MODELLING DISTRIBUTED MANUFACTURING SYSTEMS VIA FIRST ORDER HYBRID PETRI NETS

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Abstract: A Distributed Manufacturing System (DMS) is a collection of independent companies possessing complementary skills and integrated with transportation and storage systems. This paper proposes a new model for DMS employing first order hybrid Petri nets, i.e., Petri nets based on first order fluid approximation. More precisely, transporters and manufacturers are described by continuous transitions, buffers are continuous places and products are represented by continuous flows routing from manufacturers, buffers and transporters. Moreover, discrete events occurring stochastically in the system are considered to take into account the start of the retailer requests and the blocking of transports and raw material supply. With the aim of showing the model effectiveness, a DMS example is modelled and simulated under two different operative conditions. *Copyright © 2006 IFAC*

Keywords: manufacturing systems, Petri-nets, dynamic models, simulation, performance indices.

1. INTRODUCTION

A Distributed Manufacturing System (DMS) is a collection of independent companies possessing complementary skills and integrated with transportation and storage systems (Viswanadham and Raghavan 2000). Appropriate modelling and analysis of these highly complex systems are crucial for performance evaluation and comparison of competing DMS. However, few contributions face the problem of modelling the DMS in order to analyze the system performance measures and to optimize its functional objectives. Viswanadham and Raghavan (2000) model the system as a Discrete Event Dynamical System (DEDS), in which the evolution depends on the interaction of discrete events. Generalized Stochastic Petri Nets (GSPN) model a particular example of SC and determine the decoupling point location, i.e., the facility from which all finished goods are assembled after customer order confirmation. Moreover, in (Desrochers *et al.* 2005) a two product SC is modelled by complex-valued token Petri nets and the performance measures are determined by simulation. In addition, Dotoli and Fanti (2005) propose a GSPN model in order to describe in a modular way a generic DMS. However, the limit of this formalism is the modelling of products by means of discrete quantities (i.e., tokens). This assumption is not realistic in large systems with a huge amount of material flow. Since DMS are DEDS whose number of reachable states is very large, approximating fluid models can be used in this context as in manufacturing systems.

The aim of the paper is to propose a new model for DMS employing First Order Hybrid Petri Nets (FOHPN, Balduzzi *et al.* 2000), that include continuous places holding fluid and discrete places containing a non-negative integer number of tokens and transitions, where the latter are either discrete or continuous. Such a hybrid Petri net model is based on the framework proposed by Alla and David (1998) and presents the main key feature of allowing the Instantaneous Firing Speeds (IFS) of the continuous transitions to be chosen in a given range by a control agent. Moreover, the set of all admissible IFS vectors is explicitly characterized by the feasible solutions of a linear constraint set. Furthermore, an optimal IFS vector can be chosen according to a given objective function. Using such a modelling approach, this paper develops an FOHPN model of DMS by means of first-order fluid approximations. In particular, transporters and manufacturers are described by continuous transitions, buffers are continuous places and products are represented by continuous flows (fluids) routing from manufacturers, buffers and transporters. The model is built by using a modular approach based on the idea of the bottom-up methodology (Zhou and Venkatesh, 1998). A representative example, including the typical DMS elements, shows the effectiveness of the modelling technique that allows us to evaluate the system performance indices by the simulation.

The paper is structured as follows. Section 2 describes the structure and the dynamics of a generic DMS. Section 3 reports a brief overview of the FOHPN modelling formalism and Section 4 presents the modular DMS model. Section 5 describes and analyzes an example of DMS and a conclusion section closes the paper.

2. THE SYSTEM DESCRIPTION

A DMS may be described as a set of facilities with materials that flow from the sources of raw materials to subassembly producers and onwards to manufacturers and consumers of finished products. The DMS facilities are connected by transporters of materials, semi-finished goods and finished products. More precisely, the entities of a DMS can be summarized as follows.

