HYBRID CONSTRAINED FORMATION FLYING CONTROL OF MICRO-SATELLITES

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Abstract: This paper proposes a centralized solution to the problem of formation reconfiguration and keeping for fleets of satellites in the presence of persistent disturbances and under input-saturation and formation accuracy constraints. Relative position and attitude dynamics are considered. For suitable initial conditions the proposed control scheme produces system evolutions that fulfill the coordination constraints at any time and satisfy desirable control performance. This is accomplished by using a bank of Command Governor units in a hybrid framework. An example is provided in order to exhibit the effectiveness of the technique.

Keywords: formation flying control, leader following, saturation constraints, accuracy constraints, hybrid command governor.

1. INTRODUCTION

In recent years an increasing number of space missions have used small satellites. As a consequence of the significant reduction of the costs related to launches, maneuvers and maintenance, it is expected that a great deal of the future space systems will be based in fleets of micro-satellites (Esper *et al.*, 2003). Indeed, it can increase the overall efficiency, performance and survivability, in comparison to single, large vehicles, even in the presence of large instrumentation and payloads.

The aim of the present work is to propose a control technique for relative positions and attitudes in a fleet of micro-satellites with respect to a formation center, using a leader-following approach. The geometry of a formation can be subject to changes, and the relative states of the satellites can be required to fulfill stringent accuracy constraints. Thus, constrained control techniques might be necessary (Yeh *et al.*, 2000). Moreover, actuator saturation is inevitably present and has to be taken into account with small space vehicles, since small size thrusters cannot supply large torques (Hu and Lin, 2001).

Several control techniques based on convex optimization have been proposed to face this kind of problems, see e.g. (Tillerson *et al.*, 2002). However, a control strategy based on conceptual tools of model-based predictive control (MPC) (Manikonda *et al.*, 1991; Mayne *et al.*, 2000) appears particularly suitable. In this work we consider one of the simplest MPC techniques referred as the command governor approach (Bemporad *et al.*, 1997; Casavola *et al.*, 2000; Casavola *et al.*, 2006). It consists of adding to a primal compensated system a nonlinear device, called Command Governor (CG), whose action is based on the current state, set-point, and the prescribed

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constraints. The CG selects at each time instant a virtual sequence among a family of linearly parameterized command sequences, by solving a convex constrained quadratic optimization problem, and feeds the primal system according to a receding horizon control philosophy.

In (Bacconi *et al.*, 2004) we illustrated the advantages related to this approach with respect to others MPC control techniques. There, we assumed small angular displacements between the satellites and the formation center. However, this is acceptable only in the presence of small angle maneuvers. Here, we remove this assumption and describe the attitude of each member of the fleet by a nonlinear model. Since single CG based control laws cannot handle the complete range of possible angles, we consider a bank of controllers, each one designed with respect to a pre-established orbital reference frame. A centralized hybrid control scheme is then adopted.

Thus, the whole scheme retains the properties of a supervisory switching control and the presence of the CGs aims at enlarging the dynamic range where each compensated system can operate linearly. Moreover, the CGs do not modify the primal control system dynamics, since they operate only on the input signal, whenever necessary.

This produces a particularly simple structure, especially suitable for small formations. On the other hand, with large signals, there is a performance degradation. However, in the presence of limited computing power and energy, when the direct use of a bank of predictive controllers is not allowed, since it requires a quite massive amount of flops per sampling time, the use of the present CG hybrid schemes is widely justified.

The paper is organized as follows: in section 2 the mathematical model of the satellites is described. In section 3 the control problem is stated. The Command Governor approach and some variations suitable for the problem at hand are described in section 4. Moreover, a hybrid CG technique is stated in section 5 and a supervisory switching logic is introduced as well. Finally, in section 6 an example is presented where a coordinated large angle maneuver is requested to a couple of micro-satellites in an Earth Observation mission.

