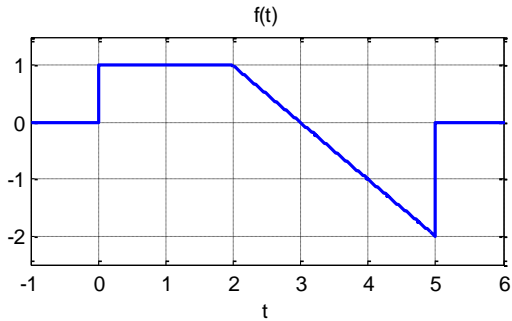

Testo A

**** Esercizio 1

** 1. (a)

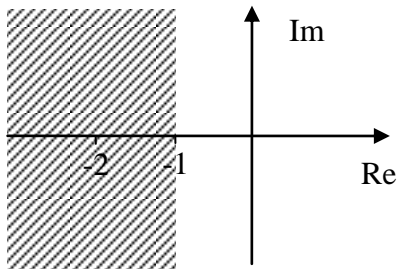


** 1. (b)

- lineare per $\alpha = 0$;
- stazionario per ogni valore di α ;

** 1. (c)

$t_{a5\%} = 3 \tau < 3; \implies \tau < 1$
 $\implies \tau = -1/\alpha < 1$ (NB: $\alpha < 0$)
 $\implies \alpha < -1$



**** Esercizio 2

2. (a)

$$y_{elle} = \exp(-t) + 2t \exp(-t)$$

2. (b)

$$y_{elle} = \exp(-(t-3)) + 2(t-3) \exp(-(t-3))$$

**** Esercizio 3

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

** 4. (a)

Autovalori
 $\lambda_1 = 3$
 $\lambda_2 = -2$

Autovettori

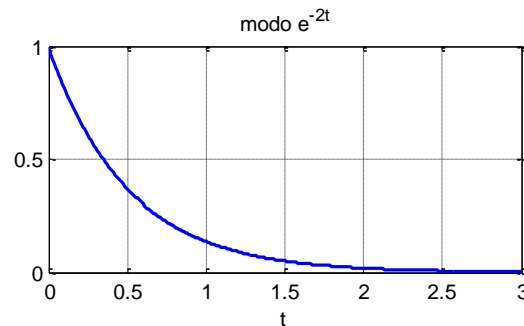
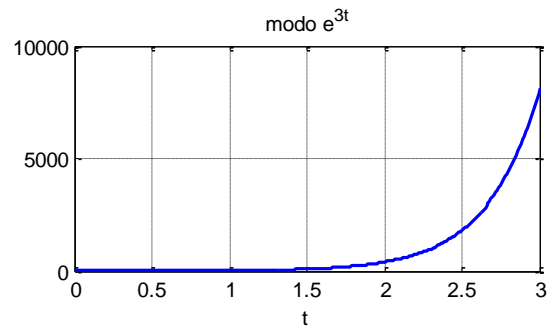
$$v_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

** 4. (b)

$$\text{Modi: } = \exp(3t), \exp(-2t)$$

Il secondo modo ha $t_{a5\%} = 3\tau = 3/2 = 1.5$



** 4. (c)

$$e_{At} = \begin{bmatrix} 2/5 \exp(-2t) + 3/5 \exp(3t), \\ 3/5 \exp(3t) - 3/5 \exp(-2t) \\ 2/5 \exp(3t) - 2/5 \exp(-2t), \\ 3/5 \exp(-2t) + 2/5 \exp(3t) \end{bmatrix}$$

** 4. (d)

La matrice di ordine 2 ha due autovalori distinti e dunque due autovettori linearmente indipendenti. Essa puo' quindi essere diagonalizzata.

$$V = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$$

$$\text{inv}V = \begin{bmatrix} 0.2 & 0.2 \\ 0.4 & -0.6 \end{bmatrix}$$

$$A_{\text{primo}} = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$$

$$B_{\text{primo}} = \begin{bmatrix} 0.6 \\ 0.2 \end{bmatrix}$$

$$C_{\text{primo}} = \begin{bmatrix} 6 & -3 \end{bmatrix}$$

$$D_{\text{primo}} = 0$$

** 4. (3)

$$z_{elle} = \begin{bmatrix} \exp(3t) \\ \exp(-2t) \end{bmatrix}$$

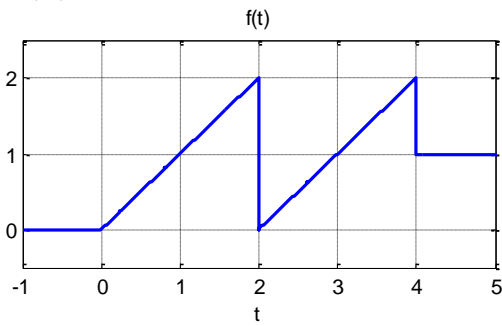
$$z_f = \begin{bmatrix} 0.4(\exp(3t) - 1) \\ 0.2(1 - \exp(-2t)) \end{bmatrix}$$

