Modeling and Control of Inventory Management Policies Using First–Order Hybrid Petri Nets

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Abstract

In this paper we use First–Order Hybrid Petri nets (FOHPN), a class of nets that combines fluid and discrete event dynamics, to describe inventory control systems. This hybrid model answers a need, deeply felt in the field of inventory control systems, for a formal tool that integrates the different phases of design, analysis and control of dynamical systems. We also highlight how costs relative to the different management policies can be easily evaluated by adding appropriate FOHPN models.

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1 Introduction

Inventory control is needed in every organization. A typical manufacturing company holds 20% of its production as stock, and this has annual costs of around 25% of value. All organizations, not just manufacturers, hold stocks of some kind and these represent a major investment which should be managed efficiently. If stocks are not controlled properly the costs can become excessive and reduce an organization's ability to compete. Efficient inventory control then becomes a real factor in an organization's long–term survival.

There are basically two approaches to inventory control [8]: a) independent demand systems, which use quantitative models to relate forecast demand, order size and costs; b) dependent demand systems, which use production plans directly to calculate stock requirements. In this paper we only consider systems belonging to the first class. However, all the results presented here may be easily extended to more general systems in the second class.

Independent demand systems assume that the demand for an item is independent of the demand for any other item. Then the aggregate demand for an item is made up of many independent demands for separate customers. In these circumstances the only reasonable approach to forecasting aggregate future demand is to project historic trends. Inventory control is then based on the quantitative models which relate demand, costs, and other variables, to find optimal values for order quantities, timing of orders, and so on.

Inventory Management Systems (IMS) are discrete event dynamic systems whose number of reachable states is typically very large, thus the analysis and optimization of these systems require large amount of computational efforts and problems of realistic scale quickly become analytically and computationally intractable.

To cope with this problem, *fluid models* which are continuous–dynamics approximations of discrete systems, may be successfully developed and applied to the inventory management domain. This has several advantages. First, there is the possibility of considerable increase in computational efficiency, because the simulation of fluid models can often be done much more efficiently. Second, fluid approximations provide an aggregated formulation to deal with complex systems, thus reducing the dimension of the state space. Their simple structures allow explicit computation and performance optimization. Third, the design parameters in fluid models are continuous (e.g., storage areas and order sizes), hence there is the possibility of using gradient information to speed up optimization and perform sensitivity analysis.

It should be noted that in general different fluid approximations are necessary to describe the same IMS, depending on its discrete state: new orders required or only demand satisfaction, storage full or empty, and so on. Thus, the resulting model can be better described as an *hybrid model*, where different dynamics are associated to each discrete state.

In this paper we use *First–Order Hybrid Petri Nets* (FOHPN), a class of nets that combines fluid and discrete event dynamics, to describe inventory control systems. This hybrid model was originally presented in [4] and adds to the formalism described by David and Alla [1, 2] linear algebraic tools for the analysis and control of the model. In this preliminary paper many of the features of FOHPN are not used because we are mainly interested in simulation. Thus we were able to use the simulator SYRPHICO [5] developed for the model of [1, 2] with suitable modifications, as described in Section 5, to bypass the problems connected to the token reservation policy used by SYRPHICO.

The choice of FOHPN as a modeling tool for IMS presents many advantages: FOHPN are both a graphical and mathematical modeling tool able to simulate the dynamic concurrent activities of IMS. FOHPN enable a modular representation of an IMS, i.e., if a system is composed by many subsystems interacting among them, it is possible to model all subsystems with sub–nets and put them together to get the model of the whole IMS. Finally, as shown in [4], the FOHPN model is amenable to sensitivity analysis, i.e., it can be used to obtain information about the degrees of freedom that can be exploited when making performance optimization or optimal design of the system parameters configuration. This feature, that is not used in this work, will be the object of future research.

In this paper we show how some independent demand IMS can be modeled with FOHPN. In particular, we associate to each management policy a different FOHPN net (a module). We consider fixed order quantity systems and periodic review systems and also show how costs relative to the different management policies can be easily evaluated by adding appropriate FOHPN modules to the corresponding IMS module. Finally, we present a numerical example to demonstrate how all these modules can be put together and then implemented by means of an appropriate simulator [5]. The numerical example also show how FOHPN can be used via simulation as an efficient tool for the solution of some numerical optimization problems. Note that while the classic analysis of inventory control provides immediate solution to deterministic optimization problems related to single processes, it may not be useful when dealing with stochastic processes or with interconnected processes. On the contrary, simulation with FOHPN always result to be a valid tool for performance evaluation.

