SIMULATION OF RAILWAY STATIONS BASED ON HYBRID PETRI NETS

Fateh KAAKAI[(1)(2)], Said HAYAT [(1)], Abdellah El MOUDNI [(2)]

(1) Institut National de Recherche sur les Transports et leur Sécurité (INRETS)
20, rue Elisée Reclus, BP : 317, F-59666 VILLENEUVE d'ASCQ Cedex FRANCE.
Phone : (+33) (0) 3 20 43 83 11 - Fax : (+33) (0) 3 20 43 83 59 - Email : said.hayat@inrets.fr

(2) Université de Technologie de Belfort-Montbéliard (UTBM)
Laboratoire Systèmes et Transports (S.e.T) Site de Belfort, 90010 BELFORT Cedex France
Phone : (+33) 3 84 58 33 42, Fax : (+33)3 84 58 33 42
Email : fateh.kaakai@utbm.fr, abdellah.elmoudni@utbm.fr

Abstract: The purpose of this paper is to present a simulation model for Railway Stations (RS). Firstly, through a systemic description, we have shown that a RS can be considered as a Hybrid Dynamical System (HDS). Then, in order to represent the main intrinsic characteristics of RS, we have built a hybrid Petri net-based model. This model has been analyzed for i) checking purposes, and ii) evaluating some qualitative properties of the system. Finally, for illustrating the suggested methodology of study, a typical subway station has been simulated through a rush period scenario. Owing to a quantitative analysis of the marking evolution of the hybrid model, it is possible to make a performance evaluation of the station facilities and to underline some structural and functional limits of the studied station.

Keywords: Transport, stations, hybrid, Petri-nets, model, performance evaluation, design.

1. INTRODUCTION

The standard EN 13816 published in 2002 by the European Committee for Standardization (ECS) and the Transit Capacity and Quality of service Manual [2] are some of the works which deal with the *service quality in public transportation networks.* According to these, the maximal density of travellers into public transports or on waiting/queuing areas of station platforms must not be greater than 4 persons per m² for comfort and security purposes.

Nevertheless, we can observe that some Railway Stations (RS), especially into busy multimodal hubs, don't respect this limit during rush hours. When the travelers' concentration becomes too much high – for instance, 670 000 travelers/day transit into of the biggest multimodal hubs of Paris ("Chatelet-Les-Halles") (Luquet, 1998) - the most fragile persons (children, old people, etc.) can be seriously affected by the movements of the crowd when the subway or the train arrives at the station: *jostles, discomforts, falls, trampling,* etc. Anyway, these dangerous situations deteriorate the service quality of public transports and make them less attractive.

In order to avoid serious accidents and respect the service quality standard, it is necessary to have an efficient tool able to evaluate the performances of existing stations and to design more precisely new secure facilities able to absorb these peaks of travelers during rush periods. Therefore, the first step is to well understand, thanks to an appropriate model, the *internal operation* of RS and the *behavior of this system* when peaks of travelers are applied as input data. Then, this model must be analyzed in order to establish formal methods able to evaluate structural and behavioral properties of the station facilities.

2. SYSTEMIC DESCRIPTION OF RAILWAY STATIONS

Railway stations are stop points of a railway transportation line where a whole of facilities is available in order to allow the parking of the public transport and the landing and the boarding of travelers.

2.1. Structural description

A "railway station" consists of (i) a platform with its input and output gates, and (ii) a public transport (train, subway, tram, etc.) when it is well parked at the platform level. The *platform* is characterized by its surface capacity C_{SMax}, the number of input (resp. output) gates N_{IG} (resp. N_{OG}) and the capacity of an input gate C_{IG} (resp. output gate C_{OG}) which corresponds to the maximal number of travelers who can simultaneously go through this gate.

The *public transport* is characterized by the number of its vehicles N_V, the maximal capacity of a vehicle C_{VMax} expressed in number of travellers, the number of landing-boarding gates per vehicle $N_{LBG/V}$, and the capacity of a landing-boarding gate C_{LBG}. The maximal capacity of the public transport, also expressed in number of travellers, is given by $C_{PTMax} = N_V \cdot C_{VMax}$.

