MODELLING AND CONTROL OF 3D OVERHEAD CRANES

A. Giua, M. Sanna, C. Seatzu
Dip. di Ingegneria Elettrica ed Elettronica, Università di Cagliari, Italy
email: {giua,seatzu}@diee.unica.it.

Abstract. In this paper we deal with the problem of designing a controller for a three–dimensional overhead crane. We consider a linear model of the crane where the length of the suspending rope is a time–varying parameter. The set of models given by frozen values of the rope length can be reduced to a single time–invariant reference model using suitable time scalings. A controller for the reference model can be designed by assigning the desired closed loop eigenvalues for the system. The time scaling relations can be used to derive a control law for the time–varying system that implements an implicit gain–scheduling.

1. Introduction

The swinging of an object suspended from an overhead crane is an undesirable result of the crane movement and serious damage could occur during the load transport. Therefore, a satisfactory control scheme is desirable in a crane design to suppress the load swing.

Several control methodologies have been proposed in the literature [1, 3, 4, 6]. However, in quite all these cases planar cranes have been considered, i.e., it has been assumed that the movement of the load lies within a plane. On the contrary, in this paper we deal with a three–dimensional overhead crane and we propose the design of an observer–controller that aims to minimize the load swinging, while moving it to the desired position as fast as possible.

We first develop a non–linear model of the overhead crane which takes into account simultaneous travel and transverse motions. Then, under appropriate simplifying assumptions (namely, small angles, constant rope velocity, force applied by the rope equal to the weight of the load and no external force acting on the load) a linear time–varying model of the crane is obtained, where the time–varying parameter is the length of the rope that sustains the load. The linearized model has order eight and its dynamic can be described as two decoupled fourth–order systems.

The controller design is realized by first considering the set of frozen models given by different constant values of the rope length. Using two suitable time scalings, one for each sub–system, all these models can be reduced to a single time–invariant reference model that does not depend on the value of the rope length. Then, the pole placement technique enables us to design a satisfactory controller for the reference model. Finally, by inverting the time–scalings, these constant feedback gains give the corresponding time–varying gains that implement an implicit gain–scheduling.

In this paper we introduce a further improvement wrt previous works [3, 6] where a gain–scheduling approach has been adopted: a double gain–scheduling has been introduced. It consists of a variation of the desired eigenvalues of the reference stationary system depending on the load mass and on the lowering/lifting movement.

An important aspect in the approach we propose has to be mentioned: the state–feedback gains are expressed in a parametrized form, as a symbolic function of the desired closed–loop dynamics (i.e., the eigenvalues of the reference closed–loop system), rope length, rope velocity, trolley and load mass. As these parameters vary, the gains need not be recomputed by reapplying the whole design procedure but can simply be obtained by function evaluation.

A final remarks, concerning stability, needs to be done. As it is well known, gain–scheduling does not guarantee the stability of the closed–loop time–varying system. However, there exist appropriate methodologies [3], based on a Lyapunov–like theorem [7], that enables us to find upper bounds on the rate of change of the varying parameter to ensure stability. In [5] it has been shown that in the applicative case examined, this approach gives sufficiently large bounds on the rope velocity to ensure stability of the time–varying system in all nominal conditions.

2. Linear time–varying model and time scaling

A three–dimensional overhead crane is constituted by a bridge and a trolley: the trolley moves on the bridge rails and contains the motor and all the other mechanisms necessary for the movement of the load; the bridge moves in the orthogonal direction thanks to appropriate wheels located on the end truck. In this paper we will consider a three–dimensional overhead crane, whose model is sketched in figure 1. The following notation is used: m_T, m_B are the mass of the trolley and that of the bridge, respectively; $m_C = m_T + m_B$ is total mass of the crane; m_L is the mass of the load; L is the length of the suspending rope; x_T, z_T denote the displacement of the trolley with respect to (wrt) a fixed coordinate system; x_L, z_L denote the displacement of the load wrt a fixed coordinate system; $x_C = (m_T x_T + m_L x_L)/(m_T + m_L)$, $z_C = (m_C z_T + m_L z_L)/(m_C + m_L)$ denote the displacement of the center of gravity of the overall system wrt a fixed coordinate system; φ is the angle between the suspending rope and the vertical; θ is the angle between the oscillation plane of the load and the XY plane, taken as positive when clockwise; $x_V = x_T - x_L = L\, sin\varphi\, cos\theta$, $z_V = z_T - z_L = L\, cos\varphi\, sin\theta$ denote the displacement of the load wrt the vertical; f_x and f_z are the control forces applied to the trolley and to the bridge, respectively; g is the gravitation constant.