1- *Suppliers:* a supplier is a facility that provides raw materials, components and semi-finished products to manufacturers that make use of them.
2- *Manufacturers and assemblers*: manufacturers and assemblers are facilities that transform input raw materials/components into desired output products.
3- *Logistics and transporters*: storage systems and transporters play a critical role in distributed manufacturing. The attributes of logistics facilities are storage and handling capacities, transportation times, operation and inventory costs.
4- *Retailers or customers*: retailers or customers are sink nodes of material flows.

Here, part of the logistics, such as storage buffers, is considered pertaining to manufacturers, suppliers and customers. Moreover, transporters connect the different stages of the production process.

The dynamics of the distributed production system is traced by the flow of products between facilities and transporters. Because of the large amount of material flowing in the system, we model a DMS as a hybrid system: the continuous dynamics models the flow of products in the DMS, the manufacturing and the assembling of different products and its storage in appropriate buffers. Hence, resources with limited capacities are represented by continuous states describing the amount of fluid material that the resource stores.

Moreover, we consider also discrete events occurring stochastically in the system, such as:

a) the blocking of the raw material supply, e.g. modelling the occurrence of labour strikes, accidents or stops due to the shifts;
b) the blocking of the transport operations due to the shifts or to unpredictable events such as jamming of transportation routes, accidents, strikes of transporters etc.;
c) the start of a request from the retailers.

3. FIRST-ORDER HYBRID PETRI NETS

3.1 The net structure and marking.

This section recalls the First Order Hybrid Petri Nets (FOHPN) formalism used in the following (Balduzzi *et al.* 2000).

A FOHPN is a bipartite digraph described by the six-tuple $PN=(P, T, Pre, Post, D, F)$. The set of places $P=P_d \cup P_c$ is partitioned into a set of discrete places P_d (represented by circles) and a set of continuous places (represented by double circles).

The set of transitions $T=T_d \cup T_c$ is partitioned into a set of discrete transitions T_d and a set of continuous transitions T_c (represented by double boxes). Moreover, the set of discrete transitions $T_d=T_I \cup T_E$ is further partitioned into a set of immediate transitions T_I (represented by bars) and a set of exponentially distributed transitions T_E (represented by boxes).

The matrices *Pre* and *Post* are the pre-incidence and the post-incidence matrices, respectively, of dimension $|P| \times |T|$. Note that the symbol $|A|$ denotes the cardinality of set A. Such matrices specify the net digraph arcs and are defined as follows:

$$Pre, Post : \begin{cases} P_c \times T \rightarrow \mathbb{R}^+ \\ P_d \times T \rightarrow \mathbb{N} \end{cases}.$$

We require that for all $t \in T_c$ and for all $p \in P_d$ it holds $Pre(p,t)=Post(p,t)$ (*well-formed nets*).

The function D: $T_t \rightarrow R^+$ specifies the timing associated to exponentially distributed timed transition $t_j \in T_E$. More precisely, we associate to each $t_j \in T_E$ the average firing delay $\delta_j=D(t_j)$. Moreover, the function F : $T_c \rightarrow R^+ \times R_\infty^+$ specifies the firing speeds associated to continuous transitions (we denote $R_\infty^+=R^+ \cup \{\infty\}$). For any continuous transition $t_j \in T_c$ we let $F(t_j)=(V_{mj}, V_{Mj})$, with $V_{mj} \leq V_{Mj}$. Here, V_{mj} represents the minimum firing speed (mfs) and V_{Mj} the Maximum Firing Speed (MFS) of the generic continuous transition.

Given a FOHPN and a transition $t \in T$, the following sets of places may be defined: $\bullet t=\{p \in P: Pre(p,t)>0\}$, named pre-set of t; $t\bullet=\{p \in P: Post(p,t)>0\}$, named post-set of t. Moreover, the corresponding restrictions to continuous or discrete places are defined as $^{(d)}t=\bullet t \cap P_d$ or $^{(c)}t=\bullet t \cap P_c$. Similar notations may be used for pre-sets and post-sets of places. The incidence matrix of the net is defined as $\mathbf{C}(p,t)=Post(p,t)-Pre(p,t)$. The restriction of $\mathbf{C}$ to P_X and T_X (with X,Y$\in$ {c, d}) is denoted by $\mathbf{C}_{XY}$.