2.2. Functional description

Firstly, we can observe that a railway station has a periodic operation as shown in Fig. 1. The operating period H_k of a station corresponds to the public transport headway, i.e. the time interval between two successive arrivals (at t_k and t_{k+1}) of the public transport at the considered station.

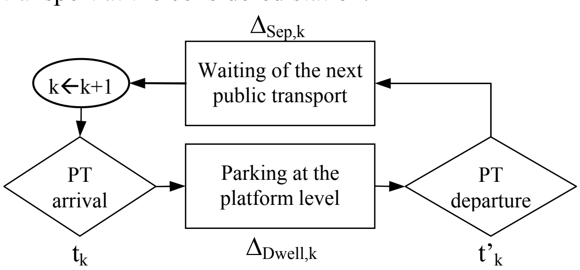

Fig. 1. Periodical operation of a railway station.

The duration of an operating period is given by (1):

$$\tau_k = t_{k+1} - t_k = \tau_{Dwell,k} + \tau_{Sep,k} \quad (1)$$

where $\tau_{Dwell,k}$ and $\tau_{Sep,k}$ are the duration of the subphases $\Delta_{Dwell,k}$ and $\Delta_{Sep,k}$ respectively (Fig. 1).

Consequently, it is sufficient to study this system on only one operating period $H_k = [t_k \;\; t_{k+1}[$ which will be identified in the rest of the paper by the index k.

Now, let us describe in details the successive steps (represented in Fig. 1 and Fig. 2) of an operating period H_k:

At $t = t_k$, the public transport arrives and parks into the station. It is characterized by an initial occupancy rate $\eta_{Busy,k} = C_{Busy,k}/C_{PTMax,k}$ and a landing rate $\eta_{Land,k} = C_{Land,k}/C_{Busy,k}$. The number of waiting travelers into the public transport who don't land (because they have another destination) is equal to (2):

$$C_{WTV,k} = [(1 - \eta_{Land,k}) \cdot \eta_{Busy,k} \cdot C_{PTMax,k}] \quad (2)$$

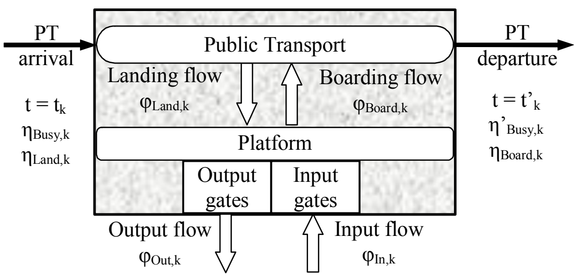

Fig. 2. Functional description of a railway station during an operating period H_k.

In the same way, $C_{WTP,k}$ denotes the initial number of travelers on the platform who are waiting in order to board into the public transport.

During the dwell time $\Delta_{Dwell,k} =]t_k , t'_k[$, an exchange of travelers takes place: $C_{Land,k} = (\eta_{Land,k} \cdot \eta_{Busy,k} \cdot C_{PTMax,k})$ travelers are going to land on the platform at a certain flow denoted $\varphi_{Land,k}$ [travelers / time unit] and then leave the station through the output gates at a certain flow $\varphi_{Out,k}$. At the same time, travelers enter into the station at a certain flow denoted $\varphi_{In,k}$ in order to board into the public transport at a certain flow $\varphi_{Board,k}$. Since the public transport has a limited capacity, the number of boarded travelers must respect the *maximal boarding constraint* (3) obtained by considering the free capacity of the public transport when the passengers' landing is completed:

$$C_{Board,k} \leq C_{BMax,k} \quad \text{with} \quad C_{BMax,k} = [1 - \eta_{Busy,k} \cdot (1 - \eta_{Land,k})] \cdot C_{PTMax,k} \quad (3)$$

At $t = t'_k$, the public transport leaves the station with a final occupancy rate $\eta'_{Busy,k} = C'_{Busy,k}/C_{Max,k}$ and a boarding rate $\eta_{Board,k} = C_{Board,k}/C'_{Busy,k}$.