2 First–Order Hybrid Petri Nets

We recall the Petri net formalism used in this paper following [4]. A First–Order Hybrid Petri Net (FOHPN) is a structure $N = (P, T, Pre, Post, \mathcal{D}, \mathcal{C})$.

The set of *places* $P = P_d \cup P_c$ is partitioned into a set of *discrete* places P_d (represented as circles) and a set of *continuous* places P_c (represented as double circles).

The set of *transitions* $T = T_d \cup T_c$ is partitioned into a set of discrete transitions T_d and a set of continuous transitions T_c (represented as double boxes). The set $T_d = T_I \cup T_D \cup T_E$ is further partitioned into a set of *immediate* transitions T_I (represented as bars), a set of *deterministic timed* transitions T_D (represented as black boxes), and a set of *exponentially distributed timed* transitions T_E (represented as white boxes).

The *pre-* and *post-incidence functions* that specify the arcs are (here $\mathbb{R}_0^+ = \mathbb{R}^+ \cup \{0\}$): $Pre, Post : P_d \times T \rightarrow \mathbb{N},\ P_c \times T \rightarrow \mathbb{R}_0^+$. We require (*well-formed nets*) that for all $t \in T_c$ and for all $p \in P_d$, $Pre(p,t) = Post(p,t)$.

The function $\mathcal{D} : T_t \rightarrow \mathbb{R}^+$ specifies the delay d associated to deterministic discrete transitions and the firing rate λ associated to exponentially distributed transitions.

For any continuous transition $t_j \in T_c$ we let $\mathcal{C}(t_j) = (V_j', V_j)$, with $V_j' \leq V_j$. Here V_j' represents the *minimum firing speed* (mfs) and V_j represents the *maximum firing speed* (MFS). In the following, unless explicitly specified, the mfs of a continuous transition will be $V_j' = 0$.

The *incidence matrix* of the net is defined as $\boldsymbol{C}(p,t) = Post(p,t) - Pre(p,t)$. The restriction of $\boldsymbol{C}$ to P_X and T_Y $(X, Y \in \{c, d\})$ is denoted $\boldsymbol{C}_{XY}$. Note that by the well-formedness hypothesis $\boldsymbol{C}_{dc} = 0$.

We denote the preset (postset) of transition t as ${}^\bullet t$ ($t^\bullet$) and its restriction to continuous or discrete places as ${}^{(d)}t = {}^\bullet t \cap P_d$ or ${}^{(c)}t = {}^\bullet t \cap P_c$.

A *marking* $\boldsymbol{m} : P_d \to \mathbb{N},\ P_c \to \mathbb{R}_0^+$ is a function that assigns to each discrete place a non-negative number of tokens, represented by black dots and assigns to each continuous place a fluid volume; m_i denotes the marking of place p_i. The value of a marking at time τ is denoted $\boldsymbol{m}(\tau)$. A *FOHPN system* $\langle N, \boldsymbol{m}(\tau_0)\rangle$ is a FOHPN N with an initial marking $\boldsymbol{m}(\tau_0)$.

The enabling of a discrete transition depends on the marking of all its input places, both discrete and continuous.

A discrete transition t is *enabled* at $\boldsymbol{m}(\tau)$ if for all $p_i \in {}^\bullet t$, $m_i(\tau) \geq Pre(p_i, t)$ and it may fire yielding

$$\boldsymbol{m}(\tau) = \boldsymbol{m}(\tau^-) + \begin{bmatrix} \boldsymbol{C}_{cd} \\ \boldsymbol{C}_{dd} \end{bmatrix} \boldsymbol{\sigma}_t$$

where $\sigma_t(t) = 1$ and $\sigma_t(t') = 0$ if $t \neq t'$.

A continuous transition is enabled only by the marking of its input discrete places. The marking of its input continuous places, however, is used to distinguish between strongly and weakly enabling.