A marking

$$\mathbf{m} : \begin{cases} P_d \rightarrow \mathbb{N} \\ P_c \rightarrow \mathbb{R}^+ \end{cases}$$

is a function that assigns to each discrete place a non-negative number of tokens, represented by black dots, and to each continuous place a fluid volume; m_i denotes the marking of place p_i. The value of a marking at time τ is denoted by $\mathbf{m}(\tau)$. The restriction of $\mathbf{m}$ to P_d and P_c are denoted by $\mathbf{m}^d$ and $\mathbf{m}^c$, respectively. A FOHPN system $<PN,\mathbf{m}(\tau_0)>$ is a FOHPN with initial marking $\mathbf{m}(\tau_0)$.

The following statements rule the firing of the continuous and discrete transitions:

1- a discrete transition $t\in T_d$ is enabled at $\mathbf{m}$ if for all $p_i\in \bullet t$, $m_i \geq Pre(p_i,t)$;

2- a continuous transition $t\in T_c$ is enabled at $\mathbf{m}$ if for all $p_i\in {}^{(d)}t$, $m_i \geq Pre(p_i,t)$.

Moreover, we say that an enabled transition $t\in T_c$ is strongly enabled at $\mathbf{m}$ if for all places $p_i\in {}^{(c)}t$, $m_i>0$; we say that transition $t\in T_C$ is weakly enabled at $\mathbf{m}$ if for some $p_i\in {}^{(c)}t$, $m_i=0$.

In addition, if $<PN,\mathbf{m}>$ is an FOHPN system and $t_j\in T_c$ with instantaneous firing speed (IFS) v_j, it holds:

1- if t_j is not enabled then $v_i=0$;

2- if t_i is strongly enabled, then it may fire with any firing speed $v_j\in[V_{mj},V_{Mj}]$;

3- if t_j is weakly enabled, then it may fire with any firing speed $v_j\in[V_{mj},V_j]$, where $V_j\leq V_{Mj}$ depends on the amount of fluid entering the empty input continuous place of t_i.

We denote by $\mathbf{v}(\tau)\in[v_1(\tau)\ v_2(\tau)\ldots\ v_{|Tc|}(\tau)]^T$ the IFS vector at time τ. Any admissible IFS vector $\mathbf{v}$ at $\mathbf{m}$ is a feasible solution of the following linear set:

$$\begin{aligned}
&V_{Mj}-v_j\geq 0 && \forall t_j\in T_\varepsilon(\mathbf{m})\\
&v_j-V_{mj}\geq 0 && \forall t_j\in T_\varepsilon(\mathbf{m})\\
&v_j=0 && \forall t_j\in T_\upsilon(\mathbf{m})\\
&\sum_{t_j\in T_\varepsilon} C(p,t_j)v_j\geq 0\ \forall p\in P_\varepsilon(\mathbf{m}),
\end{aligned} \tag{1}$$

where $T_\varepsilon(\mathbf{m})\subset T_c$ ($T_\upsilon(\mathbf{m})\subset T_c$) is the subset of continuous transitions that are enabled (not enabled) at $\mathbf{m}$ and $P_\varepsilon(\mathbf{m})=\{p_i\in P_c \mid m_i=0\}$ is the subset of empty continuous places.

3.2 The net dynamics.

The hybrid dynamics of the net combines both time-driven and event-driven dynamics. We define *macro events* the events that occur when: i) a discrete transition fires or the enabling/disabling of a continuous transition takes place; ii) a continuous place becomes empty.

