Proceedings of the 2001 IEEE International
Conference on Control Applications
September 5—7, 2001 • México City, México

Modeling Production Systems with Inventory Using Hybrid Petri Nets

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Abstract— **In this paper we deal with the problem of modeling production systems where inventory management is predominant with respect to other aspects of the production cycle. The model we use, called First–Order Hybrid Petri Nets, is an hybrid model that combines fluid and discrete event dynamics and enables us to simulate the dynamic concurrent activities of inventory management systems (IMS). It also provides a modular representation of an IMS, thus making it useful even when dealing with large dimension systems. A real application case, a cheese factory, is finally considered: all numerical data are relative to an existing plant, and simulation is carried out with an available software SIRPHYCO.**

I. Introduction

Every organization holds inventories, i.e., stocks, of some kind. These stocks are in general expensive, but allow for variations and uncertainty in supply and demand: they provide a buffer between suppliers and customers [7].

Inventory Management Systems (IMS) are discrete event dynamic systems whose number of reachable states is typically very large, thus the simulation, analysis and optimization of these systems require large amount of computational efforts and problems of realistic scale quickly become analytically and computationally intractable. To cope with this problem, *fluid models* which are continuous–dynamics approximations of discrete systems, may be successfully developed and applied to the inventory management domain [4]. In general different fluid approximations are necessary to describe the same IMS, depending on its discrete state: new orders required or only demand satisfaction, storage full or empty, and so on. Thus, the resulting model can be better described as an *hybrid model*, where different dynamics are associated to each discrete state.

In [4] we have shown how some independent demand IMS can be modeled with *First–Order Hybrid Petri Nets* (FOHPN), a class of hybrid Petri nets originally presented in [3], that adds to the original formalism of David and Alla [1], [2] linear algebraic tools for the analysis and control of the model. In particular, we associated to each management policy a different FOHPN net (a module). We considered fixed order quantity systems (with finite lead time, fixed reorder level, and instantaneous replenishment) and periodic review systems. We also showed how costs relative to the different management policies can be easily evaluated by adding appropriate net structures to the corresponding modules. A numerical example also demonstrated how FOHPN can be used via simulation as an efficient tool for the solution of some numerical optimization problems.

Note that while the classic analysis of inventory control provides immediate solution to deterministic optimization problems related to single processes, it may not be useful when dealing with stochastic processes or with interconnected processes. On the contrary, simulation with FOHPN always result to be a valid tool for performance evaluation. We also mention a related approach by Lefebvre [6], where continuous Petri nets are used and compared with standard IMS models.

In this paper we consider more complex systems, where the production process is deeply interleaved with the inventory management process. Systems of this type are common in the food industry, where the processing of different products require different stages of maturing and are managed with different inventory management policies.

In particular, we first recall the basic module of a *fixed order quantity system* (FOQS) with finite lead time and instantaneous replenishment presented in [4]. Then, we introduce the FOHPN models of two other IMS, i.e., FOQS with finite replenishment rate and FOQS with back–orders. Moreover, we derive a FOHPN model for maturing stores, showing how storage times can be modeled with an arbitrarily high precision by simply increasing the number of continuous places in the net. Then, we show how appropriate FOHPN modules, that can be directly included in all the previous nets, can be introduced to simulate time–varying demand either stochastic or piece-wise constant.

Finally, we deal with a real application case, a cheese factory, and show how even such a complex system can be easily modeled with FOHPN, thanks to their modularity property. The model and the data used for simulation are relative to a real plant in Sardinia, Italy while the hybrid net simulator used was the software SIRPHYCO [5] developed at LAG, France. In the examples, the results of simulation only show the *inventory levels*. Adding to this model the appropriate net structure (described in [4]) it is also possibile to compute the *costs associated to the factory operation*, and to optimize, via simulation, some system parameters. This is not shown in this paper, for space limitations but we refer the interested reader to [4].

Our results show that Hybrid Petri Nets are a viable approach to build in short time models of real size complex systems, and to study their behavior via simulation using standard tools.

II. First–Order Hybrid Petri Nets

We recall the Petri net formalism used in this paper following [3]. A First–Order Hybrid Petri Net (FOHPN) is a structure $N = (P, T, Pre, Post, \mathcal{D}, \mathcal{C})$.

The set of *places* $P = P_d \cup P_c$ is partitioned into a set of *discrete* places P_d (represented as circles) and a set of *continuous* places P_c (represented as double circles).

The set of *transitions* $T = T_d \cup T_c$ is partitioned into a set of discrete transitions T_d and a set of continuous transitions T_c (represented as double boxes). The set $T_d = T_I \cup T_D \cup T_E$ is further partitioned into a set of *immediate* transitions T_I (represented as bars), a set of *deterministic timed* transitions T_D (represented as black boxes), and a set of *exponentially distributed timed* transitions T_E (represented as white boxes).