2. MATHEMATICAL MODEL

We consider a formation of micro-satellites in LEO orbit, using a description based on the leader following approach. The mathematical model of each member of the formation must consequently describe the attitude of a reference frame $\{B\}$ fixed on the vehicle's body, with respect to an appropriately defined orbiting reference frame $\{O\}$ and the position of $\{B\}$ into another, possibly coinciding with $\{O\}$, orbiting reference frame $\{C\}$ (Fig. 1). The latter is assumed to be centered on

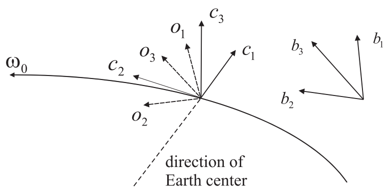

Fig. 1. Reference frames.

an orbiting point, intended as the formation center (virtual satellite). In this paper, the formation center reference frame $\{C\}$ is selected with the first axis in the opposite direction of Earth center. The second axis is assumed aligned with the orbit and the third axis completes an orthonormal right oriented versors set. Moreover, it is assumed no rotational motion of $\{O\}$ in $\{C\}$.

We introduced a linear model for the attitude of each member of the fleet in (Bacconi *et al.*, 2004) by means of the Euler's angles θ_i describing the orientation of $\{O\}$ in the body frame $\{B\}$ (Sidi, 1997), under the hypothesis of small angle maneuvers. Following the same procedure, the absolute angular velocity of $\{B\}$ can be rewritten as the sum of the velocity of $\{B\}$ with respect to $\{O\}$ and the velocity of $\{O\}$ with respect to ECI, defined as $\omega^o = [0, 0, \omega_0]'$ (all the variables expressed in $\{B\}$ coordinates):

$$\omega^b = \omega^{bo} + \omega^o = \omega^{bo''} + \omega^{o''o'} + \omega^{o'o} + \omega^o$$

with O' and O'' the reference frames obtained from O after a sequence of rotations of angles θ_1 and θ_2, respectively. By means of the rotation matrix $\mathcal{R}^{bo}$, that leads $\{O\}$ coinciding with $\{B\}$, we obtain (Bacconi *et al.*, 2004)

$$\omega^{bo} = \left[s_3\dot{\theta}_2 + c_2 c_3\dot{\theta}_1, c_3\dot{\theta}_2 - c_2 s_3\dot{\theta}_1, \dot{\theta}_3 + s_2\dot{\theta}_1\right] \tag{1}$$

For brevity we have replaced $\sin(\theta_i) = s_i$ and $\cos(\theta_i) = c_i$. Furthermore, also ω^o needs to be expressed in body coordinates. On this subject, notice that $\omega^o = \omega^c$. Hence,

$$\omega^o = \mathcal{R}^{bo}(\mathcal{R}^{oc} \cdot [0,\ 0,\ \omega_0]')$$

with $\mathcal{R}^{bo}$ and $\mathcal{R}^{oc}$ the two rotation matrices that lead $\{C\}$ coinciding with $\{O\}$ and then $\{O\}$ coinciding with $\{B\}$. Now, since both the velocity ω_0 of $\{C\}$ in ECI and the attitude of $\{O\}$ in $\{C\}$ are assumed constant and known, we can define the angular velocity of $\{C\}$ in $\{O\}$ coordinates as

$$\omega_R = \mathcal{R}^{oc} \cdot [0,\ 0,\ \omega_0]'$$

Consequently

$$\omega^o = \mathcal{R}^{bo}\omega_R = \mathcal{R}^{bo} \cdot [\omega_{R1},\ \omega_{R2},\ \omega_{R3}]'$$

It follows that ω^o corresponds to:

$$
\begin{bmatrix} c_2c_3\omega_{R1} + (c_1s_3 + s_1s_2c_3)\omega_{R2} + (s_1s_3 - c_1s_2c_3)\omega_{R3} \\ -c_2s_3\omega_{R1} + (c_1c_3 - s_1s_2s_3)\omega_{R1} + (s_1c_3 + c_1s_2s_3)\omega_{R3} \\ s_2\omega_{R1} - s_1c_2\omega_{R2} + c_1c_2\omega_{R3} \end{bmatrix} \tag{2}
$$