If the load is heavy enough, it is possible to consider the suspending rope as a rigid rod. Under appropriate simplifying assumptions (namely, small angles, force applied by the rope equal to the weight of the load and

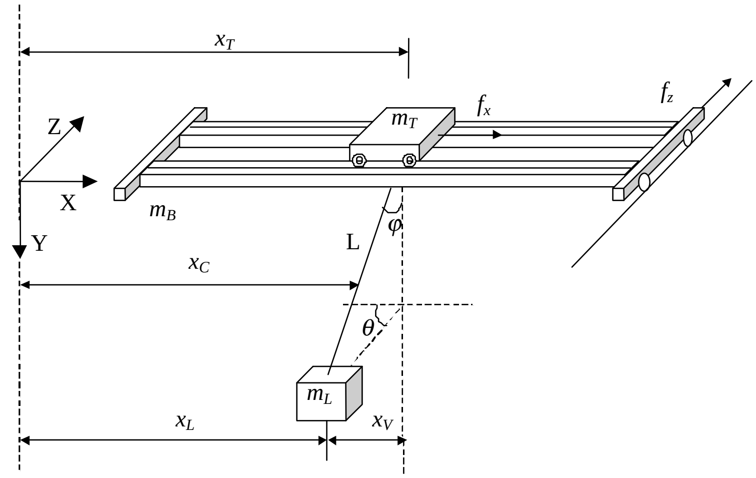

Figure 1: *Model of the 3D crane.*

no disturbance acting on the system) we obtain [5] the linearized model described by

$$\begin{cases} \ddot{x}_V + \dfrac{g(m_T + m_L)}{m_T L} x_V = \dfrac{f_x}{m_T}, \\[2ex] \ddot{x}_C = \dfrac{f_x}{m_T + m_L}, \\[2ex] \ddot{z}_V + \dfrac{g(m_C + m_L}{m_C L} z_V = \dfrac{f_z}{m_C}, \\[2ex] \ddot{z}_C = \dfrac{f_z}{m_C + m_L}. \end{cases} \tag{1}$$

Choosing the following state variables:

$$\begin{array}{llll} x_1(t) = x_V(t), & x_2(t) = x_C(t) & x_3(t) = \dot{x}_V(t), & x_4(t) = \dot{x}_C(t) \\ x_5(t) = z_V(t), & x_6(t) = z_C(t) & x_7(t) = \dot{z}_V(t), & x_8(t) = \dot{z}_C(t) \end{array} \tag{2}$$

and denoting

$$\omega_x(t) \equiv \omega_x(L(t)) = \left(\frac{g(m_T + m_L)}{m_T L(t)} \right)^{0.5}, \quad \omega_z(t) \equiv \omega_z(L(t)) = \left(\frac{g(m_C + m_L)}{m_C L(t)} \right)^{0.5}, \tag{3}$$

we get from (1) the following state variable equation:

$$\dot{\boldsymbol{x}}_t = \boldsymbol{A}_t \boldsymbol{x}_t + \boldsymbol{B}_t \boldsymbol{u}_t \tag{4}$$

with

$$\boldsymbol{x}_t = \begin{bmatrix} x_1(t) \\ \vdots \\ x_8(t) \end{bmatrix}, \quad \boldsymbol{u}_t = \begin{bmatrix} f_x(t) \\ f_z(t) \end{bmatrix},$$

$$\boldsymbol{A}_t = \left[\begin{array}{cccc|cccc} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\omega_x^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\omega_z^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right], \quad \boldsymbol{B}_t = \left[\begin{array}{c|c} 0 & 0 \\ 0 & 0 \\ 1/m_T & 0 \\ 1/(m_T + m_L) & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 & 1/m_C \\ 0 & 1/(m_C + m_L) \end{array} \right].$$