Lastly, during the separation phase $\Delta_{Sep,k} =]t'_k , t_{k+1}[$ until the next public transport arrival at $t = t_{k+1}$, the station is only composed of the platform with its input and output gates. Thus, only two dynamics animate the system: the inflow of travelers ($\varphi_{In,k}$) and the outflow of travelers ($\varphi_{Out,k}$).

Table 1: Distinct groups of travelers into the station

Station	Groups of travelers	Abbrev.	Symbol
Public Transport (PT)	G_{T1} : travelers who are going to land on the platform	Landing travelers	Lng
	G_{T2} : travelers who have boarded into the PT	Boarded travelers	Bed
	G_{T3} : travelers who don't land because they have another destination	Waiting travelers	Wng
Platform	G_{P1} : travelers who have landed on the platform	Landed travelers	Led
	G_{P2} : travelers who are going to board into the PT	Boarding travelers	Bng

Four intrinsic dynamics which completely characterize the station operation have been identified: (i) *inflow* of travelers into the station, (ii) *boarding* of travelers into the public transport, (iii) *landing* of travelers on the platform, and (iv) *outflow* of travelers from the station. These different dynamics contribute to generate distinct groups of travelers into the public transport (3 groups) and on the platform (2 groups), as shown in Table 1.

2.3. Synthesis of the systemic description

Through this systemic description, we have observed that the operation of railway stations is governed by two kinds of phenomenon: (i) *external discrete events* corresponding to the successive arrivals and departures of the public transport (timetable), and (ii) *continuous dynamics* of the travelers into the station. Moreover, we can remark that some interactions exist between the discrete events and the continuous dynamics. For example, the public transport arrivals and departures control the start and the end of the landing and boarding flows of travelers. Thus, by definition, a railway station is a Hybrid Dynamical System (HDS). Furthermore, several intrinsic characteristics of this hybrid dynamical system have been described (see Fig. 2), notably the functional aspects of *synchronization* (of the public transport on the timetable), *parallelism* (of the flows of travellers), *resource sharing* and *limited capacities* of the public transport and the platform.

Among the several models dedicated to HDS (Antsaklis, *et al.*, 1996; Zaytoon, 2001; Engell, *et al.*, 2002), Hybrid Petri Nets constitute a suitable model for our study because they allow: (i) a graphical and modular representation of the different parts of the system, (ii) an easy and efficient modeling of the intrinsic characteristics quoted above, (iii) a qualitative study of the structural and behavioral properties of the model, and (iii) a quantitative performance evaluation which does not require an exhaustive enumeration of the state space.

3. MODELING

Firstly, we are going to precise the modeling framework of this paper. Then, we will present the suggested hybrid Petri net-based model. The basic concepts of hybrid Petri nets are assumed to be known to the reader. Otherwise, the reader may refer to (David, 1997; Alla and David, 1998; David and Alla, 2001).

3.1 Modeling framework

In this paper, we do not consider the disturbances which can affect the transportation network. Thus, the public transports are supposed to *respect* their timetables. We also consider that the landing and the boarding of travelers take place *simultaneously* (parallel approach) during the dwell phase ($\Delta_{Dwell,k}$) of the public transport at the platform level.

3.2 Hybrid Petri net of railway stations

The suggested hybrid Petri net-based model of railway stations is shown in Fig. 3. The Table 2 gives the initial markings of the discrete and continuous places and the expressions of the weights ($\neq 1$) associated to some arcs of the net. Below, a description of the nodes of the hybrid model is given.

A. Public transport arrivals and departures

T_{Arr} (resp. T_{Dep}) is a timed discrete transition which represents the successive public transport arrivals at the station (resp. the successive public transport departures from the station). The time $\tau_{Sep,k}$ associated with T_{Arr} corresponds to the separation time introduced in section 2.2 (see Fig. 1). The time $\tau_{Dwell,k}$ associated with T_{Dep} represents the duration of the public transport dwell phase (see Fig. 1). These times $\tau_{Sep,k}$ and $\tau_{Dwell,k}$ can be considered as constant or variable during the period of study of the station (hour, day, week, etc.).

P_{Sep} and P_{Dwell} are two discrete places which models the periodicity of the station operation. A token in P_{Sep} corresponds to the phase $\Delta_{Sep,k}$ of Fig. 1 whereas a token in P_{Dwell} corresponds to the phase $\Delta_{Dwell,k}$ of Fig. 1.