The equation that governs the time-driven evolution of the marking of a place $p_i\in P_C$ is:

$$\dot{m}_i(\tau)=\sum_{t_j\in T_C} C(p_i,t_j)v_j(\tau)\,. \tag{2}$$

Now, if τ_k and τ_{k+1} are the occurrence times of two macro-events, we assume that within the time interval $[\tau_k,\tau_{k+1})$ (macro period) the IFS vector $v(\tau_k)$ is constant. Then the continuous behaviour of an FOHPN for $\tau\in[\tau_k,\tau_{k+1})$ is described by:

$$\begin{aligned}
&\mathbf{m^c}(\tau)=\mathbf{m^c}(\tau_k)+\mathbf{C_{cc}}\mathbf{v}(\tau_k)(\tau-\tau_k)\\
&\mathbf{m^d}(\tau)=\mathbf{m^d}(\tau_k)
\end{aligned}\ . \tag{3}$$

The evolution of the net at the occurrence of the macro-events is described by:

$$\begin{aligned}
&\mathbf{m^c}(\tau_k)=\mathbf{m^c}(\tau_k^-)+\mathbf{C_{cd}}\sigma(\tau_k)\\
&\mathbf{m^d}(\tau_k)=\mathbf{m^d}(\tau_k^-)+\mathbf{C_{dd}}\sigma(\tau_k)
\end{aligned}, \tag{4}$$

where $\sigma(\tau)$ is the firing count vector associated to the firing of the discrete transition t_j.

4. THE MODULAR DMS MODEL

Petri net modelling and synthesis is a very important research area that attracted much attention in the past (Zhou and Venkatesh, 1998). Based on the idea of the bottom-up approach, this section proposes a modular FOHPN model to describe a DMS. Such a method can be summarized in two steps: decomposition and composition. Decomposition involves dividing a system into several subsystems. In DMS this division can be performed based on the determination of distributed system facilities (i.e., suppliers, manufacturers, transporters and customers). All these subsystems are modelled by FOHPN. On the other hand, composition involves the interacting of these sub-models into a complete model, representing the whole DMS. The following FOHPN modules model each individual subsystem composing the DMS.

1- *The supplier module.* The supplier is modelled as a continuous transition and two continuous places (see Fig 1). The continuous place p_S represents the row material buffer of finite capacity C_S and p'_S represents the corresponding available capacity such that $m_S+m'_S=C_S$. Moreover, the continuous transition t_S models the arrival of raw material in the system at a bounded rate v_S with $F(t_S)=(V_{Smin},V_{Smax})$. In addition, we consider the possibility that the providing of raw material is blocked. This situation is represented by a discrete event modelled by two exponentially distributed transitions and two discrete places. In particular, place $p_k\in P_d$ models the operative state of the supplier and $p'_k\in P_d$ is the non-operative state (see Fig 1). The blocking and the restoration of the raw material supply corresponds to the firing of transitions t_k and t'_k, respectively. For the sake of clarity, Fig. 1 depicts the transition t_T modelling the transport operation.

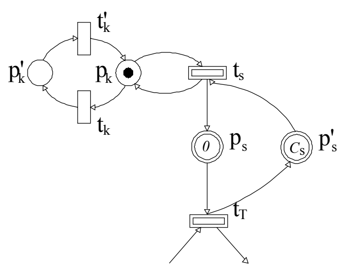

Fig. 1. The FOHPN modelling the supplier.

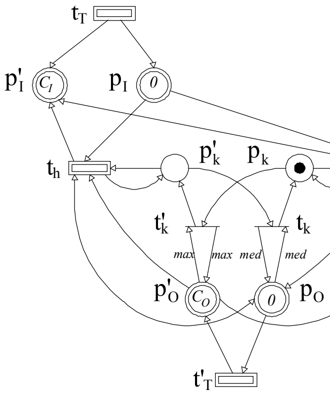

Fig. 2. The FOHPN modelling the manufacturer and the assembler.

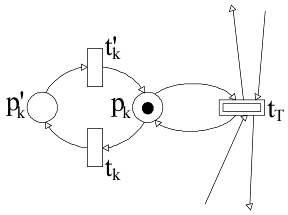

Fig. 3. The FOHPN modelling logistics.