The *pre-* and *post-incidence functions* that specify the arcs are (here $\mathbb{R}_0^+ = \mathbb{R}^+ \cup \{0\}$): $Pre, Post : P_d \times T \to \mathbb{N},\ P_c \times T \to \mathbb{R}_0^+$. We require (*well-formed nets*) that for all $t \in T_c$ and for all $p \in P_d$, $Pre(p,t) = Post(p,t)$.

The function $\mathcal{D} : T_t \to \mathbb{R}^+$ specifies the delay d associated to deterministic discrete transitions and the firing rate λ associated to exponentially distributed transitions.

For any continuous transition $t_j \in T_c$ we let $\mathcal{C}(t_j) = (V'_j, V_j)$, with $V'_j \leq V_j$. Here V'_j represents the *minimum firing speed* (mfs) and V_j represents the *maximum firing speed* (MFS). In the following, unless explicitly specified, the mfs of a continuous transition will be $V'_j = 0$.

The *incidence matrix* of the net is defined as $\boldsymbol{C}(p,t) = Post(p,t) - Pre(p,t)$. The restriction of $\boldsymbol{C}$ to P_X and T_Y $(X, Y \in \{c, d\})$ is denoted $\boldsymbol{C}_{XY}$. Note that by the well-formedness hypothesis $\boldsymbol{C}_{dc} = 0$.

We denote the preset (postset) of transition t as ${}^\bullet t$ ($t^\bullet$) and its restriction to continuous or discrete places as ${}^{(d)}t = {}^\bullet t \cap P_d$ or ${}^{(c)}t = {}^\bullet t \cap P_c$.

A *marking* $\boldsymbol{m} : P_d \to \mathbb{N},\ P_c \to \mathbb{R}_0^+$ is a function that assigns to each discrete place a non-negative number of tokens, represented by black dots and assigns to each continuous place a fluid volume; m_i ($\overline{m}_i$) denotes the marking of place p_i ($\overline{p}_i$). The value of a marking at time τ is denoted $\boldsymbol{m}(\tau)$. A *FOHPN system* $\langle N, \boldsymbol{m}(\tau_0) \rangle$ is a FOHPN N with an initial marking $\boldsymbol{m}(\tau_0)$.

The enabling of a discrete transition depends on the marking of all its input places, both discrete and continuous.

A discrete transition t is *enabled* at $\boldsymbol{m}(\tau)$ if for all $p_i \in {}^\bullet t$, $m_i(\tau) \geq Pre(p_i, t)$ and it may fire yielding

$$\boldsymbol{m}(\tau) = \boldsymbol{m}(\tau^-) + \begin{bmatrix} \boldsymbol{C}_{cd} \\ \boldsymbol{C}_{dd} \end{bmatrix} \boldsymbol{\sigma}_t$$

where $\sigma_t(t) = 1$ and $\sigma_t(t') = 0$ if $t \neq t'$.

A continuous transition is enabled only by the marking of its input discrete places. The marking of its input continuous places, however, is used to distinguish between strongly and weakly enabling.

Let $\langle N, \boldsymbol{m} \rangle$ be a FOHPN system. A continuous transition t is *enabled* at $\boldsymbol{m}$ if for all $p_i \in {}^{(d)}t$, $m_i \geq Pre(p_i, t)$.

We say that an enabled transition $t \in T_c$ is: *strongly enabled* at $\boldsymbol{m}$ if for all places $p_i \in {}^{(c)}t$, $m_i > 0$; *weakly enabled* at $\boldsymbol{m}$ if for some $p_i \in {}^{(c)}t$, $m_i = 0$.

The enabling state of a continuous transition t_i defines its admissible *instantaneous firing speed* v_i. If t_i is not enabled then $v_i = 0$. If t_i is strongly enabled, then it may fire with any firing speed $v_i \in [V'_i, V_i]$. Finally, if t_i is weakly enabled, then it may fire with any firing speed $v_i \in [V'_i, \overline{V}_i]$, where $\overline{V}_i \leq V_i$ depends on the amount of fluid entering the empty input continuous place(s) of t_i. In fact, the transition cannot remove more fluid from any empty input continuous place $\overline{p}$ than the quantity entered in $\overline{p}$ by other transitions.

The *instantaneous firing speed* (IFS) at time τ of a transition $t_j \in T_c$ is denoted $v_j(\tau)$ and $\boldsymbol{v}(\tau) = [v_1(\tau), \ldots, v_{n_c}(\tau)]^T$ is the IFS vector at time τ (n_c is the number of continuous places).

We use linear inequalities to characterize the set of *all* admissible firing speed vectors $\mathcal{S}$. Each IFS vector $\boldsymbol{v} \in \mathcal{S}$ represents a particular mode of operation of the system described by the net, and among all possible modes of operation, the system operator may choose the best according to a given objective. In all the examples considered in this paper we implicitly assume that the performance index to be optimized is the sum of the firing speeds of continuous transitions. This implies that, whenever a continuous transition is strongly enabled, then it fires at its maximum firing speed.