Finally, adding (1) and (2), taking the first derivative of ω^b and substituting into the well known Euler's equations, yields to a nonlinear model $\ddot{\theta} = f(\theta, \dot{\theta}, \omega_R, \tau)$. We do not assume small angular deviation between the formation center $\{C\}$ and $\{B\}$. On the other hand we assume small angles between $\{O\}$ and $\{B\}$ aiming at describing all the possible attitudes of a member of the fleet with a suitable set of frames $\{O_i\}$. As will be described in Sect. 5, it allows at addressing the whole problem in a hybrid framework. Thus, considering $\cos(\theta_i) \simeq 1$, $\sin(\theta_i) \simeq \theta_i$, and disregarding nonlinear terms, the Euler's equations become

$$
\begin{aligned}
\ddot{\theta}_1 =& (\omega_{R3}^2 - \omega_{R2}^2)J_1\theta_1 + \omega_{R3}(1 + J_1)\dot{\theta}_2 + \omega_{R1}\omega_{R2}J_1\theta_2 \\
&+ \omega_{R2}(J_1 - 1)\dot{\theta}_3 - \omega_{R1}\omega_{R3}J_1\theta_3 + \tau_1 + M_1 \\
\ddot{\theta}_2 =& \omega_{R3}(J_2 - 1)\dot{\theta}_1 - \omega_{R1}\omega_{R2}J_2\theta_1 + (\omega_{R1}^2 - \omega_{R3}^2)J_2\theta_2 \\
&+ \omega_{R1}(1 + J_2)\dot{\theta}_3 + \omega_{R2}\omega_{R3}J_2\theta_3 + \tau_2 + M_2 \\
\ddot{\theta}_3 =& \omega_{R2}(1 + J_3)\dot{\theta}_1 + \omega_{R1}\omega_{R3}J_3\theta_1 + \omega_{R1}(J_3 - 1)\dot{\theta}_2 \\
&- \omega_{R2}\omega_{R3}J_3\theta_2 + (\omega_{R2}^2 - \omega_{R1}^2)J_3\theta_3 + \tau_3 + M_3
\end{aligned} \tag{3}
$$

where $J_i = (I_j - I_k)/I_i$ $(i, j, k \in \underline{3},\ i \neq j \neq k$ and I_i are the principal moments of inertia of the spacecraft).

It can be rewritten in a more compact way, that will be useful in the following, as

$$
\dot{\theta} = \Phi_\theta \theta + G_\theta(\tau + M) \tag{4}
$$

introducing $\theta = [\dot{\theta}_1\ \theta_1\ \dot{\theta}_2\ \theta_2\ \dot{\theta}_3\ \theta_3]'$ as the state space vector corresponding to the Euler's angles, $\tau = [\tau_1\ \tau_2\ \tau_3]'$ as the control torques vector and $M = [M_1\ M_2\ M_3]'$ as the disturbance torques vector.

Next, we consider the position model of each member of the formation. We take into account fleets of micro-satellites in low orbits and close proximity. Thus, the motion of each spacecraft with respect to the formation center, can be described by the Hill's equations (Sidi, 1997; Tillerson *et al.*, 2002). They consist in the following linear model

$$
\begin{aligned}
m\ddot{p}_1 &= 3\omega_0^2 m p_1 + 2\omega_0 m\dot{p}_2 + f_1 + n_1 \\
m\ddot{p}_2 &= -2\omega_0 m\dot{p}_1 + f_2 + n_2 \\
m\ddot{p}_3 &= -\omega_0^2 m p_3 + f_3 + n_3
\end{aligned} \tag{5}
$$

where m is the mass of the satellite. For simplicity, acting as above, we summarize (5) in the state space equation

$$
\dot{p} = \Phi_p p + G_p(f + n) \tag{6}
$$

where $p = [\dot{p}_1\ p_1\ \dot{p}_2\ p_2\ \dot{p}_3\ p_3]'$ is the state space vector corresponding to the relative-coordinates of the satellite, $f = [f_1\ f_2\ f_3]'$ are the actuator forces acting along the positive axes directions and $n = [n_1\ n_2\ n_3]'$ are the components of disturbance forces.

It is worth pointing out that here, with the assumption of small displacements between $\{B\}$ and $\{O\}$, there is no difference in representing inputs and disturbances either in body or orbital coordinates. Moreover, we neglect the effects of components $p_1 \neq 0$ on the angular velocity. Hence, ω_0 is assumed constant.