Let $\langle N, \boldsymbol{m}\rangle$ be a FOHPN system. A continuous transition t is *enabled* at $\boldsymbol{m}$ if for all $p_i \in {}^{(d)}t$, $m_i \geq Pre(p_i, t)$.

We say that an enabled transition $t \in T_c$ is: *strongly enabled* at $\boldsymbol{m}$ if for all places $p_i \in {}^{(c)}t$, $m_i > 0$; *weakly enabled* at $\boldsymbol{m}$ if for some $p_i \in {}^{(c)}t$, $m_i = 0$.

The enabling state of a continuous transition t_i defines its admissible *instantaneous firing speed* v_i. If t_i is not enabled then $v_i = 0$. If t_i is strongly enabled, then it may fire with any firing speed $v_i \in [V_i', V_i]$. Finally, if t_i is weakly enabled, then it may fire with any firing speed $v_i \in [V_i', \overline{V}_i]$, where $\overline{V}_i \leq V_i$ depends on the amount of fluid entering the empty input continuous place(s) of t_i. In fact, the transition cannot remove more fluid from any empty input continuous place $\overline{p}$ than the quantity entered in $\overline{p}$ by other transitions.

The *instantaneous firing speed* (IFS) at time τ of a transition $t_j \in T_c$ is denoted $v_j(\tau)$ and $\boldsymbol{v}(\tau) = [v_1(\tau), \ldots, v_{n_c}(\tau)]^T$ is the IFS vector at time τ (n_c is the number of continuous places).

We use linear inequalities to characterize the set of *all* admissible firing speed vectors $\mathcal{S}$. Each IFS vector $\boldsymbol{v} \in \mathcal{S}$ represents a particular mode of operation of the system described by the net, and among all possible modes of operation, the system operator may choose the best according to a given objective. In all the examples considered in this paper we implicitly assume that the performance index to be optimized is the sum of the firing speeds of continuous transitions. This implies that, whenever a continuous transition is strongly enabled, then it fires at its maximum firing speed.

As $\boldsymbol{m}$ changes the IFS vector may vary as well. In particular it changes at the occurrence of the following *macro–events*: (a) a discrete transition fires, thus changing the discrete marking and enabling/disabling a continuous transition; (b) a continuous place becomes empty, thus changing the enabling state of a continuous transition from strong to weak.

Let τ_k and τ_{k+1} be the occurrence times of two consecutive macro–events of this kind; we assume that within the interval of time $[\tau_k, \tau_{k+1})$ the IFS vector is constant and we denote it $\boldsymbol{v}(\tau_k)$. Then the continuous behavior of a FOHPN for $\tau \in [\tau_k, \tau_{k+1})$ is described by

$$\begin{cases} \boldsymbol{m}^c(\tau) & = \boldsymbol{m}^c(\tau_k) + \boldsymbol{C}_{cc}\boldsymbol{v}(\tau_k)(\tau - \tau_k) \\ \boldsymbol{m}^d(\tau) & = \boldsymbol{m}^d(\tau_k). \end{cases} \tag{1}$$

3 Modeling inventory control systems with FOHPN

Independent demand systems can use either fixed order quantities or periodic reviews [8]. Fixed order quantity systems place an order of fixed size whenever stock falls to a certain level. Such systems need continuous monitoring of stock levels and are better suited to low, irregular demand for relatively expensive items. Periodic review systems place orders of varying size at regular intervals to raise the stock level to a specified value. The operating cost of this system is lower and it is better suited to high, regular demand of low–value items.

3.1 Fixed order quantity systems

Let us consider a fixed order quantity system with a finite lead time and a fixed reorder level. Under the assumption of continuous and constant demand, the stock level of an item varies with a typical pattern shown in figure 1.b where the following notation has been used: $L(\tau)$ is the *stock level* at the generic time instant τ; Q is the fixed *order quantity*; LT is the *lead time*, i.e., the delay between placing an order and receiving the goods in stock; T is the *cycle time*, i.e., the time between two consecutive replenishment; D is the *demand* and coincides with the constant slope (taken as positive) of the curves