As $\boldsymbol{m}$ changes the IFS vector may vary as well. In particular it changes at the occurrence of the following *macro–events*: (a) a discrete transition fires, thus changing the discrete marking and enabling/disabling a continuous transition; (b) a continuous place becomes empty, thus changing the enabling state of a continuous transition from strong to weak.

Let τ_k and τ_{k+1} be the occurrence times of two consecutive macro–events of this kind; we assume that within the interval of time $[\tau_k, \tau_{k+1})$ the IFS vector is constant and we denote it $\boldsymbol{v}(\tau_k)$. Then the continuous behavior of a FOHPN for $\tau \in [\tau_k, \tau_{k+1})$ is described by

$$\begin{cases} \boldsymbol{m}^c(\tau) &= \boldsymbol{m}^c(\tau_k) + \boldsymbol{C}_{cc}\boldsymbol{v}(\tau_k)(\tau - \tau_k) \\ \boldsymbol{m}^d(\tau) &= \boldsymbol{m}^d(\tau_k). \end{cases} \quad (1)$$

III. Fixed order quantity systems (FOQS)

Fixed order quantity systems place an order of fixed size whenever stock falls to a certain level. Such systems need continuous monitoring of stock levels and are better suited to low, irregular demand for relatively expensive items.

In this section we present the most important models of IMS characterized by a fixed order quantity. In particular, we consider FOQS with finite lead time, finite replenishment rate, and back–orders.

For simplicity, all models are presented assuming a constant demand. Then, in section V we show how the use of appropriate FOHPN modules enables us to simulate time–varying and even stochastic demand.

A. FOQS with finite lead time

In this subsection we consider FOQS with a finite lead time and a fixed reorder level. We also assume an instantaneous replenishment and no shortages in nominal operating conditions [4].

Under the assumption of continuous and constant demand, the stock level of an item varies with a typical pattern shown in figure 1.a where the following notation has been used:

- $L(\tau)$ is the *stock level* at the generic time instant τ;
- Q is the fixed *order quantity*;
- LT is the *lead time*, i.e., the delay between placing an order and receiving the goods in stock;
- T is the *cycle time*, i.e., the time between two consecutive replenishment;
- D is the *demand* and coincides with the constant slope (taken as positive) of the curves in figure 1.a; it denotes

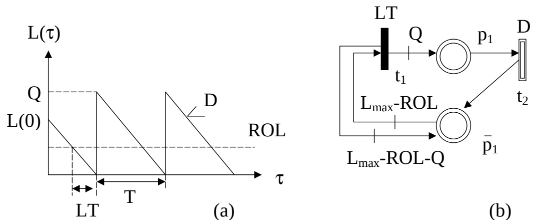

Fig. 1. Fixed order quantity system with finite lead time and instantaneous replenishment: regular pattern (a) and the FOHPN model (b).

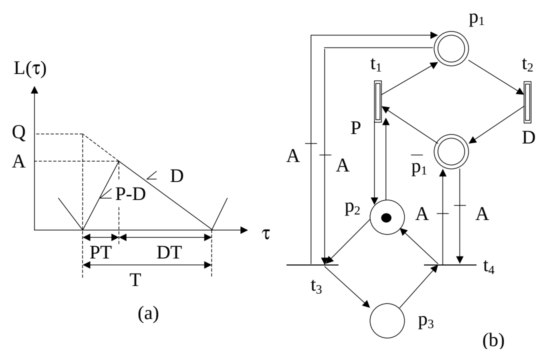

Fig. 2. Fixed order quantity system with finite replenishment rate: regular pattern (a) and the FOHPN model (b).

the number of units to be supplied from stock in a given time period.

In fixed order quantity systems new orders take place whenever the stock level falls to the reorder level ROL.

In figure 1.b the FOHPN model for this kind of systems is presented. The marking of continuous place p_1 represents the stock level while the complementary place $\overline{p}_1$ represents the available space in the storage area. By construction, at each time instant τ, the sum of the marking in p_1 and in its complementary place $\overline{p}_1$ always keeps to a constant value L_{max}. L_{max} represents the maximum capacity of the storage area that is in general much greater than the order quantity Q.

When the marking of p_1 is positive, transition t_2 may fire at its maximum firing speed D, thus reducing the marking of p_1 with a constant slope D. As soon as the marking of p_1 falls to the reorder level ROL (i.e., the marking of $\overline{p}_1$ goes over $L_{max} - ROL$) discrete transition t_1 is enabled and fires after LT time units. When t_1 fires the ordered quantity Q is received in p_1 (the stock), thus this firing produces an increasing of Q units in p_1 and a decreasing of the same entity in the complementary place $\overline{p}_1$.

B. FOQS with finite replenishment rate

In the previous subsection we dealt with a typical situation met by wholesalers: a large delivery of an item instantaneously raises the stock level and then the demand reduces it. Now, let us consider the stock of finished goods at the end of a production line.