The subscript t has been introduced to recall that the variables are functions of time. The model given by (4) is time–varying because both ω_x and ω_z are functions of $L(t)$. If we consider a given constant value of both ω_x and ω_z, i.e., if we consider the system (4) for a frozen value of L, we can consider the following transformations:

$$\tau_x = \omega_x t, \qquad \tau_z = \omega_z t. \tag{5}$$

These transformations define a time scaling that enable us to rewrite (2) as:

$$\boldsymbol{x}_t = \boldsymbol{N}\boldsymbol{x}_\tau \tag{6}$$

where

$$\begin{array}{l} \boldsymbol{N} = \mathrm{diag}\{\ 1,\quad 1,\quad \omega_x,\quad \omega_x,\quad 1,\quad 1,\quad \omega_z,\quad \omega_z\ \} \\ \boldsymbol{x}_\tau = [\ x_1(\tau_x)\quad x_2(\tau_x)\quad x_3(\tau_x)\quad x_4(\tau_x)\quad x_5(\tau_z)\quad x_6(\tau_z)\quad x_7(\tau_z)\quad x_8(\tau_z)\ ]^T. \end{array} \tag{7}$$

Moreover, we may also write

$$\dot{\boldsymbol{x}}_t = \boldsymbol{\Omega}\boldsymbol{N}\dot{\boldsymbol{x}}_\tau \tag{8}$$

where $\dot{\boldsymbol{x}}_\tau$ is the derivative of $\boldsymbol{x}_\tau$ wrt τ_x for the first four components and wrt τ_z for the remaining ones. It has been assumed

$$\boldsymbol{\Omega} = \mathrm{diag}\{\ \omega_x,\quad \omega_x,\quad \omega_x,\quad \omega_x,\quad \omega_z,\quad \omega_z,\quad \omega_z,\quad \omega_z\ \}. \tag{9}$$

Using (6) and (8), it is possible to rewrite the equation (4) as

$$\dot{\boldsymbol{x}}_\tau = \boldsymbol{A}_\tau\boldsymbol{x}_\tau + \boldsymbol{B}_\tau\boldsymbol{u}_\tau \tag{10}$$

with

$$\boldsymbol{u}_\tau = \begin{bmatrix} \dfrac{1}{\omega_x^2} & 0 \\ 0 & \dfrac{1}{\omega_z^2} \end{bmatrix} \boldsymbol{u}_t = \boldsymbol{N}_u^{-1}\boldsymbol{u}_t = \begin{bmatrix} \dfrac{f_x}{\omega_x^2} \\ \dfrac{f_z}{\omega_z^2} \end{bmatrix}, \tag{11}$$

$$\boldsymbol{A}_\tau = \boldsymbol{N}^{-1}\boldsymbol{\Omega}^{-1}\boldsymbol{A}_t\boldsymbol{N} = \left[\begin{array}{cccc|cccc} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \tag{12}$$

$$\boldsymbol{B}_\tau = \boldsymbol{N}^{-1}\boldsymbol{\Omega}^{-1}\boldsymbol{B}_t\boldsymbol{N}_u = \left[\begin{array}{cccc|cccc} 0 & 0 & 1/m_T & 1/(m_T+m_L) & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1/m_C & 1/(m_C+m_L) \end{array}\right]^T. \tag{13}$$

The representation given by (10) is time–invariant and does not depend on the frozen value of L in (4).