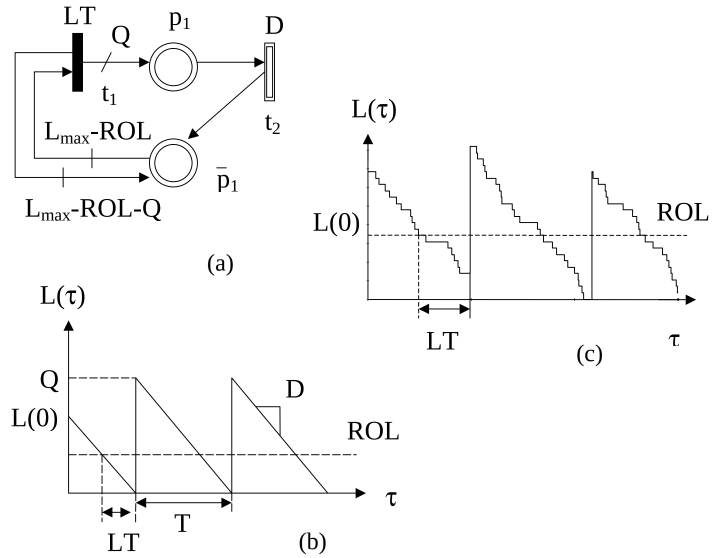

Figure 1: *The model of a fixed order quantity system with constant demand (a). Stock level with constant (b) and stochastic (c) demand.*

in figure 1.b; it denotes the number of units to be supplied from stock in a given time period.

In fixed order quantity systems new orders take place whenever the stock level falls to the reorder level ROL.

In figure 1.a the FOHPN model for this kind of systems is presented. In the figure, the marking of continuous place p_1 represents the stock level while the complementary place $\bar{p}_1$ represents the available space in the storage area. By construction, at each time instant τ, the sum of the marking in p_1 and in its complementary place $\bar{p}_1$ always keeps to a constant value L_{max}. L_{max} represents the maximum capacity of the storage area that is in general much greater than the order quantity Q.

When the marking of p_1 is positive, transition t_2 may fire at its maximum firing speed D, thus reducing the marking of p_1 with a constant slope D. As soon as the marking of p_1 falls to the reorder level ROL (i.e., the marking of $\bar{p}_1$ goes over $L_{max} - ROL$) discrete transition t_1 is enabled and fires after LT time units. When t_1 fires the ordered quantity Q is received in p_1 (the stock), thus this firing produces an increasing of Q units in p_1 and a decreasing of the same entity in the complementary place $\bar{p}_1$.

In the case of stochastic demand, the continuous transition t_2 has to be replaced by an exponentially distributed timed transition with an appropriate value of the average firing rate. To emphasize the stochastic effect of the demand, we assume that the arcs between

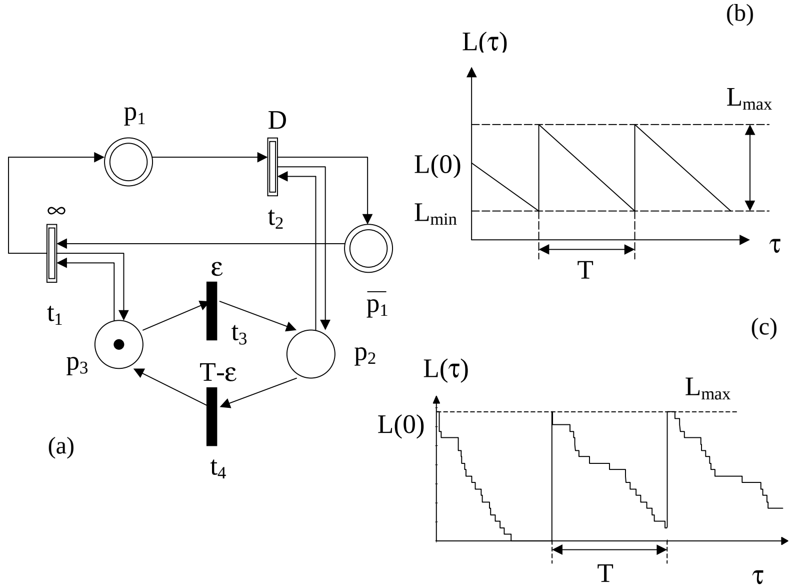

Figure 2: *The model of a periodic review system with constant demand (a). Stock level with constant (b) and stochastic (c) demand.*

p_1, $\bar{p}_1$ and t_2 have weight α (we assume Q is a multiple of α) while the firing rate of transition t_2 is $\lambda = D/\alpha$ so that on the average the stochastic demand is equal to D. Figure 1.c shows the example of a regular pattern of stock level in the case of stochastic demand. Note that in such a case shortage may occur, i.e., there may exist some intervals of time with a null stock level.