If the rate of production is greater than demand, the stock level rises at a rate which is the difference between production and demand. If we call the rate of production P, stocks will build up at a rate $P - D$, as shown in figure 2.a. Stock will continue to accumulate as long as production continues. After some time, PT, a decision is made to stop production. Then, stock is used to meet demand and declines at a rate D. After some further time, DT, all stock has been used and production must start again. Thus, a decision must be made at some point to stop production of this item and transfer facilities to making other items [7].

The resulting variation in stock level is shown in figure 2.a where $A = (P - D) \cdot PT$, and L, Q, T have the same physical meaning as in the previous subsection.

In figure 2.b the FOHPN model for this kind of systems is shown. The marking of continuous place p_1 denotes the stock level, while $\overline{p}_1$ is its complementary place and at each time instant $m_1 + \overline{m}_1 = A$. When the discrete place p_2 is marked, as in figure 2.b, and p_1 is not empty, continuous transitions t_1 and t_2 may fire at their maximum firing speeds, P and D respectively. Assuming $P > D$, the fluid content of continuous place p_1 increases with a constant slope equal to $P - D$, while the fluid content of $\overline{p}_1$ decreases at the same constant slope. As soon as $m_1 = A$, transition t_3 fires thus moving the token from p_2 to p_3. This produces the disabling of transition t_1, and the stock level starts decreasing at a constant slope equal to the demand D. Such a decreasing proceeds until p_1 gets empty, and the content of its complementary place $\overline{p}_1$ is equal to A, thus producing the firing of transition t_4. Then the cycle repeats unaltered.

C. FOQS with back–orders

The models described so far have assumed that all demand must be met. The implication is that shortages are very expensive and must be avoided. There are however, situations where planned shortages are beneficial, and an obvious example of this occurs when the cost of keeping an item in stock is higher than the gross profit made from selling it. When there is customer demand for an item which cannot be met immediately there are shortages, and each customer has a choice: he can wait for an item to come into stock, in which case it is met by a back–order, or he can withdraw his order and go to another supplier, in which case there are lost sales [7]. In this subsection we shall look at the first case.

Under the assumption of continuous and constant demand, the stock level of an item varies with a typical pattern shown in figure 3.a. where back–orders are shown as negative stock levels. Let $L(0)$ be the initial stock level. At time $\tau = L(0)/D$, the stock gets empty and the following demand cannot be met, thus producing shortages. When shortages reach a certain value S, a new order Q is immediately supplied: S units are used to satisfy the unmet demand, while $Q - S$ units are stored in the stock. And the process repeats periodically.

In figure 3.b the FOHPN model for this kind of systems is reported. The fluid content of continuous place p_1 represents the stock level, while place $\overline{p}_1$ is its complementary place and at each time instant $m_1 + \overline{m}_1 = Q - S$. Let us assume that the initial discrete marking is that in figure 3.b, while $m_1(0) = L(0)$. Continuous transition t_2 may fire at its maximum firing speed D. As soon as p_1 gets empty, i.e., $\overline{m}_1 = Q - S$, the immediate transition t_4 fires thus marking discrete place p_5. This enables the firing of continuous transition t_3 with a firing speed that is equal to the demand D. As soon as p_3, whose fluid content represents the amount of unmet demand, is equal to S, transition t_1 fires and places p_3, p_5 and $\overline{p}_1$ get empty, while an amount

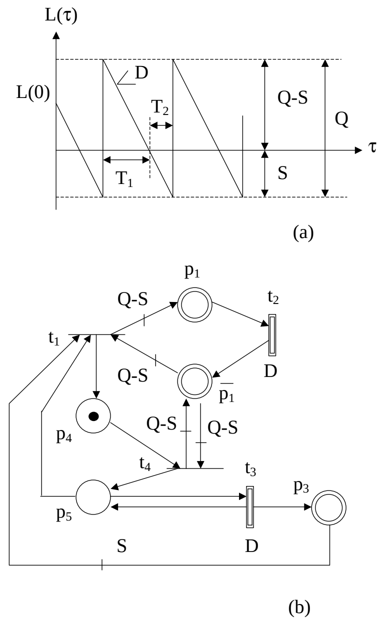

Fig. 3. Fixed order quantity system with back–orders: regular pattern (a) and the FOHPN model (b).

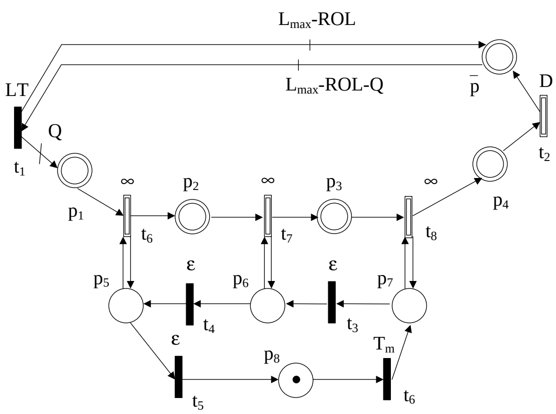

Fig. 4. The FOHPN of a maturing store.

of fluid equal to $Q - S$ is supplied to continuous place p_1. At this point, the demand can be satisfied again and the process repeats cyclically.

