main goals of our future research in this topic.

5. Synthesis of decentralized GMECs

Different criteria can be followed to determine the vectors l_i 's and the constants k_i 's of the decentralized GMEC.

A very natural choice is that of assuming that each vector l_i is obtained by simply projecting the vector l of the global GMEC on the support P_i of the i -th decentralized GMEC, thus leaving the constants k_i 's as the only unknowns of the problem. This is the case considered in this paper, namely we assume that

$$(A1) \quad l_i(p) = l(p) \quad \text{if } p \in P_i, \quad l_i(p) = 0 \quad \text{otherwise.}$$

Under the above assumption the following result clearly holds.

Corollary 1 *Let (l, k) be a global GMEC over a net system $\langle N, \mathbf{m}_{p0} \rangle$. Let P_1, \dots, P_{n_r} be a given partition of places as described in Section 4. Assume that the vectors l_i 's of the n_r decentralized GMECs are chosen as in Assumption (A1). The following implication holds*

$$(l_1 \mathbf{m} \leq k_1) \wedge \dots \wedge (l_{n_r} \mathbf{m} \leq k_{n_r}) \quad \Rightarrow \quad l \mathbf{m} \leq k$$

if $\sum_{i=1}^{n_r} k_i \leq k$ with $k_i \in \mathbb{N}$ for all $i = 1, \dots, n_r$.

Obviously, different values of k_i 's provide different sets of legal markings. Our goal here is that of trying to determine a systematic criterion to select the values of k_i 's in order to maximize the cardinality of the set of markings that are legal under the decentralized control, namely the cardinality of the set

$$\cap_{i=1}^{n_r} \mathcal{M}(l_i, k_i).$$

The solution we propose here is based on intuitive geometrical considerations that lead to an optimality criterion if the constraint $\mathbf{m} \in \mathbb{N}^m$ is relaxed to $\mathbf{m} \in (\mathbb{R}_0^+)^m$. In particular, the solution we propose is based on the following result.

Lemma 2 *Let*

$$\sum_{i=1}^m w_i x_i \leq b, \quad w_i, x_i, b \in \mathbb{R}_0^+$$

be a convex region in the m -dimensional space. The (generalized) volume of this region, that we denote as V_m , is equal to

$$V_m = \frac{1}{m!} \frac{b^m}{\prod_{i=1}^m w_i}.$$

Proof: If $m = 1$ the volume is the length of the segment described by the equation $x_1 \leq b/w_1$, thus the statement clearly holds.

If $m = 2$ the volume is the area of a strait-angled triangle having base equal to b/w_1 and height equal to b/w_2 , thus

$$V_2 = \frac{1}{2} \frac{b^2}{w_1 w_2}.$$

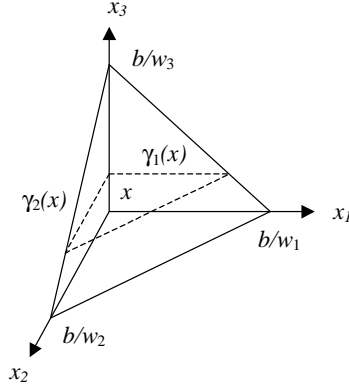


Figure 1. Proof of Lemma 2 in the case of $m = 3$.

If $m = 3$ the volume can be easily computed by simple integration, namely using the notation of Figure 1:

$$\begin{aligned} V_3 &= \frac{1}{2} \int_0^{b/w_3} \gamma_1(x) \gamma_2(x) dx \\ &= \frac{1}{2} \int_0^{b/w_3} \frac{(b - w_3 x)^2}{w_1 w_2} dx = \frac{1}{3!} \frac{b^3}{w_1 w_2 w_3}. \end{aligned}$$

Generalizing, the volume V_m can be obtained as

$$\int_0^{b/w_m} \frac{1}{(m-1)!} \frac{(b - w_m x)^{m-1}}{\prod_{i=1}^{m-1} w_i} dx = \frac{1}{m!} \frac{b^m}{\prod_{i=1}^m w_i},$$

thus proving the statement. □

Using Lemma 2 we can prove the following proposition.