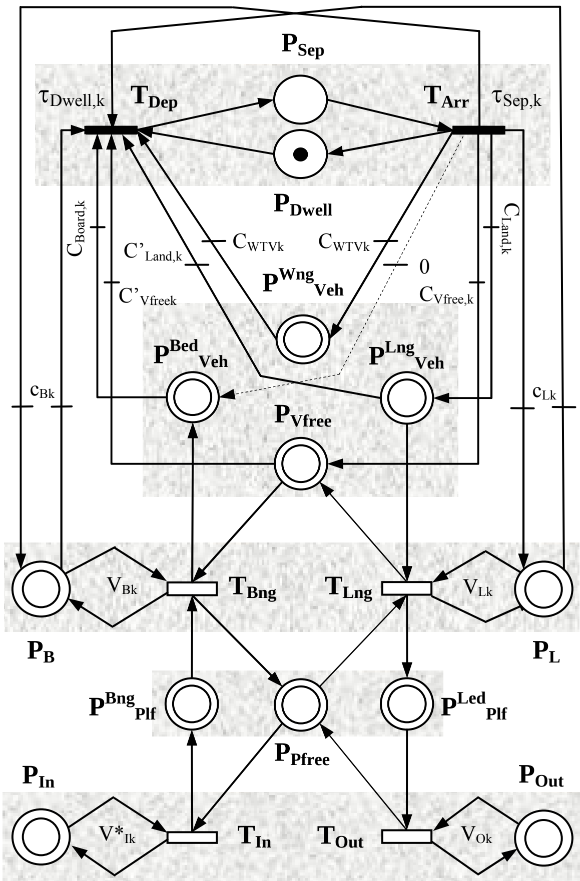

Fig. 3. Suggested hybrid Petri net of railway stations. The dotted arc between T_{Arr} and P^{Bed}_{Veh} has a weight equal to zero. It has been introduced in the net just for underlining the symmetry of the model.

Table 2 : Initial marking and weights at $t = t_k$

	Public Transport	Platform
Initial Marking	$m^{Lng}_{Veh} = \eta_{Land,k} \cdot C_{PTMax,k}$	$m^{Led}_{Plf} = 0$
	$m^{Bed}_{Veh} = 0$	$m^{Bng}_{Plf} = C_{WTPk}$
	$m^{Wng}_{Veh} = C_{WTVk} = C_{Busy,k} - C_{Land,k}$	$m_{Pfree} = C_{SMax} - C_{WTPk}$
	$m_{Vfree} = C_{PTMax,k} - C_{Busy,k}$	$m_{In} = N_{IG} \cdot N_{IT/G}$
	$m_L = m_B = ½ \cdot N_{Vk} \cdot N_{LBG/V} \cdot C_{LBG}$	$m_{Out} = N_{OG} \cdot N_{OT/G}$
	$m_{Sep} = 0$, $m_{Dwell} = 1$	
Weights	$C_{Land,k} = \eta_{Land,k} \cdot C_{PTMax,k}$	
	$C_{WTVk} = C_{Busy,k} - C_{Land,k}$	
	$C_{Board,k}$ = value of m^B_{Veh} at $t = t'_k$	
	$C_{Vfree,k} = C_{PTMax,k} - C_{Busy,k}$	
	$C'_{Vfree,k}$ = value of m_{Vfree} at $t = t'_k$	
	$C'_{Land,k}$ = value of m^L_{Veh} at $t = t'_k$	
	$C_{Lk} = C_{Bk} = ½ \cdot N_{Vk} \cdot N_{LBG/V} \cdot C_{LBG}$	

B. Capacity of the public transport

The global capacity of the public transport (continuous place P_{Veh} which doesn't appear in the net) can be divided into four subcapacities (four continuous subplaces) according to the distinct groups of travelers shown in Table 1 and such that:

$$\begin{cases} P_{Veh} = P_{Veh}^{Lng} \cup P_{Veh}^{Bed} \cup P_{Veh}^{Wng} \cup P_{Vfree} \\ P_{Veh}^{Lng} \cap P_{Veh}^{Bed} \cap P_{Veh}^{Wng} \cap P_{Vfree} = \{\emptyset\} \end{cases} \quad (4)$$

where

- P^{Lng}_{Veh} corresponds to the public transport capacity taken up by the landing travelers.
- P^{Bed}_{Veh} corresponds to the public transport capacity taken up by the boarded travelers.
- P^{Wng}_{Veh} corresponds to the public transport capacity taken up by the waiting travelers.
- P_{Vfree} is a continuous place which represents the free welcome capacity ($C_{Vfree,k}$) of the public transport expressed in number of travelers.