IV. Maturing stores

As well known, many products, especially in the food field, need a certain maturing time before getting ready to be sold. This fact obviously needs to be taken into account when solving related inventory control problems.

In this section we show how FOHPN can be used as a modeling tool for maturing stores. The corresponding module is shown in figure 4 where we assumed the fixed order quantity policy with non–null reorder level and finite lead time discussed in subsection III-A. Note however, that whatever inventory management policy may be considered as well.

In this model the stock level is no more equal to the fluid content within a single continuous place. It is given by the sum of the fluid content within four places: p_1, p_2, p_3, and p_4. Continuous place $\overline{p}$ is the complementary place of all the above four places, i.e., at each time instant $m_1 + m_2 + m_3 + m_4 + \overline{m} = L_{max}$, where L_{max} denotes the maximum capacity of the storage area.

The net evolution can be briefly summarized as follows. As soon as $\overline{m} = L_{max} - ROL$ (i.e., $m_1 + m_2 + m_3 + m_4 = ROL$) transition t_1 is enabled and, after a time interval LT, it fires, thus introducing Q further units within the storage area and more precisely, within place p_1. Now, let us consider the cycle $p_8, t_6, p_7, t_3, p_6, t_4, p_5, t_5, p_8$. Since $\varepsilon \approx 0$, we can assume that whenever a new order Q is supplied, only discrete place p_8 is marked, while places p_5, p_6 and p_7 are empty. In particular, two extreme cases may occur: it may happen that either a new order arrives when place p_8 has been just marked, or place p_8 is marked since a time period that is quite equal to T_m. In the former case the new order will remain in p_1 for T_m time instants, while in the latter case, the new order will immediately leave p_1. On the average, the new order Q will stay in p_1 for a time period that is equal to $T_m/2$. Moreover, it will remain in each one of the other $n-1$ stores (places p_2 and p_3) for a time period T_m. Thus, we can conclude that the average maturing time is $\overline{T}_m = T_m/2 + (n-1) \cdot T_m$ where n is the number n of continuous places whose marking denote the level of maturing products (in the case of figure 4, $n = 3$), and the maximum error we can commit in evaluating the maturing period is $T_m/2$.

Note that the precision in modeling the maturing process can be arbitrarily improved by simply increasing the number of continuous places.

V. Time–varying demands

In all the above sections we have assumed that the demand is continuous and constant. Obviously, this is only a simplifying assumption that does not find its counterpart in real applications. In this section we provide new FOHPN modules that can be directly included in all the previous nets, replacing all continuous transitions with maximum firing speed D.

In the case of stochastic demand, all continuous transitions with maximum firing speed D have to be replaced by an exponentially distributed timed transition with an appropriate value of the average firing rate.

For simplicity of presentation, we focus our attention on a particular IMS model. As an example, let us consider the fixed order quantity system in subsection III-A. In this case, continuous transition t_2 should be replaced by an exponentially distributed timed transition. Moreover, to emphasize the stochastic effect of the demand, we assume that the arcs between p_1, $\overline{p}_1$ and t_2 have weight α (we assume that Q is a multiple of α) while the firing rate of transition t_2 is $\lambda = D/\alpha$ so that on the average the stochastic demand is equal to D. As an example, figure 5.a shows a regular pattern of stock level in the case of stochastic demand.

It often happens that demand periodically varies, but it keeps constant during certain sub–periods. This can be easily simulated by the FOHPN module within dashed line in figure 6.a. Transitions t_2, t_3 and t_4 produce a decreasing in the marking of continuous place p_1 with constant slope D_2, D_3 and D_4, respectively. The enabling of these transitions occur during consecutive time intervals of length d_5, d_6 and d_7, respectively. Note that for simplicity, we have not reported the module for the management costs computation. Nevertheless, the new FOHPN module should also

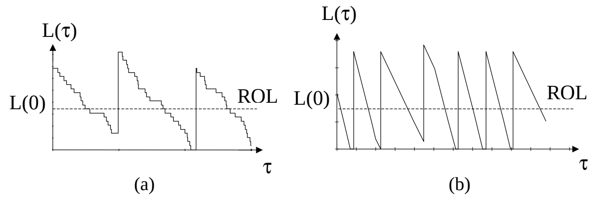

Fig. 5. Stock level with stochastic demand (a) and piecewise constant demand (b) in the case of the IMS in subsection III-A.