3. Controller design

Let us consider a linear and time–invariant system of the form (10). If the couple $(\boldsymbol{A}_\tau, \boldsymbol{B}_\tau)$ is controllable [2], then a regulator can be designed by imposing the closed loop poles to system (10), finding a control law of the form

$$\boldsymbol{u}_\tau = -\boldsymbol{K}_\tau\boldsymbol{x}_\tau \tag{14}$$

where $\boldsymbol{K}_\tau$ is a constant matrix and does not depend on the value of L. The above equation can be transformed, using (6) and (11), into a corresponding law for the frozen system (4) that gives:

$$\boldsymbol{u}_t = -\boldsymbol{K}_t\boldsymbol{x}_t, \qquad \boldsymbol{K}_t = \boldsymbol{N}_u\boldsymbol{K}_\tau\boldsymbol{N}^{-1}. \tag{15}$$

The feedback laws (14) and (15) lead to closed loop systems whose characteristic matrices are:

$$\bar{\boldsymbol{A}}_\tau = \boldsymbol{A}_\tau - \boldsymbol{B}_\tau\boldsymbol{K}_\tau, \qquad \bar{\boldsymbol{A}}_t = \boldsymbol{A}_t - \boldsymbol{B}_t\boldsymbol{K}_t. \tag{16}$$

Note that also the above matrices can be rewritten as

$$\bar{\boldsymbol{A}}_t = \begin{bmatrix} \bar{\boldsymbol{A}}_{t,x} & \boldsymbol{0}_{4,4} \\ \boldsymbol{0}_{4,4} & \bar{\boldsymbol{A}}_{t,z} \end{bmatrix} \qquad \bar{\boldsymbol{A}}_\tau = \begin{bmatrix} \bar{\boldsymbol{A}}_{\tau,x} & \boldsymbol{0}_{4,4} \\ \boldsymbol{0}_{4,4} & \bar{\boldsymbol{A}}_{\tau,z} \end{bmatrix}.$$

For a stationary system it is easy to find a feedback control law by imposing the closed loop eigenvalues following the procedure presented in [2]. Let us denote as $s^4 + a_{x,3}s^3 + a_{x,2}s^2 + a_{x,1}s + a_{x,0}$ and $s^4 + a_{z,3}s^3 + a_{z,2}s^2 + a_{z,1}s + a_{z,0}$ the open loop characteristic polynomials relative to matrices $A_{\tau,x}$ and $A_{\tau,z}$, respectively. Then, let $s^4 + p_{x,3}s^3 + p_{x,2}s^2 + p_{x,1}s + p_{x,0}$ and $s^4 + p_{z,3}s^3 + p_{z,2}s^2 + p_{z,1}s + p_{z,0}$ be the desired closed loop characteristic polynomials relative to matrices $\bar{\boldsymbol{A}}_{\tau,x}$ and $\bar{\boldsymbol{A}}_{\tau,z}$, respectively. Therefore, the time–invariant control law is [2]:

$$\boldsymbol{K}_\tau = \begin{bmatrix} \boldsymbol{K}_{\tau,x} & \boldsymbol{0}_{1,4} \\ \boldsymbol{0}_{1,4} & \boldsymbol{K}_{\tau,z} \end{bmatrix} \boldsymbol{P}_c^{-1} \tag{17}$$

where

$$\boldsymbol{K}_{\tau,x} = [\ p_{x,0} - a_{x,0} \quad p_{x,1} - a_{x,1} \quad p_{x,2} - a_{x,2} \quad p_{x,3} - a_{x,3}\ ],$$

$$\boldsymbol{K}_{\tau,z} = [\ p_{z,0} - a_{z,0} \quad p_{z,1} - a_{z,1} \quad p_{z,2} - a_{z,2} \quad p_{z,3} - a_{z,3}\ ],$$