3. PROBLEM FORMULATION

Combining equations (4) and (6) gives a linear time-continuous system:

$$
\dot{s} = A_c s + B_c(u + \xi) \tag{7}
$$

where $s = [p'\ \theta']' \in \mathbb{R}^{12}$, $u = [f'\ \tau']' \in \mathbb{R}^6$, $\xi = [n'\ M']' \in \mathbb{R}^6$ and matrices A_c and B_c direct consequence. Hence, with sampling period T, the ZOH sampled dynamical model of each satellite takes the form

$$
s(t+1) = As(t) + B(u(t) + d(t)) \tag{8}
$$

where $d(t)$ represents relative disturbance forces and torques accumulated during a sampling period.

The objective of the control problem is to drive each component of the fleet to a desired position and attitude defined in $\{C\}$ along a pre-specified path. Notice that it encompasses collision avoidance constraints. Further, maneuvers are made thanks to a combination of small jet actuators subject to input saturation constraints of the form

$$
|u_i(t)| \leq u_{i_{max}}, \quad i = 1, \cdots, 6 \tag{9}
$$

Furthermore, we want to handle formation accuracy constraints, i.e. state-related constraints

$$
|y_i(t) - r_i(t)| < \varepsilon, \quad i = 1, \cdots, 6 \tag{10}
$$

$\forall\ t \in \mathbb{Z}_+$, with r the reference signal and y some suitably selected output. Consequently, we propose a control strategy based on a Command Governor approach. It consists in designing a primal control law that does not take the constraints into account, and an external unit capable of taking care of constraints fulfillment by modifying, whenever necessary, the reference. Since the primal controller is designed for the linear model (8), any simple control strategy can be selected. In this paper we solve the unconstrained control problem using a simple linear quadratic LQ regulator (Mosca, 1995). Therefore, our attention can be focused on the closed-loop unconstrained system

$$
\begin{cases} x(t+1) = \Phi x(t) + Gr(t) + G_d d(t) \\ \quad\ y(t) = H_y x(t) \end{cases} \tag{11}
$$

4. COMMAND GOVERNOR APPROACH

The basis theory related to Command Governor is extensively described in (Bemporad *et al.*, 1997;

Casavola *et al.*, 2000; Albertoni *et al.*, 2003) and (Bacconi *et al.*, 2004). Briefly, the closed-loop state-space description of a plant regulated by a primal controller and CG unit is

$$\begin{cases} x(t+1) = \Phi x(t) + Gg(t) + G_d d(t) \\ y(t) = H_y x(t) \\ c(t) = H_c x(t) + Lg(t) + L_d d(t) \end{cases} \tag{12}$$

In particular, $x(t) \in \mathbb{R}^n$ is the state which includes plant and compensator states (if any), $g(t) \in \mathbb{R}^m$, which would be typically $g(t) = r(t)$ if no constraints were present (no CG present), is the CG output, viz. a suitably modified version of the reference signal $r(t) \in \mathbb{R}^m$. Moreover, $d(t) \in \mathbb{R}^{n_d}$ is an exogenous disturbance satisfying $d(t) \in \mathcal{D}$, $\forall\, t \in \mathbb{Z}_+$, with $\mathcal{D}$ a specified convex and compact set such that $0_{n_d} \in \mathcal{D}$, $y(t) \in \mathbb{R}^m$ is the output, viz. a performance related signal which is required to track $r(t)$ and $c(t) \in \mathbb{R}^{n_c}$ is the vector to be constrained.
The main idea of the CG technique is to choose at each time instant a constant *virtual command* $v(\cdot) \equiv w$, with minimal distance from the reference of value $r(t)$, such that the corresponding virtual evolution fulfill the constraints with a certain margin δ over a semi-infinite horizon. It can be summarized in the following problem: solve