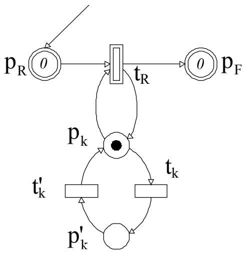

Fig. 4. The FOHPN modelling the retailer.

2- *The manufacturer and assembler module.* The manufacturers and assemblers are modelled by the FOHPN shown by Fig. 2. More precisely, place p_I is the input buffer of finite capacity C_I storing the input goods of a particular type. The corresponding place p'_I models the available buffer space so that $m_I+m'_I=C_I$. Analogously, the output buffer of capacity C_O storing the output product of a particular type is modelled by the place p_O representing the occupied buffer level and by place p'_O modelling the corresponding available capacity, with $m_O+m'_O=C_O$. To manage the production in function of the output inventory of the product, we assume that the production rate of the facility changes in function of the available space in the output buffer. Indeed, two different rates are considered for the production: a nominal rate $F(t_n)=(0,V_{mn})$ associated with the continuous transition t_n and a high rate $F(t_h)=(V_{mn},V_{Mh})$ associated with the continuous transition t_h. If it holds $m'_O(\tau)<max$ (i.e., the inventory is high enough and the buffer free space is not too high) then transition t'_k is disabled and the manufacturer works at the nominal rate with m_k=1 and $m_{k'}$=0 with $p_k,p_{k'}\in P_d$ as depicted in Fig. 2. On the contrary, when the buffer level of output parts decreases and hence the available buffer space reaches the maximum value $m'_O(\tau)=max$, the immediate transition t'_k is enabled and can fire. After the firing of t'_k, it holds m_k=0 and $m_{k'}$=1, so that the facility works at the higher rate associated with the continuous transition t_h and the output buffer p_O is replenished more rapidly. Analogously, if the level of this buffer reaches the medium value $m_O(\tau)=med$, then the transition t_k is enabled and can fire, leading the facility rate at the nominal value. Note that transitions t_T and t'_T of Fig. 2 depicts the transports facilities.

3- *The logistics module.* The transporters connecting the different facilities are modelled by a continuous transition t_T that describes the flow of material from a facility to a subsequent one at a bounded rate $0\leq v_T\leq V_{MT}$. Moreover, the random stop of the material transport is represented by two places $p_k,p'_k\in P_d$ and two exponentially distributed transitions $t_k,t'_k\in T_d$. If place $p_k\in P_d$ is marked, then the transport is operative. On the contrary, if transition t_k fires, then the transporters are not operative and the place $p'_k\in P_d$ becomes marked. When transition t'_k fires, the transporters are established again.

4- *The retailer module.* Finally, we consider the model of the retailers, represented by a continuous buffer place p_R of infinite capacity associated with each final product type. Moreover, a continuous transition t_R models the acquisition of final products by the retailer. However, we consider that the requests of the products are managed by discrete random events expressed by two discrete timed transitions t_k and t'_k and two places p_k and p'_k. If m_k=1 then the flow of material is enabled. On the contrary, if transition t_k fires, then a token in p'_k means that the retailer does not require any product and t_R is inhibited. The continuous place p_F models the system output and collects all the products requested by the retailer. Figure 4 depicts the retailer module.

5. AN EXAMPLE OF DMS

We consider a system producing two product types E and F by the four stages depicted in Fig. 5. The first stage includes two component suppliers S1 and S2, the second stage is composed by two subassembly manufacturers M1 and M2, the third stage is composed by two assemblers A1 and A2 and the last stage is constituted by the retailers R1 and R2. Moreover, six logistics service providers T1 to T6 suitably connect the DMS facilities that are located in different geographical sites. The buyers order two brands of products (E and F). Such products are obtained by two assemblers that assemble two types of products (C and D) obtained from two manufacturers. The subassemblies C and D are in turn produced by the manufacturers of the second stage, which receives the components of type A and B by the first supplier stage.