C. Gates of the public transport

T_{Ldg} (resp. T_{Bdg}) is a continuous transition which models the travelers' landing (resp. the travelers' boarding) through the public transport gates. The maximal firing speed of T_{Ldg} (resp. T_{Bdg}) is denoted V_{Lk} (resp. V_{Bk}).

P_L (resp. P_B) is a continuous place which represents the capacity of the whole landing (resp. boarding) gates of the public transport $C_{Lk} = ½ \cdot N_{Vk} \cdot N_{LBG/V} \cdot C_{LBG}$ (resp. $C_{Bk} = ½ \cdot N_{Vk} \cdot N_{LBG/V} \cdot C_{LBG}$).

D. Platform capacity

P_{Plf} is a continuous place which represents the station platform capacity; this global continuous place (which doesn't appear in the model) can be divided into three continuous subplaces according to Table 1 and such that:

$$\begin{cases} P_{Plf} = P_{Plf}^{Led} \cup P_{Plf}^{Bng} \cup P_{Pfree} \\ P_{Plf}^{Led} \cap P_{Plf}^{Bng} \cap P_{Pfree} = \{\emptyset\} \end{cases} \quad (5)$$

where

- P^{Led}_{Plf} corresponds to the platform surface taken up by the *landed travelers*.
- P^{Bng}_{Plf} corresponds to the platform surface taken up by the *boarding travelers*.
- P_{Pfree} is a continuous place corresponding to the free welcome surface (C_{Sfree}) of the platform.

E. Gates of the platform

T_{In} (resp. T_{Out}) is a continuous transition which models the mean inflow (resp. the outflow) of travelers into (resp. from) the station. The maximal firing speed of T_{In} (resp. T_{Out}) is denoted V_{Ik} (resp. V_{Ok}).

P_{In} (resp. P_{Out}) is a continuous place which represents the maximal capacity of the whole input gates: $C_I = N_{IG} \cdot C_{IG}$ (resp. output gates: $C_O = N_{OG} \cdot C_{OG}$, see section 2).

3.3 Firing speeds of the continuous transitions

Several algorithms have been proposed in the literature for computing the instantaneous speeds of continuous transitions (David and Alla, 2001). In this study, we will use the CCPN-based algorithm for which the firing speed vector of the HPN remains constant during each invariant behavior phase (i.e. time interval between two events which change the behavior of the model). For more details, see (Alla and David, 1998).

Concerning the maximal firing speeds, they often only depend on structural and/or operating parameters of the *studied system.* In our case, we have identified two kinds of parameters which limit the amplitude of the travelers' flows into the station: the capacities of the gates and the crossing time (τ_{Ck}) which is the time required by a traveler for going through a gate. The crossing time will only be studied according to the period nature: it will be *minimum* ($\tau_{CMin,k}$) during off-peak periods and *maximal* ($\tau_{CMax,k}$) for rush periods. Average values of $\tau_{CMin,k}$ and $\tau_{CMax,k}$ must be measured on the field.

Moreover, in order to consider the travelers' inflow variations during the day, especially the difference between rush hours and off-peak periods, a real random variable denoted $A^* \in [0\ 1]$ has been introduced at the level of the input transition T_{In}. This random coefficient will take, according to the period nature, the following suggested values:

Off-peak period $$A^* = r(1)/K_1 \quad (6)$$

Rush period $$A^* = K_2 + r(1 - K_2) \quad (7)$$

where

- r is a generator function of uniformly distributed random numbers such that $0 \leq r(x) \leq x$.
- K_1 and K_2 are two real numbers. A typical value for K_1 is $\sqrt{2}$ whereas K_2 must be comprised between 0 and 1.