$$g(t) = \arg \min_{w \in \mathcal{V}(x(t))} \|w - r(t)\|^2_\Psi \tag{13}$$

at any time instant. Here, $\Psi = \Psi' > 0_p$, $\|w\|^2_\Psi := x'\Psi x$ and $\mathcal{V}(x(t))$ is the set of signals w such that the virtual evolution of the system satisfy the constraints for any time instant. Such a command is applied, a new state is measured and the procedure is repeated.
It has been shown in (Bemporad *et al.*, 1997) that the problem is convex, the minimizer in (13) uniquely exists at each $t \in \mathbb{Z}_+$ and the overall system is asymptotically stable.
Moreover, in order to accommodate constraints in the form of (10) that transform the third equation in (12) in

$$c(t) = H_c x(t) + L_g g(t) + L_d d(t) + L_r r(t) \tag{14}$$

we introduced in (Bacconi *et al.*, 2004) a suitable parameterization of the reference trajectory

$$\alpha(t) = \alpha(t-1) + \Delta(t) \qquad \Delta(t) \in [0, 1] \tag{15}$$

with $r(\alpha(t))$ any point $\in \mathbb{R}^m$ between $r(t-1)$ and $r(t)$ along the reference trajectory. In particular, $g(t) = r(\alpha(t))$ corresponding to the nominal point $r(t)$ in the reference trajectory, when $\Delta(t) = 1$, i.e. no constraints present.
The new system, arising from (12) after the introduction of (15) is

$$\begin{cases} x(t+1) = \Phi x(t) + Gr(\alpha(t)) + G_d d(t) \\ y(t) = H_y x(t) \\ c(t) = H_c x(t) + Lr(\alpha(t)) + L_d d(t) \\ \alpha(t) = \alpha(t-1) + \Delta(t) \end{cases} \tag{16}$$

where $L = L_g + L_r$. Therefore, the CG problem is

$$\begin{aligned} \Delta(t) &:= \arg \max_{r(\alpha(t-1)+\Delta) \in \mathcal{V}(x(t))} \Delta \\ w(t) &:= r(\alpha(t-1) + \Delta(t)) \end{aligned} \tag{17}$$

and all the properties pertaining the CG approach described above are restored.
Finally, notice that for the problem at hand, attitude paths are defined in combination with position paths. Thus, the applied control law has to select the most restrictive Δ between the one resulting from the attitude maneuver and the one resulting from the position maneuver.

5. HYBRID COMMAND GOVERNOR

The assumption of small angles, that leads to system (3), might not match with some demanding applications. Experimental results show that the linearized models have significant discrepancy with respect to non linear models for deviations of Euler's angles larger than 10 degrees.
Hence, when the small-angles assumption does not hold, we propose the use of a bank of linearized models in the form of (3), each one representing the attitude relative to a specific reference frame $\{O_i\}$, described in $\{C\}$ by a rotation matrix $\mathcal{R}^{o_i c}$. The position components are not affected by this problem and just one single model can be used along with each member of the given set of attitude models.
Consequently, referring to each vehicle in the fleet, a single CG unit can be designed for each linearized model and a suitably designed supervisory unit can take care of orchestrating the switching among the CG candidates during the on-line operations. The overall technique il termed *hybrid CG control scheme* (HCG). A similar approach have been previously used in (Albertoni *et al.*, 2003). In order to provide a control law for every possible situation, it requires a correct definition of the linearized systems.
Consider the following set of reference set-points r which are desired to be tracked without offset

$$r \in \Xi \subset \mathbb{R}^m$$

Assume that $\Xi \not\subset \mathcal{W}^\delta$, where $\mathcal{W}^\delta$ is the set of signals w such that the steady state virtual evolution of the system satisfy the constraints with a margin δ (Bemporad *et al.*, 1997). Thus, the requirement that all set-points in Ξ will be tracked without error cannot be satisfied. A way to overcome this limitation is that of covering the set Ξ with a collection of $\mathcal{W}^\delta_i$, $i = 1, \cdots, l$ with overlapping interior corresponding to l different CGs such that