Proposition 3 *Let (\mathbf{l}, k) be a global GMEC over a given net system. Let P_1, \dots, P_{n_r} be a given partition of places as described in Section 4. Assume that the vectors \mathbf{l}_i 's of the n_r decentralized GMECs are chosen as in Assumption (A1). Assume that the integrality constraint on the marking \mathbf{m} , i.e., $\mathbf{m} \in \mathbb{N}^m$, is relaxed to $\mathbf{m} \in (\mathbb{R}_0^+)^m$.*

Let $k_i, i = 1, \dots, n_r$, be a solution of the following nonlinear integer programming problem:

$$\left\{ \begin{array}{ll} \max & \prod_{i=1}^{n_r} k_i^{n_i} \\ \text{s.t.} & \sum_{i=1}^{n_r} k_i = k \\ & k_i \in \mathbb{N}, \quad i = 1, \dots, n_r. \end{array} \right. \quad (a) \quad (1)$$

The (generalized) volume of the convex set

$$\cap_{i=1}^{n_r} \mathcal{M}_R(\mathbf{l}_i, k_i),$$

where

$$\mathcal{M}_R(\mathbf{l}_i, k_i) = \{\mathbf{m} \in (\mathbb{R}_0^+)^m \mid \mathbf{l}_i \mathbf{m} \leq k_i\}$$

is the set of relaxed markings that are consistent with the GMEC (\mathbf{l}_i, k_i) , is greater or equal to the volume of any other convex set $\cap_{i=1}^{n_r} \mathcal{M}_R(\mathbf{l}_i, k'_i) \subseteq \mathcal{M}_R(\mathbf{l}, k)$ with $k'_i \neq k_i$ for some $i = 1, \dots, m$.

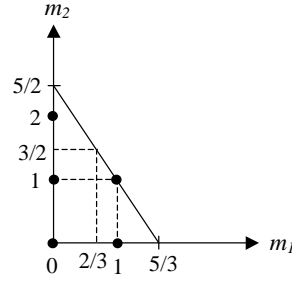


Figure 2. The numerical Example 4.

Proof: By Lemma 2 each decentralized GMEC (l_i, k_i) defines a convex region $\mathcal{M}_R(l_i, k_i)$ in the n_i -dimensional space having a volume equal to

$$\frac{1}{n_i!} \frac{k_i^{n_i}}{\prod_{i=1}^{n_i} l_i(p_i)}.$$

Moreover, the region defined by the intersection of $\mathcal{M}_R(l_i, k_i)$ for $i = 1, \dots, n_r$, has a volume that is equal to the product of the n_r volumes.

Thus, in order to maximize the whole volume we look for the constants k_i 's such that $\prod_{i=1}^{n_r} k_i^{n_i}$ is maximum, being the other terms constant with respect to k_i 's.

Finally, by Corollary 1, (a) is the less restrictive constraint on k_i 's ensuring that the global GMEC (l, k) is satisfied. \square

Example 4 Let (l, k) be a global GMEC over a given P/T system where $l = [3 \ 2]^T$ and $k = 5$. Assume $P = \{p_1, p_2\}$ and $P_1 = \{p_1\}$, $P_2 = \{p_2\}$.

The decentralized GMECs are $3m_1 \leq k_1$ and $2m_2 \leq k_2$ where k_1 and k_2 have to be determined.

Assume that the integrality constraint on m is relaxed. In such a case the decentralized GMECs (l_1, k_1) and (l_2, k_2) that maximize the (generalized) volume $\mathcal{M}_R(l_1, k_1) \cap \mathcal{M}_R(l_2, k_2)$ can be found by solving the nonlinear integer programming problem:

$$\begin{cases} \max & k_1 k_2 \\ \text{s.t.} & k_1 + k_2 = 5 \\ & k_1, k_2 \in \mathbb{N}. \end{cases}$$

The above problem has two different optimal solutions, namely $k_1' = 2, k_2' = 3$, and $k_1'' = 3, k_2'' = 2$.

In Figure 2 we can see the set $\mathcal{M}_R(l, k)$, i.e., the triangle of base $5/3$ and height $5/2$; the set $\mathcal{M}_R(l_1, k_1') \cap \mathcal{M}_R(l_2, k_2')$, i.e., the square of unitary sides; and the set $\mathcal{M}_R(l_1, k_1'') \cap \mathcal{M}_R(l_2, k_2'')$, i.e., the rectangle of base $2/3$ and height $3/2$.