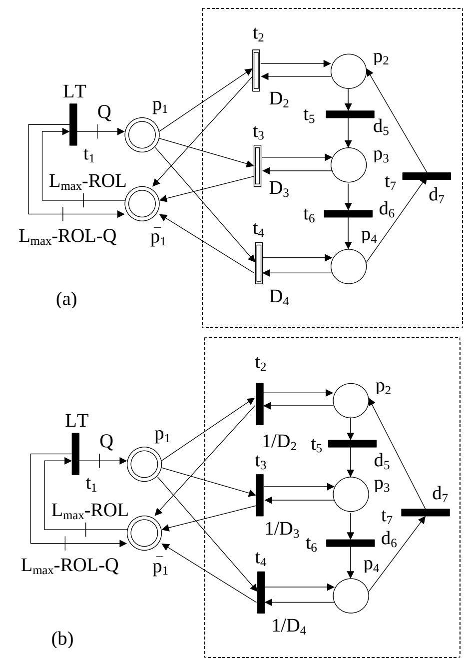

Fig. 6. FOHPN module (a) and discrete FOHPN module (b) of a piecewise constant demand.

replace the continuous transition relative to the holding cost component. As an example, figure 5.b shows a regular pattern of stock level in the case of piecewise constant demand.

A discrete FOHPN of piecewise constant demand can also be given. It is shown in figure 6.b. The only difference with respect to the previous one consists in replacing the continuous transitions t_2, t_3, and t_4 with discrete timed transitions whose time delays are equal to $1/D_2$, $1/D_3$ and $1/D_4$, respectively. Note that in this case a conflict needs to be solved at each discrete place. So as to obtain the desired behaviour of the net, we have to impose priority to transitions t_2, t_3 and t_4 with respect to t_5, t_6 and t_7, respectively.

VI. A real application case: the FOHPN model of a cheese factory

In this section we deal with a real application case. In particular, we consider a cheese factory in Sardinia, Italy.

The most significant production consists in two different kinds of cheese: ripe cheese and soft cheese. For both kinds of product, the production process can be divided in three consecutive phases:

- *milk supply*: every day a certain amount of milk is supplied. Such an amount in not constant and depends on the current month following a characteristic curve (the milk–curve); it reaches the highest values in spring (March, April, May), negligible values in autumn (October, November and December), and intermediate values in the other months;
- *production*: this phase both includes the actual transformation of milk in cheese and the maturing process; obviously, the time required for this phase depends on the kind of cheese and is approximately equal to 76 days for ripe cheese, and 20 days for soft cheese;
- *storage of finished products*: as soon as the maturing process is finished, goods are transferred to appropriate storage areas and are ready to be sold. Clearly, the time that ripe and soft cheese may spend within these areas are different, and strict constraints exist in the latter case.

The adopted production strategy consists in giving priority to the production of soft cheese. The amount of soft cheese that has to be prepared is evaluated on the base of both the current demand and the stock level in the storage area. All the remaining milk is used for the production of ripe cheese. Note that this policy originates from the consideration that in the case of ripe cheese, large amount of storage may be kept for even long periods of time. So, even if priority is given to the production of soft cheese, demand of ripe cheese may be satisfied as well.

Finally, let us observe that the demand of both products largely varies during the year, and sometimes this may produce serious difficulties in making good sales forecasts.

In the following we illustrate how some of the previous FOHPN modules can be assembled so as to model the whole production system. As it can be seen in figure 7 four elementary modules are required:
1. *milk supply*,
2. *production and maturing of soft cheese*,
3. *production and maturing of ripe cheese*,
4. *demand of finished products*.

1. Milk supply has been modeled with an elementary FOHPN module obtained by slightly modifying the discrete piecewise constant demand model in figure 6.b. While the module in figure 6.b. represents the withdrawal of a certain good from a given storage area, in the actual case we need to model the inlet of a certain good (the milk) within a given storage area. This only requires the inversion of all arcs between discrete transitions and continuous place.

Milk supply is modeled by the firing of discrete transitions $t_{M,Jan-Feb}$, $t_{M,Mar}$, $\cdots$, $t_{M,Sep}$, $t_{M,Oct-Dec}$ whose firing delays are all equal to 1 (we assume the day as the time unit). The content of continuous place p_M represents the milk level in the warehouse, while the weights of the arcs from the above transitions to p_M denote the daily amount of milk supplied during the different months. No milk supply occurs during the last three months of the year.

2. The second elementary module represents the production and maturing of soft cheese and has been already presented in section IV. For soft cheese the required time for this phase is equal to 20 days and this has been modeled by assuming $n = 3$, $T_m = 8$ and $\varepsilon = 0.01$. In such a way the maximum error we can commit in evaluating the maturing period is equal to $T_m/2 = 4$ days. Note that in this case the maturing store is a FOQS with instantaneous replenishment, $LT = 1$ day, $Q = 875$, $ROL = 35000$, and $L_{max} = 40600$ cheese.