$$\Xi \subset \bigcup_{i=1}^{l} \mathcal{W}^\delta_i \tag{18}$$

and $\text{Interior}\{\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta\} \neq 0$, for at least a pair $(i,j) \in \{1,\dots l\}$. Clearly, CG_i operates properly when initial and final set-points belong to $\mathcal{W}_i^\delta$. If the final set-point belongs to a different set $\mathcal{W}_j^\delta$, a procedure for switching between CG_i and CG_j has to be defined. To this end, let us consider the output admissible set $\mathcal{Z}_i^\delta \subset \mathbb{R}^m \times \mathbb{R}^n$ for CG_i. It consists of the set of all pairs $[r,x]'$ whose evolutions satisfy the constraints for all $t \in \mathbb{Z}_+$. Hence, we can define the set of all states which can be steered to feasible equilibrium points without constraints violation

$$\mathcal{X}_j^\delta := \{x \in \mathbb{R}^n : \begin{bmatrix} w \\ x \end{bmatrix} \in \mathcal{Z}_i^\delta \text{ for at least one } w\}$$

Now, if (i,j) is such that $\text{Interior}\{\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta\} \neq 0$ then also $\text{Interior}\{\mathcal{X}_i^\delta \cap \mathcal{X}_j^\delta\} \neq 0$. Thus, one can a-priori define a convenient *transition reference* $r_{ij} \in \text{Interior}\{\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta\}$ such that $\overline{x}_{ij} \in \text{Interior}\{\mathcal{X}_i^\delta \cap \mathcal{X}_j^\delta\}$, where $\overline{x}_{ij}$ is the equilibrium disturbance-free steady-state corresponding to r_{ij} (using a worst case approach). Finally, $[r_{ij}, \overline{x}_{ij}]' \in \{\mathcal{Z}_i^\delta \cap \mathcal{Z}_j^\delta\}$ and the transfer strategy is simply defined. Assume to be at instant $\bar{t}$, be using CG_i and let $r(\bar{t}) \in \mathcal{W}_i^\delta$, $r(\bar{t}+1) \in \mathcal{W}_j^\delta$ with $\{\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta\} \neq 0$. Hence, a possible switching logic is as follows:

1. Solve and apply

$$g(\bar{t}+k) = \arg \min_{w \in \mathcal{V}_i(x(\bar{t}+k))} \|w - r(\bar{t})\|_\Psi^2,\ k = 1,\dots,\bar{k}$$

2. At $t = \bar{t} + \bar{k}$, as soon as

$$x(t) \in \text{Interior}\{\mathcal{X}_i^\delta \cap \mathcal{X}_j^\delta\} \tag{19}$$

switch to CG_j and solve

$$g(t) = \arg \min_{w \in \mathcal{V}_j(x(t))} \|w - r(\bar{t}+1)\|_\Psi^2,\ t \geq \bar{t}+1+\bar{k}$$

The illustrated scheme, inspired by (Gilbert and Kolmanovsky, 1999), is motivated by the fact that for any $x \in \mathbb{R}^n$ the state evolution will enter in $\text{Interior}\{\mathcal{X}_i^\delta \cap \mathcal{X}_j^\delta\}$ within a finite number of time instants.

Finally, we propose a criterion to select the CG_j at time instant $t = \bar{t} + \bar{k}$, based on the Euclidean norm between the state and the linearization point. The supervisor switches to model K_j centered in $\{O_j\}$ where CG_j corresponds to

$$j = \min_j \|\theta(\{O_j\}) - \theta(t)\| \tag{20}$$

An analysis of the properties of the proposed switching criterion is under development. Of course, other possibilities for the switching logic exist, which could be more effective for some applications.

6. SIMULATIONS

We refer to a reconfiguration maneuver as depicted in Fig. 2 regarding three vehicles orbiting around Earth at a distance of about 600 Km (LEO) at the velocity $\omega_0 = 0.0011\ rad/s$. They could form, as an example, an Earth Observing System with two slaves satellites pointing to a master, e.g. number III. For each micro-satellite

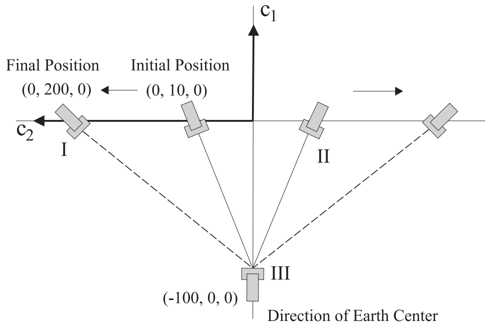

Fig. 2. Reconfiguration maneuver.

of the formation, we assume $m = 150\ Kg$ and $I = diag(35, 16, 25)\ Kg{\cdot}m^2$. Further the following saturation constraints are supposed: $|f|_{max} = 5 \cdot 10^{-2}N$, $|\tau|_{max} = 2 \cdot 10^{-3}Nm$. The maximum amplitudes of disturbances are assumed to be $|n|_{max} = 6 \cdot 10^{-3}N$ and $|M|_{max} = 2 \cdot 10^{-4}Nm$ and a value of $\delta = 10^{-4}$ is selected.