The maximal firing speeds of the hybrid Petri net of Fig. 3 are given by (8), (9) and (10):

$$V_{Lk} = V_{Bk} = ½ \cdot N_{Vk} \cdot N_{LBG/V} \cdot C_{LBG} / \tau_{Ck} \quad (8)$$

$$V^*_{Ik} = A^* \cdot N_{IG} \cdot C_{IG} / \tau_{Ck} \quad (9)$$

$$V_{Ok} = N_{OG} \cdot C_{OG} / \tau_{Ck} \quad (10)$$

Now, we are going to analyze the hybrid Petri net for, on the one hand, checking the validity of the suggested model and its modeling framework (coherence of the assumptions on which the model is based) and on the other hand, finding some important qualitative properties of the modeled system.

4. ANALYSIS OF THE HYBRID PETRI NET

In order to prove that the assumptions on which is based the suggested hybrid model are coherent with the real operation of a station, we can at first verify the global conservation of the number of travelers (marks) into the model. In other terms, we have to prove the following implications:

$$\begin{aligned}&\forall t \in \Delta_{Dwell,k} = [t_k \quad t'_k[,\\ &(4) \Rightarrow m_{Veh}^{Lng} + m_{Veh}^{Bed} + m_{Veh}^{Wng} + m_{Vfree} = C_{PTMax,k}\end{aligned} \tag{11}$$

$$\begin{aligned}&\forall t \in H_k = [t_k \quad t_{k+1}[,\\ &(5) \Rightarrow m_{Plf}^{Led} + m_{Plf}^{Bng} + m_{Pfree} = C_{SMax}\end{aligned} \tag{12}$$

The equations (11) and (12) correspond to marking invariants which can be found owing to algebraic methods. Indeed, the marking invariants of a Petri net are the solutions of (13):

$$^{T}F \cdot W_k = 0 \tag{13}$$

with $^{T}F = (q_1, \cdots, q_7)$ a vector of positive integers and W_k the incidence matrix of the hybrid Petri net .
The details of the place invariant computation are not presented in this article since they are easy to develop. The linear combination of (11) and (12) allows concluding that the hybrid Petri net is *overall conservative* and thus *structurally bounded*. Another important issue for transportation systems consists in the research of deadlocks and the identification of states which induce deadlocks. Deadlocks can be identified when no movement of travelers (marks) is possible from a specific situation (given marking). For our model, we can identify a deadlock state which is reached when $m_{Vfree} = m_{Pfree} = 0$ at $t = t_k$. Indeed, if we consider a rush period scenario for which both the public transport and the station platform are initially saturated, then all continuous transitions are dead, and thus, the station is completely deadlocked. Fortunately, this specific situation is very rare in reality.

5. SIMULATION

We begin this section with a short recall of the simulation methodology of hybrid Petri nets. Then, the real data required by the simulation will be presented. However, most of the time, several real parameters are not available (even for public transport managers). Consequently, an artificial generation of data will be suggested and illustrated through a simulation scenario. Lastly, the simulation results will be analyzed for performance evaluation purposes.

5.1. Simulation methodology

The simulation of a hybrid Petri net is based on the determination of its Invariant Behavior States (IBS) owing to the fundamental equation (14). An IB-state is such that the marking of the discrete places and the instantaneous speed vector of the continuous transitions remain constant as long as the system is in the same IB-state.

$$\forall t \in IBS_j = [t_{ij} \quad t_{fj}[\tag{14}$$

$$m(t-t_{ij}) = m(0) + W_k \cdot \left(\sigma(t-t_{ij}) + \int_{t_{ij}}^{t_{fj}} v(u-t_{ij}) \cdot du \right)$$

In (14), t_{ij} and t_{fj} are the initial time and the final time of an IBS j. The vector $\boldsymbol{\sigma}(t-t_{ij})$ represents the firing number of each discrete transition between t_{ij} and $t \leq t_{fj}$ (the components of $\boldsymbol{\sigma}$ associated with continuous transitions are null). The vector $\mathbf{v}(u-t_{ij})$ symbolizes the instantaneous firing speeds associated with continuous transitions (the components of $\mathbf{v}$ associated with discrete transitions are null). For more details, see (David, 1997; Alla and David, 1998; David and Alla, 2001).