3.2 Periodic review systems

Let us consider a periodic review system, whose regular pattern of stock level is reported in figure 2.b. The physical meaning of variables is the same as in the previous case apart L_{min} denoting the minimum stock level during each time period T. Note however, that while in the previous model, the variable producing an event, i.e., a new order, was ROL, in the actual case the variable producing an event is the time period T.

The corresponding FOHPN model is reported in figure 2.a, where the continuous place p_1 and the continuous transition t_2 have the same physical meaning as in the previous model. Place $\bar{p}_1$ is the complementary place of p_1 and at each time instant τ the sum of their markings is equal to L_{max}. When discrete place p_3 is marked, continuous transition t_1 is enabled and it fires at an infinite speed transferring all the content of place $\bar{p}_1$ in place p_1. Obviously, the firing of t_1 with an infinite firing speed is only a simplifying assumption,

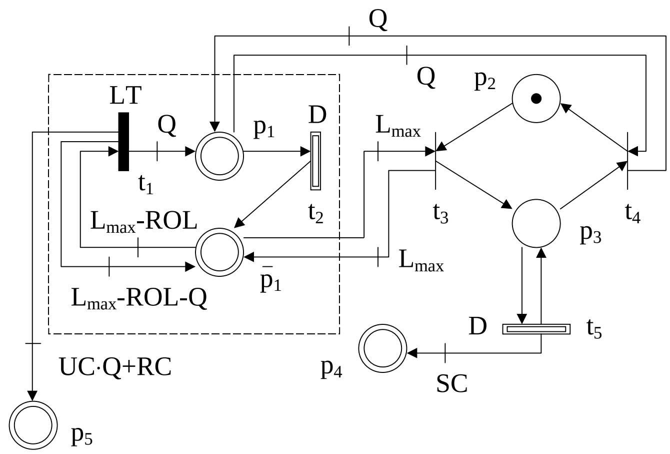

Figure 3: *The model of a fixed order quantity system and its costs evaluation module.*

whose aim is that of underlining that the time required for the transferring of goods may be neglected with respect to the time period T. Finally, the delay of discrete transition t_3 is $\varepsilon \cong 0$, i.e., it fires after a very short time period, thus enabling the timed transition t_4, whose time delay is equal to $T - \varepsilon$. And the process still proceeds unaltered.

Note that, as in the case of fixed order quantity systems, a stochastic demand can be easily simulated by replacing the continuous transition t_2 with an exponentially distributed timed transition and imposing appropriate values to the arcs between t_2 and p_1, $\bar{p}_1$. An example of regular pattern of stock level in the presence of stochastic demand is shown in figure 2.c.

4 Management costs evaluation

In this section we discuss how an appropriate FOHPN module can be used to compute the total costs related to the IMS modules previously described.

Now, let us consider a fixed order quantity system. The total cost C during an interval of time T_h may be calculated as follows:

$$C = UC \cdot Q \cdot N + RC \cdot N + HC \cdot L_{av} \cdot T_h + SC \cdot N_{sc} \tag{2}$$

where the following notation has been used: UC is the *unit cost*, i.e., the price charged by the suppliers for one unit of the item, or the total cost to the organization of acquiring one unit; N is the number of orders; RC is the *reorder cost*, i.e., the cost of placing a routine order for the item and might include allowances for drawing up an order, computer time, correspondence, telephone costs, receiving, use of equipment, expediting delivery, quantity checks, and so on; HC is the *holding cost*, i.e., the cost of holding one unit of an item in stock for one period of time; L_{av} is the *average stock level*; T_h is the *time held*; SC is the *shortage cost*: if the demand for an item cannot be met because stocks have been exhausted, there is usually some associated shortage cost. Note that the fixed order quantity policy assumes that in nominal conditions, i.e., when the demand is constant and equal to that assumed when designing the system, all demand is met. On the contrary, in the case of stochastic demand, shortage may occur producing a significant increase in the total cost.

Finally, N_{sc} is number of lost sales.