Besides saturation constraints we want to consider accuracy constraints $|p_i(t) - r_i(\alpha(t))| < 0.5\ m$ for each of the position components and $|\theta_i(t) - r_i(\alpha(t))| < 0.1\ deg$ for each of the attitude components.

The results of the application of the hybrid CG controller to the first satellite are illustrated in Fig. 3 and 4 (constraint boundaries in horizontal dash lines and instants of switching in vertical dash lines). Figures related to the second satellite

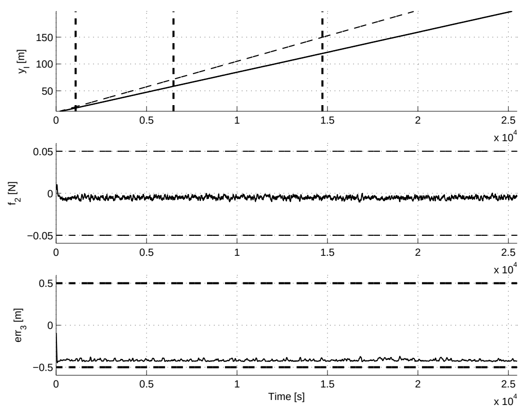

Fig. 3. Relative position, control force and error in coordination accuracy for $p_2(t)$ under HCG.

are mirror images. The constraints that influence the values of $\Delta(t)$ (17) are the ones related to the position accuracy. This is evident in Fig. 3 and Fig. 4 (bottom). Input forces and torques, on the contrary, take values close to zero during the entire reconfiguration. In the present simulation, a different linear model has been associated to the system, for Euler's angles of $(0, 0, 0)\ rad$,

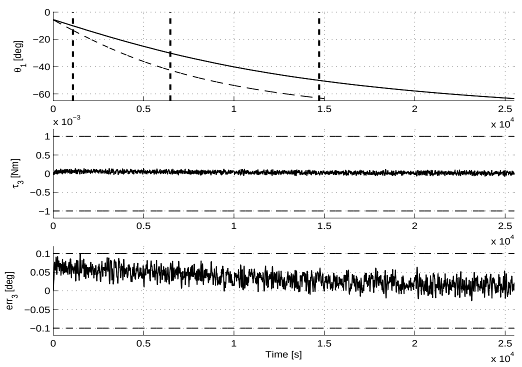

Fig. 4. Euler's angle, control torque and error in coordination accuracy for $\theta_3(t)$ under HCG.

$(0, 0, 20)$ rad, $(0, 0, 40)$ rad and so on.
Notice that, because of the use of the CG, the reference trajectory is tracked with a speed typically lower than the planned velocity. This is evident in Figs. 3 and 4.
If the hybrid CG scheme is not used, the formation accuracy constraints related to the position are not fulfilled.

7. CONCLUSIONS

The problem of reconfiguration for fleets of satellites subject to persistent disturbances and under input saturation and coordination accuracy constraints has been addressed.
The proposed control scheme is based on a bank of controllers composed by primal LQ control laws and nonlinear Command Governor units. The LQ controllers compensate the system without taking into account the presence of constraints whereas the CGs are used to generate a suitably modified version of the reference signal, capable of producing evolutions that fulfill the constraints at each time instant. A supervisory unit has been presented, capable of switching between the CG units of the bank, according to the attitude of the satellites.
Finally, the algorithm has been applied to reconfigure an Earth Observing System, forcing two satellites to cover a pre-specified trajectory defined with respect to a third one, fulfilling on it some accuracy constraints and preventing input saturation from occurring.

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