$$y_{elle} = 6\exp(3t) - 3\exp(-2t)$$

$$y_f = 2.4\exp(3t) + 0.6\exp(-2t) - 3$$

 Testo B

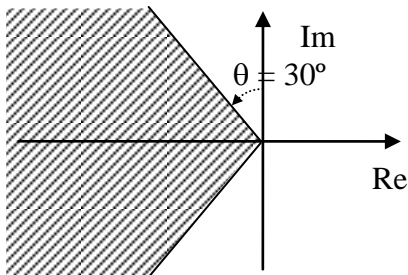
**** Esercizio 1
 ** 1. (a)



** 1. (b)
 - lineare per ogni valore di alfa;
 - stazionario per alfa = 0;

** 1. (c)

zeta = sin(theta) > 0.5;
 ==> theta > 30 gradi



**** Esercizio 2

** 2. (b)
 $y_{\text{elle}} = \sqrt{13}/3 \cdot \exp(-t) \cdot \cos(3t - 0.58)$
 $= \exp(-t) \cdot \cos(3t) + 2/3 \cdot \exp(-t) \cdot \sin(3t)$

** 2. (b)
 $y_{\text{elle}} = \sqrt{13}/3 \cdot \exp(-(t-3)) \cdot \cos(3(t-3) - 0.58)$

**** Esercizio 3

A = $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$
 B = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
 C = $\begin{bmatrix} 3 & 0 \end{bmatrix}$
 D = $\begin{bmatrix} 0 \end{bmatrix}$

** 4. (a)

Autovalori
 $\lambda_1 = 2$
 $\lambda_2 = -1$

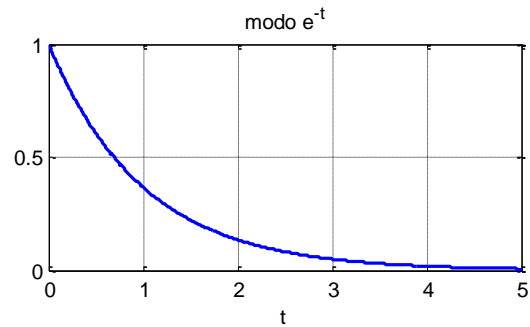
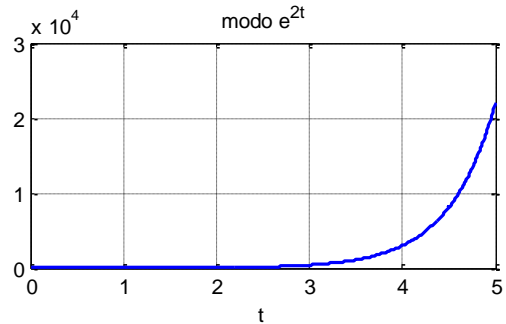
Autovettori

v1 = $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

v2 = $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

** 4. (b)
 Modi: = $\exp(2t), \exp(-t)$

Il secondo modo ha $t_{a5\%} = 3 \cdot \tau = 3$



** 4. (c)
 $e_{At} = \begin{bmatrix} 2/3 \exp(2t) + 1/3 \exp(-t), & -2/3 \exp(-t) + 2/3 \exp(2t) \\ -1/3 \exp(-t) + 1/3 \exp(2t), & 1/3 \exp(2t) + 2/3 \exp(-t) \end{bmatrix}$

** 4. (d)
 La matrice di ordine 2 ha due autovalori distinti e dunque due autovettori linearmente indipendenti. Essa puo' quindi essere diagonalizzata.

V = $\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$

invV = $\begin{bmatrix} 1/3 & 1/3 \\ 1/3 & -2/3 \end{bmatrix}$

A_primo = $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

B_primo = $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

C_primo = $\begin{bmatrix} 6 & 3 \end{bmatrix}$
 D_primo = $\begin{bmatrix} 0 \end{bmatrix}$

** 4. (3)

z_elle = $\begin{bmatrix} \exp(2t) \\ \exp(-t) \end{bmatrix}$

z_f = $\begin{bmatrix} \exp(2t) - 1 \\ 2(\exp(-t) - 1) \end{bmatrix}$

y_elle = $6 \exp(2t) + 3 \exp(-t)$

y_f = $6 \exp(2t) + 6 \exp(-t) - 12$