Thus, the total cost is given by the sum of four different costs components: the *unit cost component* $UC \cdot Q \cdot N$, the *reorder cost component* $RC \cdot N$, the *holding cost component* $HC \cdot L_{av} \cdot T_h$, and the *shortage cost component* $SC \cdot N_{sc}$.

In figure 3 we show how all the above costs can be computed via an appropriate FOHPN, thus defining a new elementary module. The net within the dashed rectangle models the fixed order quantity system and has been already discussed. The content of continuous place p_5 is equal to the sum of the unit cost component and the reorder cost component. In fact, the firing of timed transition t_1 corresponds to a new order and the weight of the arc from t_1 to p_5 is equal to $UC \cdot Q + RC$.

The holding cost component can be immediately calculated by the knowledge of the marking in p_1, being $L_{av} = 1/T_h \int_0^{T_h} m_1(\tau) d\tau$. We assume that this measure is directly available when implementing the net with an hybrid Petri net simulator. The rest of the FOHPN has been added so as to compute the shortage cost component, that is null unless unforeseen demand occurs. In normal operating condition place p_2 is marked as shown in figure 3. A shortage occurs when place p_1 becomes empty and the marking of its complementary place $\bar{p}_1$ reaches the value L_{max}, thus enabling the firing of immediate transition t_3 that moves the token from p_2 to p_3. During all time in which p_3 is marked, transition t_5 fires at its maximum firing speed D, thus increasing the marking of continuous place p_4, that represents at each instant τ the shortage cost. As soon as the marking of place p_1 comes back to Q, immediate transition t_4 fires, re-establishing the normal operating condition.

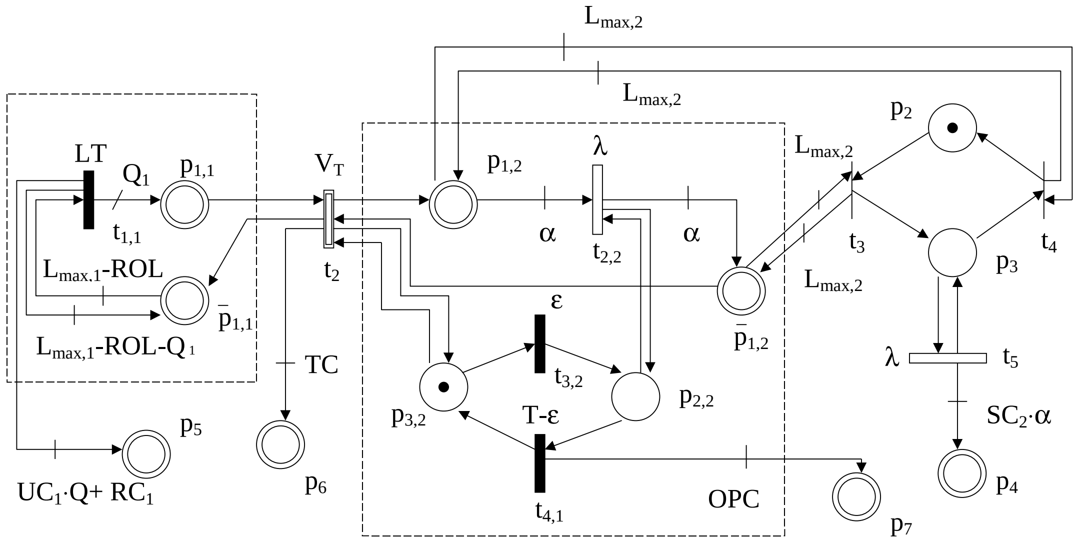

Figure 4: *A numerical example: a double–areas warehouses.*

5 A numerical example

In the literature many optimization problems concerning IMS have been studied. Bhaskaran and Malmorg [3], Malmorg [6], Malmorg and Balachandran [7] deal with optimization problems of double–areas warehouses sizing. In [3] the authors dealt with the problem of relative sizing the two areas for minimizing total costs; [6] has the purpose of formulating an integrated evaluation model which allows distribution system designers to estimate costs associated with inventory carrying, reordering and expediting, item replenishment and retrieval, shortage in the item retrieval area, and storage space. These estimates are based on management policies that address inventory management, space allocation, and storage layout. A limited computational experiment has been used to demonstrate how the model could be applied to investigate alternative policy scenarios. Although the model does not lend itself to analytical optimization methods, it does provide a means for exploring alternative facility design and operating plan combinations. In [7] the authors formulated a model to describe the cost effects of alternative warehouse layouts in a dual address order picking environment.