$$\boldsymbol{P}_c = \begin{bmatrix} (\boldsymbol{A}_{\tau,x}^3 + a_{x,3}\boldsymbol{A}_{\tau,x}^2 + a_{x,2}\boldsymbol{A}_{\tau,x} + a_{x,1}\boldsymbol{I})\boldsymbol{B}_{\tau,x} \\ (\boldsymbol{A}_{\tau,x}^2 + a_{x,3}\boldsymbol{A}_{\tau,x} + a_{x,2}\boldsymbol{I})\boldsymbol{B}_{\tau,x} \\ (\boldsymbol{A}_{\tau,x} + a_{x,3}\boldsymbol{I})\boldsymbol{B}_{\tau,x} \\ \boldsymbol{B}_{\tau,x} \\ (\boldsymbol{A}_{\tau,z}^3 + a_{z,3}\boldsymbol{A}_{\tau,z}^2 + a_{z,2}\boldsymbol{A}_{\tau,z} + a_{z,1}\boldsymbol{I})\boldsymbol{B}_{\tau,z} \\ (\boldsymbol{A}_{\tau,z}^2 + a_{z,3}\boldsymbol{A}_{\tau,z} + a_{z,2}\boldsymbol{I})\boldsymbol{B}_{\tau,z} \\ (\boldsymbol{A}_{\tau,z} + a_{z,3}\boldsymbol{I})\boldsymbol{B}_{\tau,z} \\ \boldsymbol{B}_{\tau,z} \end{bmatrix}^T,$$

and $\boldsymbol{B}_{\tau,x}$ and $\boldsymbol{B}_{\tau,z}$ are the two non–null sub–matrices of $\boldsymbol{B}_\tau$, i.e.,

$$\boldsymbol{B}_\tau = \begin{bmatrix} \boldsymbol{B}_{\tau,x} & \boldsymbol{0}_{4,1} \\ \boldsymbol{0}_{4,1} & \boldsymbol{B}_{\tau,z} \end{bmatrix}.$$

Note that, $\boldsymbol{P}_c$ is an equivalence transformation that brings the initial system into a controllable canonical form [2].

Using equation (15), we get the time–varying control law:

$$\boldsymbol{K}_t = \begin{bmatrix} \boldsymbol{K}_{t,x} & \boldsymbol{0}_{1,4} \\ \boldsymbol{0}_{1,4} & \boldsymbol{K}_{t,z} \end{bmatrix} \tag{18}$$

where

$$\boldsymbol{K}_{t,x} = [\ (p_{x,2} - p_{x,0} - 1)m_T\omega_x^2 \quad p_{x,0}(m_T + m_L)\omega_x^2 \quad p_{x,3} - p_{x,1})m_T\omega_x \quad p_{x,1}(m_T + m_L)\omega_x\ ]$$

$$\boldsymbol{K}_{t,z} = [\ (p_{z,2} - p_{z,0} - 1)m_C\omega_z^2 \quad p_{z,0}(m_C + m_L)\omega_z^2 \quad p_{z,3} - p_{z,1})m_C\omega_z \quad p_{z,1}(m_C + m_L)\omega_z\ ].$$

4. An applicative example

In this section we show how the above procedure can be applied to a real overhead crane. We consider a model produced by Munck Cranes Inc., Ontario–Canada whose load capacity varies from 1 to 50 ton. In particular, in this paper we consider an overhead crane whose trolley mass is $m_T = 4037\,$Kg and whose bridge mass is $m_B = 4112\,$Kg. We assume the length of the suspending rope to be: $L(t) \in [L_{min}, L_{max}]$, where $L_{min} = 2\,$m and $L_{max} = 10\,$m. To deduce the controller and observer gain matrices we assumed that the rope length has a constant derivative $|\dot{L}(t)| = 0.5\,$m/s. Clearly this is not true during a real movement. Therefore during numerical simulations, we have removed this assumption and we have imposed an acceleration of $\pm 0.5\,$m/s^2 at the beginning and at the end of the hoisting and lowering movement, while in the central part of the movement the velocity is constant and equal to $\pm 0.5\,$m/s.

During the simulations, we have also removed the assumption of linearity thus we used a nonlinear model of the crane derived in [5]. Numerical simulations have been carried out with the SIMULINK toolbox of MATLAB.

An important remarks needs to be done. The physical realization of such a gain–scheduling controller requires the knowledge of all state variables (centre of mass position and velocity, load displacement with respect to (wrt) the vertical and its rate of change), of the rope length and of the load weight. During numerical simulations we assume that only the trolley position and the rope length can be measured by appropriate sensors as discussed by several authors [8] and we also design a time–varying observer via gain–scheduling and pole–placement to provide an estimate of the unknown state vector. The design procedure adopted is the same as that already presented for the observer, with the only difference that in this case, desired poles are assigned to the closed–loop stationary error system that is defined by means of the same time–scaling relations used for the controller design. Details are not reported here for brevity's requirements. An interested read can look at [5] for a precise description of the problem.