Finally, the weight of the arc from continuous place p_M to discrete transition $t_{S,prod}$ has been assumed equal to

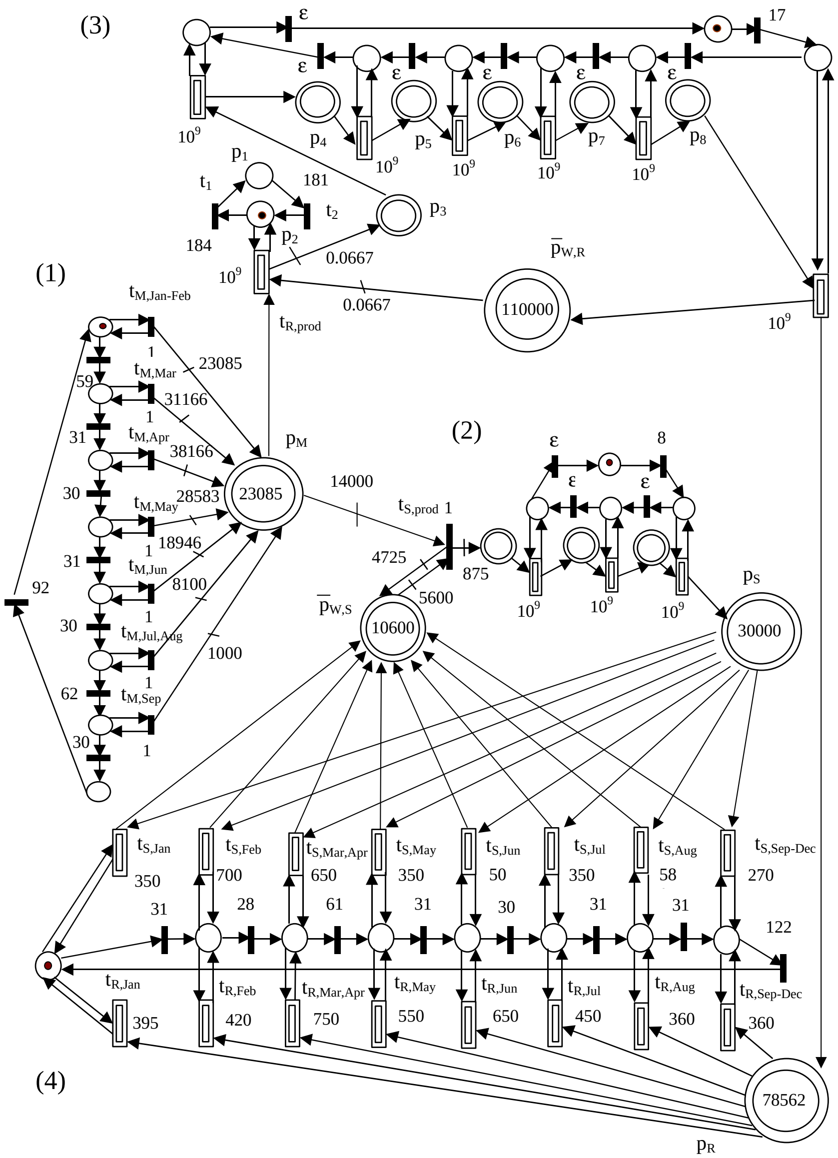

Fig. 7. The FOHPN model of a cheese factory.

14000. This originates from the fact that, from real data provided by the cheese factory, we know that 16 liters of milk are required for 1 soft cheese. Thus, so as to satisfy a new order Q, $875 \cdot 16 = 14000$ liters of milk should be supplied.

3. The third elementary module represents the production and maturing of ripe cheese and has been obtained by slightly modifying the FOHPN module in section IV. Firstly, let us consider the FOHPN module on the bottom left constituted by discrete transitions t_1, t_2 and discrete places p_1, p_2. Its goal is that of imposing the production of ripe cheese only during the first six months of the year. Moreover, it has been assumed that when continuous transition $t_{R,prod}$ is enabled, it fires with a firing speed so high (10^9) that the time required for milk supply can be neglected. Secondly, the marking of the co–buffer $\overline{p}_{W,R}$ is complementary to the sum of the marking in places p_3, p_4, $\cdots$ p_8, but does not depend on the marking of place p_R, whose content denotes the amount of finished product. This originates from the real situation of the plant. In fact, as soon as ripe cheese stops its maturing process, it is stored in an area that is independent from the maturing store and whose dimensions do not require a control on the stock level. On the contrary, more strict constraints are imposed by the dimension of the maturing store. In particular its maximum capacity is equal to 110000 cheeses.

The weight of the arc from continuous transition $t_{R,prod}$ to continuous place p_3 has been assumed equal to 0.0667. This originates from the fact that, from real data provided by the cheese factory, we know that 15 liters of milk are required for 1 ripe cheese. Thus, each liter of milk may produce $1/15 = 0.0667$ cheese.