In this subsection we consider a double–areas warehouses. The first area $\mathcal{M}_1$ is denoted as the *storage area*. As soon as products are finished they are put into $\mathcal{M}_1$ with no precise criterion. We assume that this area is managed with a fixed order quantity policy. The second area $\mathcal{M}_2$, denoted as *pick–area*, collects all products that are ready to be sold. Here products are ordered according to specific criteria, and much higher holding costs are needed. We assume that this area is managed with a periodic review policy.

As it can be seen in figure 4, the FOHPN of this system can be obtained by simply connecting in series the FOHPN modules discussed in the previous subsections where constant deterministic demand has been replaced by an exponentially distributed timed demand. The two FOHPN elementary modules are connected through transition t_2 that is assumed to may fire with a maximum firing speed V_T.

The model used for costs computation is quite similar to that already introduced in the previous section, apart from two new cost components due to transfer of goods. Two further continuous places p_6 and p_7 have been added, whose fluid content denote respectively the *transfer cost component* and the *order–picking cost component*. The first one is proportional to the amount of goods supplied, while the second one is proportional to the number of orders supplied. Therefore, in this case the total cost is given by:

$$\begin{aligned} C = UC_1 \cdot Q_1 \cdot N_1 + RC_1 \cdot N_1 + HC_1 \cdot L_{av,1} \cdot T_h + \\ HC_2 \cdot L_{av,2} \cdot T_h + SC_2 \cdot N_{sc,2} + TC \cdot N_{1,2} + OPC \cdot N_2 \end{aligned} \tag{3}$$

where subscript has been used to distinguish among variables in $\mathcal{M}_1$ and $\mathcal{M}_2$. Moreover, we denoted as $N_{1,2}$ the number of units supplied from $\mathcal{M}_1$ to $\mathcal{M}_2$ in a given time period.

Now, let us consider the following inventory control problem and let us show how its optimal solution can be determined via simulation of FOHPN.

Let us assume that all parameters relative to the storage–area $\mathcal{M}_1$ are fixed.

We want to determine the size of the storage capacity $L_{max,2}$ of the pick–area, so that it becomes empty at the end of each time period $T = D \cdot L_{max,2}$, while minimizing the total cost C. Here $L_{min,2} = 0$.

Our approach requires the implementation of the net in figure 4 for many different values of $L_{max,2}$ and thus many different values of T.

The following numerical values have been used: $L_{max,1} = 2000$ units, $Q_1 = 500$ units, $ROL = 400$ units, $LT = 10$ days, $UC_1 = 24000\,£$ a unit, $RC_1 = 125000\,£$ an order, $HC_1 = 15000\,£$ a unit a day, $HC_2 = 90000\,£$ a unit a day, $SC_2 = 200000\,£$ a unit, $TC = 2000\,£$ a unit, $OPC = 200000\,£$ an order, $V_T = 10^6$ units a day, $L_1(0) = 1550$ units, $L_2(0) = L_{max,2}$ units, $\varepsilon = 1$ hour, $T_h = 4$ years.

We use a stochastic demand (transitions $t_{2,2}$ and t_5) with exponential distribution. We assume the average demand is $D = 20$ units/day and we associate to the two stochastic transitions: firing rate $\lambda = 2$ 1/days and arc weights $\alpha = 10$ units.

The results of the simulations are reported in figure 5 where the total cost with respect to $L_{max,2}$ is plotted. The optimal value computed is $L^*_{max,2} = 200$ and the corresponding total cost is $C^* = 8.5{\cdot}10^8$ £. Such a behaviour can be easily interpreted: when $L_{max,2} \to 0$, we need a large number of order to satisfy the demand, thus greatly increasing the order–picking cost component; when $L_{max,2} \to \infty$ the average stock level in $\mathcal{M}_2$ increases, thus producing excessive the holding costs. Note that in quite all cases, the stochastic demand produces shortage.