Note that in previous works the authors used the gain–scheduling technique to derive a satisfactory control law for a given planar crane [3, 6]. In those works, even in the second one where also an observer has been designed, a single set of eigenvalues for the controller and a single one for the observer has been used. In this paper, we make a different choice motivated by the greater complexity of the system at hand. In particular, we divided the whole range of possible values of the load mass in three different intervals and we further distinguished among lowering and lifting movement. Then, we associated to each range a different set of eigenvalues for the reference stationary system and the error system. In this way we introduced a double gain–scheduling, thus producing a significant improvement in the performance of the controlled

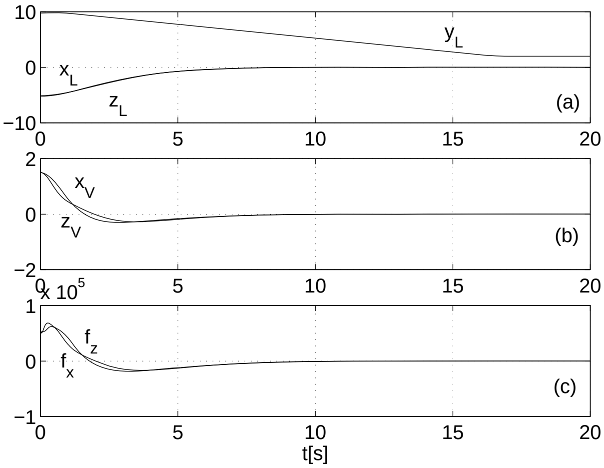

Figure 2: *The results of numerical simulation.*

system. Note that, from an applicative point of view, this does not introduce any amount in the cost of realization of the system, being the load mass assumed known [8] during each operation. These values are not reported here for brevity's sake, but are available in [5].

Now, let us present the results of a numerical simulation. We considered a load mass equal to the maximum load capacity, i.e., equal to 50 ton. The simulation was performed for a lifting movement from $L_o = 10\,\mathrm{m}$ to $L_f = 2\,\mathrm{m}$. The initial state of the crane was $x_V(0) = z_V(0) = 1.5\,\mathrm{m}$, $x_C(0) = z_C(0) = -5\,\mathrm{m}$, $\dot{x}_V(0) = \dot{x}_C(0) = \dot{z}_V(0) = \dot{z}_C(0) = 0\,\mathrm{m/s}$, while the initial state of the observer was $\hat{x}_V(0) = 1\,\mathrm{m}$, $\hat{x}_C(0) = -4.5\,\mathrm{m}$, $\hat{z}_V(0) = 2\,\mathrm{m}$, $\hat{z}_C(0) = -5.5\,\mathrm{m}$, $\hat{\dot{x}}_V(0) = \hat{\dot{x}}_C(0) = \hat{\dot{z}}_V(0) = \hat{\dot{z}}_C(0) = 0\,\mathrm{m/s}$.

In figure 2 the results of this simulation are reported. Figure (a) shows the displacement of the load wrt to a fixed coordinate system; (b) shows the displacement of the load wrt the vertical and enables us to conclude that quite no oscillation occurs during the load movement; in (c) the curves representative of the control forces are shown.

5. Conclusions

In this paper we presented a general methodology for controlling three–dimensional overhead cranes. This work is an extension of previous ones where the authors limit to consider planar cranes.

Time scaling relations have been used to reduce the original time–varying system to a stationary one. The controller design for the reference system has been carried out via pole–placement. Then, the time–scalings inversion enabled us to derive in a parametric form the time–varying gains for the controller. Note that in this paper we implemented a double gain–scheduling, being the eigenvalues of the closed–loop system dependent on the load mass and on the lowering/lifting movement.

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