5.2 Required real data for the simulation

Table 3 and
Table 4 recapitulate the required data for the simulation. The following structural and functional parameters have been introduced during the systemic description and in the modeling section.

Table 3 : Characteristics of a subway station
(Mainly taken from http://www.metro-pole.net/)

	CHARACTERISTICS	EXAMPLES OF VALUES
PUBLIC TRANSPORT	Inter-arrival time (Headway H_k)	5 min.
	Dwell time ($\tau_{Dwell,k}$)	4 min.
	Numb. of vehicles of the PT (N_{Vk})	5
	Max. density of travelers (d_{VMax})	5 trav. / m²
	Max. capacity of a vehicle (C_{VMax})	200 trav.
	Number of gates per vehicle ($N_{LBG/V}$)	3
	Capacity of a gate (C_{LBG})	3 trav.
	Max. crossing time (τ_{CMax})	10 sec.
	Min. crossing time (τ_{CMin})	3 sec.
	Occupancy rate ($\eta_{Busy,k}$)	80 %
	Landing rate ($\eta_{Land,k}$)	65 %
PLATFORM	Surface (S_{Plf})	300 m²
	Max. density of travelers (d_{PMax})	5 trav. / m²
	Number of input gates (N_{IG})	2
	Capacity of an input gate (C_{IG})	4 trav.
	Number of output gates (N_{OG})	3
	Capacity of an output gate (C_{OG})	4 trav.
	Initial number of travelers (C_{WTPk})	200 trav.
	Max. crossing time (τ_{CMax})	10 sec.
	Min. crossing time (τ_{CMin})	3 sec.

Table 4 : Input data of the hybrid Petri net

DATA	MIN	MAX	UNITS
Landing and Boarding flows $V_{Lk} = V_{Bk}$	270	900	trav./min
Input flow V_{Ik}	48	160	trav./min
Output flow V_{Ok}	72	240	trav./min
Number of landing travelers ($C_{Land,k}$)	520		trav.
Maximal boarding constraint (C_{BMax})	720		trav.

5.3 Generation of artificial data

When some real data are not easy to find, a well known solution is to generate artificial data in order to fill in this lack of information. In our case, the occupancy rates ($\eta_{Busy,k}$) and the landing rates

($\eta_{Land,k}$) are two sets of statistical data which are not always available. That's why we suggest generating them using the following relations:

$$\text{Off-peak period} \quad \begin{cases} \eta^*_{Busy,k} = r(1)/B_1 \\ \eta^*_{Land,k} = r(1)/L_1 \end{cases} \qquad (15)$$

$$\text{Rush period} \quad \begin{cases} \eta^*_{Busy,k} = B_2 + r(1-B_2) \\ \eta^*_{Land,k} = L_2 + r(1-L_2) \end{cases} \qquad (16)$$

where

- r is a generator function of uniformly distributed random numbers such that $0 \le r(x) \le x$.
- B_1, B_2, L_1 and L_2 are four real values. A typical value for B_1 and B_2 is $\sqrt{2}$ whereas L_1 and L_2 must be comprised between 0 and 1.

5.4 Application

In order to illustrate the methodology of study presented in this paper, we have simulated (with MALAB) a typical station of the subway network of Paris during one hour and through a rush period scenario. We have used the simulation data presented in Table 3 and Table 4. Moreover, the parameters used for the generation of artificial occupancy and landing rates are: $B_1 = \sqrt{2}$, $B_2 = 0.9$, $L_1 = 1$ and $L_2 = 0.8$. We can observe on Fig. 4 for this rush period scenario that the public transport could become *saturated* before the end of the dwell time because of *an insufficient welcome capacity* (number of vehicles). In the same way, because of *successive accumulations* of travelers on the platform, which underline *an insufficient evacuation capacity* of the station (insufficient number of output gates for instance), the platform could reach its capacity and could remain saturated during the 8^{th} operating period (H_8 = [35 40] (min.)). These observations allow concluding that this station is not adapted to rush periods since its facilities are not able to absorb peaks of travelers.