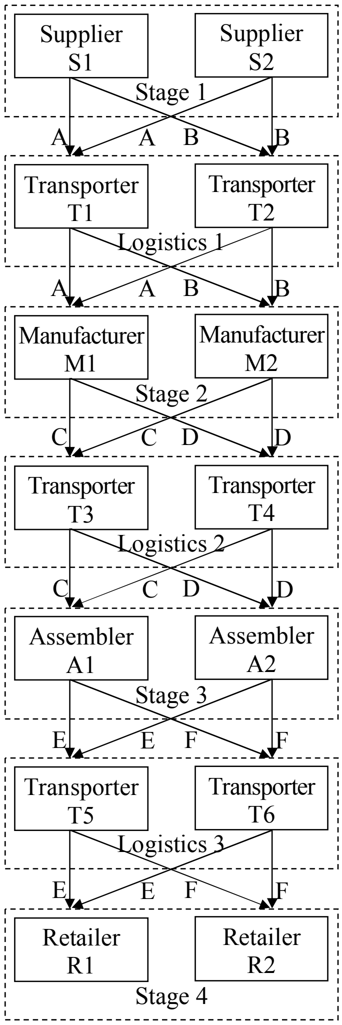

Fig. 5: The DMS configuration.

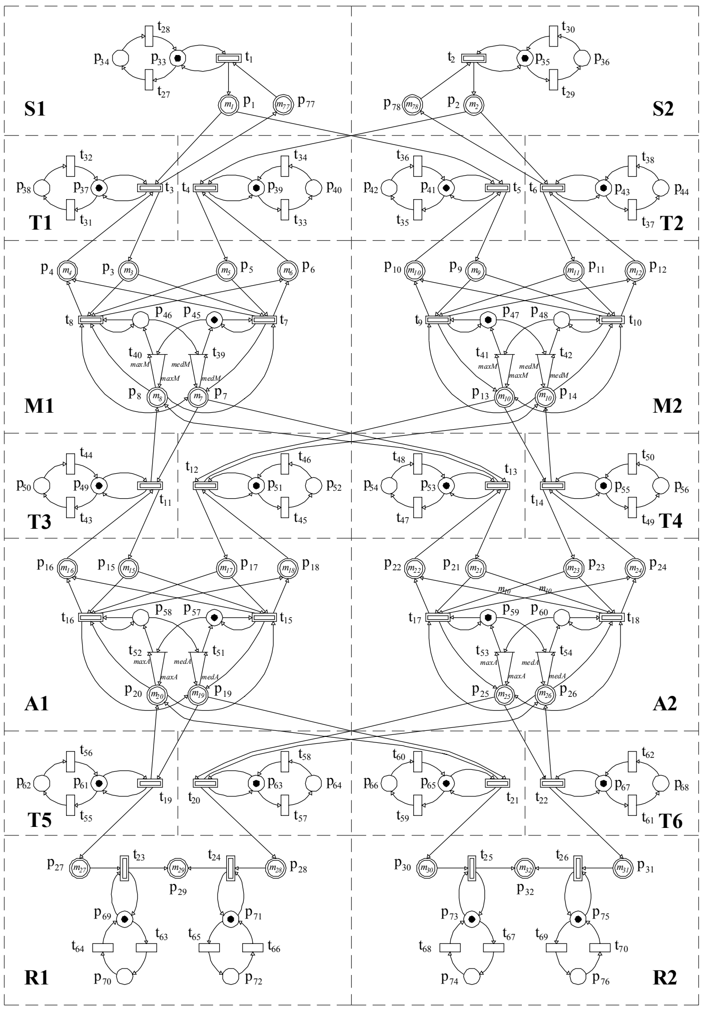

Fig. 6: The DMS model.

Table 1: Firing speeds and average firing delay of continuous and discrete transitions for Cases 1 and 2.