The volumes of the convex sets $\mathcal{M}_R(l_1, k_1') \cap \mathcal{M}_R(l_2, k_2')$ and $\mathcal{M}_R(l_1, k_1'') \cap \mathcal{M}_R(l_2, k_2'')$ are obviously the same, being both decentralized GMECs optimal with respect to the considered objective. \blacksquare

It is important to observe that Proposition 3 is no more valid if the integrality constraint $m \in \mathbb{N}^m$ is not relaxed. To prove this let us simply look at the following example.

Example 5 Let us consider again the numerical case of Example 4 with the constraint $m \in \mathbb{N}^2$.

As it can be seen in Figure 2, there are five integer markings that are consistent with the global GMEC (\mathbf{l}, k) , namely

$$\mathcal{M}(\mathbf{l}, k) = \{[0\ 0], [0\ 1], [1\ 0], [1\ 1], [2\ 0]\}.$$

Moreover,

$$\mathcal{M}(l_1, k'_1) \cap \mathcal{M}_R(l_2, k'_2) = \{[0\ 0], [0\ 1], [1\ 0], [1\ 1]\},$$

while

$$\mathcal{M}(l_1, k''_1) \cap \mathcal{M}_R(l_2, k''_2) = \{[0\ 0], [1\ 0]\},$$

thus making it evident that the second solution is not optimal. ■

At present we are not still able to extend the results of Proposition 3 to the case of $\mathbf{m} \in \mathbb{N}^m$ and arbitrary partitions of the set of places $P = P_1 \cup \dots \cup P_r$. Nevertheless, if all subsets P_i 's only contain one place, we can prove the following result, that provides an optimality criterion to select the decentralized GMECs.

Proposition 6 *Let (\mathbf{l}, k) be a global GMEC over a given net system. Using the notation of Section 4, let $n_r = m$ and $P_i = \{p_i\}$ for $i = 1, \dots, m$ (thus $n_i = 1$ for all $i = 1, \dots, n_r$). Assume that l_i 's are chosen as in Assumption (A1).*

Let $k_i, i = 1, \dots, m$, be a solution of the following nonlinear integer programming problem:

$$\left\{ \begin{array}{ll} \max & \prod_{i=1}^{n_r} k_i \\ \text{s.t.} & \sum_{i=1}^{n_r} k_i = k \quad (a) \\ & \frac{k_i}{l(p_i)} \in \mathbb{N} \quad i = 1, \dots, n_r \quad (b) \\ & k_i \in \mathbb{N}, \quad i = 1, \dots, n_r. \end{array} \right. \quad (2)$$

The cardinality of the set $\cap_{i=1}^{n_r} \mathcal{M}(l_i, k_i)$ is greater or equal to the cardinality of any other set $\cap_{i=1}^{n_r} \mathcal{M}(l_i, k'_i) \subseteq \mathcal{M}(\mathbf{l}, k)$ with $k'_i \neq k_i$.

Proof: The validity of the statement follows from Proposition 3 and the consideration that, thanks to constraint (b), we reject all those decentralized GMECs that lead to polyedra whose vertices are not characterized by integer coordinates, namely polyedra whose volume is maximum but whose number of “integer” points in their inside is not maximum. □

Example 7 Let us consider again the case of Example 4. The integer programming problem (2) only provides the solution $k_1 = 3$ and $k_2 = 2$, thus $\mathcal{M}(l_1, k_1) \cap \mathcal{M}(l_2, k_2) = \{[0\ 0], [0\ 1], [1\ 0], [1\ 1]\}$, i.e., it only provides the optimal solution. ■

6. Conclusions and future work

In this paper we investigated the problem of determining a set of decentralized GMECs (\mathbf{l}_i, k_i) that are able to impose a specification on the net behaviour given in terms of a global GMEC (\mathbf{l}, k) . In particular we assume that the set of places is partitioned into a given number of subsets P_i 's and the vectors l_i 's are taken equal to the projection of \mathbf{l} on P_i . In such a way, based on geometrical considerations and under appropriate assumptions, we suggest a procedure to compute k_i 's that requires the solution of an integer programming problem.

There are still many open questions we plan to investigate, that can be briefly summarized as follows.

- What happens if the vectors l_i are not chosen as in Assumption (A1)? Is it possible to improve the solution?
- How can we generalize the result of Proposition 6 to the case of an arbitrary partition of places?
- What happens if the partition of P is not given a priori?
- What happens if not all transitions are controllable and observable?
- Assuming that the partition of places is given a priori, is it possible to assume that each subset P_i may only observe and/or control a given subset T_i of transitions?
- How can we keep into account constraints on the net behaviour, e.g. boundness?

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