6. CONCLUSION

The contributions of this work are: (i) the physical description of railway stations as being a hybrid dynamical system, (ii) the new modeling approach based on hybrid Petri nets, and (iii) the performance evaluation of the station facilities owing to a quantitative analysis of the marking evolution of the model. The authors also believe that this simulation model can be successfully used for designing the station facilities by controlling intrinsic parameters of the model.

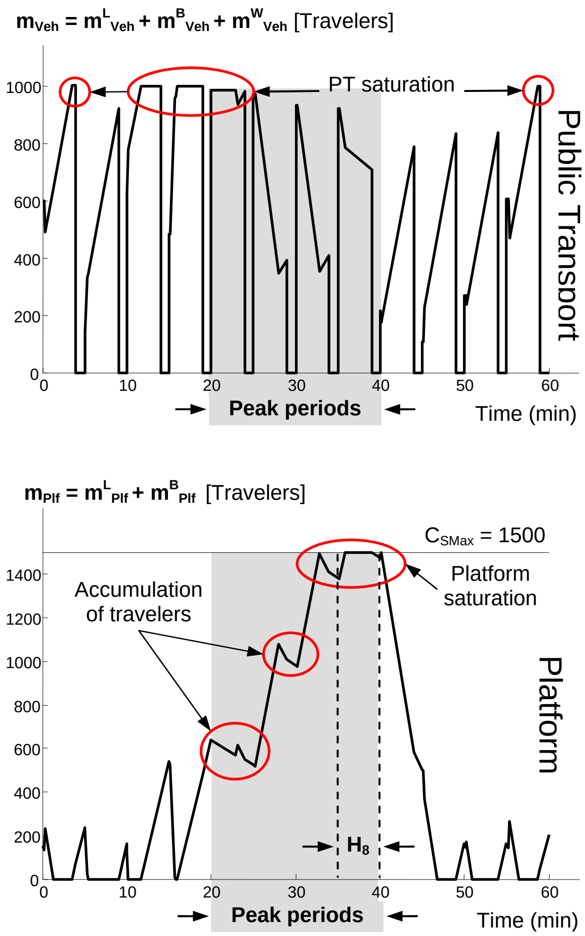

Fig. 4. Evolution of the total number of travelers into the public transport and on the platform for a rush period scenario.

REFERENCES

Alla, H. and R. David (1998). Modelling and analysis tool for discrete events systems: continuous Petri net. *Performance Evaluation*, v 33, n 3, pp. 175-199.

Antsaklis, P., X. Koutsoukos and J. Zaytoon (1996). On hybrid control of hybrid systems: a survey. *JESA*, vol. 32, no 9-10, pp. 1023-1045.

David, R. (1997). Modeling of hybrid systems using continuous and hybrid Petri nets. *International Workshop on Petri Nets and Performance Models*, pp. 47-58.

David, R. and H. Alla (2001). On Hybrid Petri Nets. *Journal of Discrete Event Dynamic Systems: Theory and Applications*, **Vol. 11**, pp. 9-40.

ECS, http://www.cenorm.be/cenorm/index.htm

Engell, S., G. Frehse and E. Schnieder (2002) *Modelling, Analysis, and Design of Hybrid Systems*. LNCIS, Ed. Springer-Verlag, Germany.

Kaakai, F., S. Hayat and A. El Moudni (2005). Formal design of public transport stations owing to Hybrid Event Graphs", in Proc. of IEEE Intelligent Transportation Systems Conf., Vienne.

Kittelson & Associates *et al.*, Transit *Capacity and Quality of service Manual* [Online]. Available: http://www4.trb.org/trb/crp.nsf/All+Projects/TCRP+A-15, 1^{nd} edition, 2003.

Luquet, F. (1998). Multimodality: urban hub management. *Revue RATP Savoir-Faire*, N° 26, pp. 31-35.

Murata, T. (1989). Petri Nets: Properties, analysis and applications. *Proc. IEEE*, vol. 77, pp. 541–580.

Zaytoon J. (2001). Systèmes dynamiques hybrides. Traité IC2. (Hermès (Ed)), Paris.