Finally, for ripe cheese the required time for the production/maturing phase is equal to 76 days and this has been modeled by assuming $n = 6$, $T_m = 14$ and $\varepsilon = 0.01$. In such a way the maximum error we can commit in evaluating the maturing period is equal to $T_m/2 = 7$ days.

Note that to ensure the priority of soft cheese production, we assign priority to the firing of discrete transition $t_{S,prod}$ with respect to continuous transition $t_{R,prod}$.

4. The demand of finished products has been modeled by slightly modifying the piecewise constant demand module in figure 6.a. We now use a single model so as to simulate the demand of two different items. In order to do this, we simply need to associate two different continuous transitions to each discrete place.

The firing of transitions $t_{R,Jan}$, $t_{R,Feb}$, $\cdots$, $t_{R,Sep-Dec}$ simulate the demand of ripe cheese, while transitions $t_{S,Jan}$, $t_{S,Feb}$, $\cdots$, $t_{S,Sep-Dec}$ simulate the demand of soft cheese. Note that in the latter case, arcs between continuous transitions and the co–buffer $\overline{p}_{W,S}$ are also required.

A. A numerical simulation

Now, let us present the result of a numerical simulation carried out with the software SIRPHYCO [5]. All numerical data have been provided by a Sardinian cheese factory.

The marking in figure 7 is representative of the initial simulating condition. In particular the fluid content of places p_S and p_R is equal to the amount of soft and ripe cheese, respectively, remained from the previous year.

The results of simulation are reported in figure 8.

Figure 8.a shows the stock level evolution in the ripe cheese warehouse (fluid content in p_R) during one year. As it can be seen, during the first 76 days the curve decreases, since no new finished product arrives. After this time period, new product is available, thus producing an increasing in the stock level. Then, after 184 more days, milk supply is interrupted. Nevertheless, for about 80 more days new product still arrives in the ripe cheese warehouse, as a result of the actual content within the maturing store. Finally, during the rest of the year the stock level decreases. As it can be seen, at the end of the year a large amount of product is still available, so as to satisfy the demand occurring at the beginning of the following year.

Figure 8.b shows the stock level evolution in the soft cheese warehouse (fluid content in p_S) during the first three years, so as to highlight the periodicity of the curve and the decreasing of the level until stocks are finished. Note that the second point is a prior requirement in the case of soft cheese whose quality is not preserved for long time periods. As in the previous case, at the beginning of each year stock level decreases since no new product is still ready. Then, at the end of each year, stock level decreases again, since no milk supply occurs.

Finally, figure 8.c shows the stock level in the milk warehouse (fluid content in place p_M). As it can be seen high stock levels should not be maintained for many consecutive days, being milk a very perishable good.

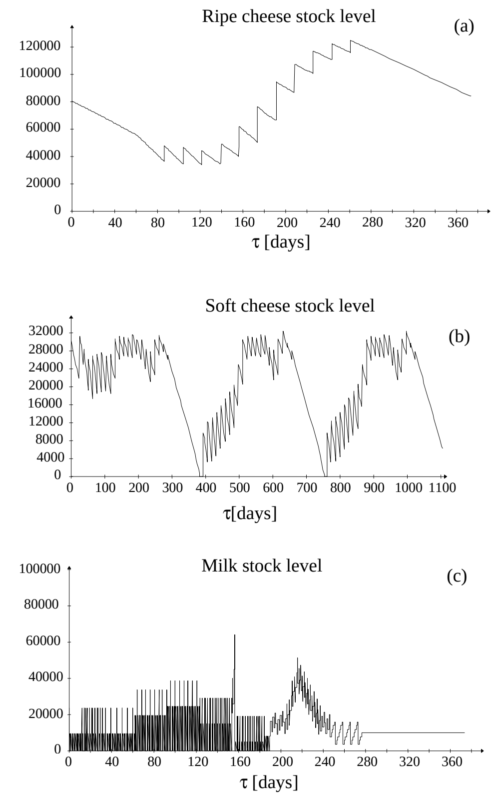

Fig. 8. The results of a numerical simulation carried out on the FOHPN model in figure 7.

VII. Conclusions

In this paper we dealt with the problem of simulating systems whose predominant aspect of the production cycle is the inventory management. The choice of FOHPN as a modeling tool presents many advantages: they can be used as a visual–communication aid; they enables us to set up mathematical models governing the behaviour of systems; with the addition of tokens, FOHPN are able to simulate the dynamic concurrent activities of IMS; finally, they enable a modular representation of an IMS, thus enabling us to deal even with very large dimension systems.

In this paper we provided the FOHPN model for three different fixed order quantity systems, for maturing stores, and for time–varying demand. A real application case, a cheese factory, has also been considered: all numerical data are relative to an existing plant, and simulation has been carried out with the software SIRPHYCO.

Our future work will be that of considering more complex production systems, also introducing appropriate FOHPN modules to compute costs relative to the different phases of management and production, so as to solve even complex numerical optimization problems, as already done in [4] for a simple application case.

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