Let us observe that the time held T_h, i.e., the time interval of simulation, has been chosen much greater than the cycle times characteristic of the IMS $\mathcal{M}_1$ and $\mathcal{M}_2$, so as to ensure that the computed costs, and the optimum as well, would not significantly depend on the initial chosen conditions. In fact, being the system steady–state evolution periodic, it can be partitioned in three different phases: an initial transient behaviour, a steady–state evolution containing the period an integer number of times, and a final zone where only a fraction of the whole period is contained. If the time held is taken significantly larger than the period, then both first and final phase can be neglected with respect to the intermediate one, that does not depend on the initial conditions.

In order to simulate the above FOHPN we have used the software SYRPHICO [5] that provides an accurate and fast simulation tool for the hybrid Petri net model of Alla and David [1, 2]. Note however, that while SYRPHICO uses a *token reservation* policy, in FOHPN a *concurrent enabling* policy is adopted (as described in Section 2). Although concurrent enabling is more general than token reservation, in these examples it was possible to adapt all the FOHPN models here described so as to implement them with SYRPHICO. For brevity's sake we do not go over the details of this burdensome (but straightforward) procedure.

6 Conclusions

In this paper we dealt with the problem of modeling and control IMS using FOHPN. The choice of FOHPN as a modeling tool presents many advantages: they can be used as a visual–communication aid; they enables us to set up equations, algebraic equations, and other mathematical models governing the behaviour of systems; with the addition

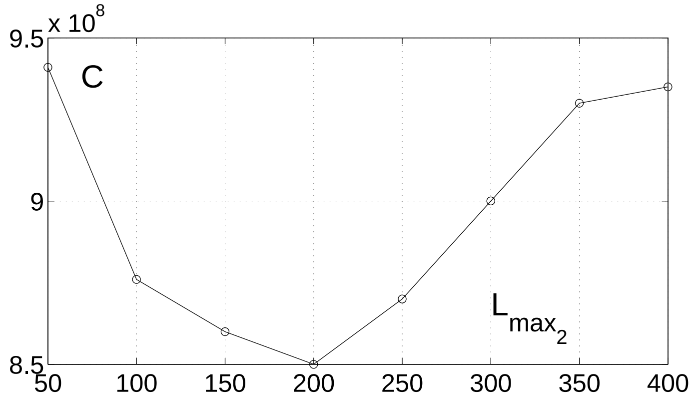

Figure 5: *The results of the numerical optimization problem.*

of tokens, FOHPN are able to simulate the dynamic concurrent activities of IMS; finally, they enable a modular representation of an IMS, thus enabling us to deal even with very large dimension systems.

In this paper we considered fixed order quantity systems and periodic review systems and showed how costs relative to different management policies can be easily evaluated by adding appropriate FOHPN modules to the corresponding IMS module.

References

[1] H. Alla, R. David, "Continuous and Hybrid Petri Nets," *Journal of Circuits, Systems, and Computers*, Vol. 8, No. 1, pp. 159-88, February 1998.

[2] H. Alla, R. David, "A modeling and analysis tool for discrete events systems: continuous Petri Net," *Performance Evaluation*, Vol. 33, No. 3, pp. 175-99, 1998.

[3] K. Bhaskaran, C.J. Malmborg, "Economic tradeoffs in sizing warehouse reserve storage area," *Applied Math. Modelling*, Vol. 14, pp. 381-5, July 1990.

[4] F. Balduzzi, A. Giua, G. Menga, "First-Order Hybrid Petri Nets: a Model for Optimization and Control," *IEEE Trans. on Robotics and Automation*, Vol. 16, August 2000.

[5] G. Guerre-Chaley, "Conception et réalisation d'un simulateur de réseaux de Petri continus et hybrides pour l'evaluation de performances des systmes discrets et/ou continus," *CNAM dissertation, Conservatoire National des Arts et Métiers*, (Grenoble, France), April 1997.

[6] C.J. Malmorg, "An integrated storage system evaluation model," *Applied Math. Modelling*, Vol. 20, pp. 359-70, May 1996.

[7] C.J. Malmorg, S. Balachandran, "A model based evaluation of a commonly used rule of thumb for warehouse layout," *Applied Math. Modelling*, Vol. 10, pp. 133-8, April 1986.

[8] C.D.J. Waters, "Inventory Control and Management," *Ed. John Wiley and Sons*, 1992.