Continuous transitions	$[V_{min}, V_{max}]$ (Case 1)	$[V_{min}, V_{max}]$ (Case 2)	Discrete transitions	Average firing delay (hours)
t_1	[0, 8]	[0, 4]	t_{27}	14
t_2	[0, 10]	[0, 5]	t_{29}	14
$t_3\ t_6\ t_{14}\ t_{22}$	[0, 10]	[0, 5]	t_{28}	10
$t_4\ t_7\ t_{13}\ t_{17}\ t_{19}$	[0,8]	[0, 3]	t_{30}	10
$t_5\ t_{12}\ t_{20}$	[0,9]	[0, 4]	$t_{31}\ t_{37}$	9
t_8	[8,20]	[0, 6]	$t_{47}\ t_{49}$	9
t_9	[0, 12]	[0, 2]	$t_{55}\ t_{57}$	9
t_{10}	[12,30]	[0, 6]	$t_{32}\ t_{38}$	15
$t_{11}\ t_{21}$	[0, 11]	[0, 3]	$t_{48}\ t_{50}$	15
t_{15}	[0, 6]	[4, 15]	$t_{56}\ t_{58}$	15
t_{16}	[6, 15]	[0, 4]	$t_{33}\ t_{35}$	10
t_{17}	[0, 8]	[4, 12]	$t_{43}\ t_{45}$	10
t_{18}	[8, 20]	[0, 3]	$t_{59}\ t_{61}$	10
t_{23}	[0, 3]	[0, 4]	$t_{34}\ t_{36}$	14
$t_{24}\ t_{25}$	[0, 4]	[4, 10]	$t_{44}\ t_{46}$	14
t_{26}	[0, 5]	[0, 6]	$t_{60}\ t_{62}$	14

We model the DMS in Fig. 5 by using in a modular way the elementary modules described in Section 3. Figure 6 shows the merged FOHPN model and depicts each facility module in dashed line squares.

To analyze the DMS behaviour, we simulate the FOHPN in two different cases that correspond to two different operative conditions. The data relative to Case 1 and 2 are shown in Table 1. In particular, Case 1 corresponds to a system with high production rate of each manufacturer and assembler. On the other hand, in Case 2 manufacturers and assemblers exhibit lower production rates than in Case 1 but the same transportation speed and buffer capacities. Moreover, Table 1 reports for each $t_j \in T_c$ the minimum and maximum firing speeds and for each $t_j \in T_E$ the average firing delay. In addition, Table 2 shows for Case 1 and Case 2 further data necessary to fully describe and simulate the system: the buffer capacities for the inventories of each stage, the initial markings of continuous places and the values of the edge weights.

To analyze the system dynamics, we define some performance indices assumed as relevant measures for the DMS analysis: i) the throughput T_i with i=1,2 of retailer Ri with i=1,2 respectively, i.e., the average number of products obtained by each retailer in a time unit; ii) the system throughput $T=T_1+T_2$; iii) the average input stocks in manufacturers Mi with i=1,2 (I_{Mi} i=1,2) and in assemblers Ai with i=1,2 (I_{Ai} i=1,2) during the run time TP; iv) the average output inventory in Mi with i=1,2 (O_{Mi} i=1,2) and in Ai with i=1,2 (O_{Ai} i=1,2) during the run time TP; v) the average system inventory SI, i.e., the amount of product storage in each buffer during the run time TP; vi) the average lead time LT that is as follows:

$$LT=SI/T \quad (5)$$

The results are obtained by simulating the FOHPN in the Matlab environment for a simulation run of TP=480 hours. In particular, Fig. 7 shows the average inventories in manufacturers and in the assemblers in Cases 1 and 2. The figure shows that, also thanks to the dependence of manufacturers and assemblers production rates with their output inventories, the DMS is able to keep stocks at a satisfactorily high level, so that the demand is efficiently satisfied and inventory is not excessive. Moreover, as expected the DMS input stocks are always higher than the corresponding output inventories. However, the inventories of Case 1 are usually higher than the inventories in Case 2 because of the higher production rates.

In addition, Table 3 reports the average lead times and the system inventories for Cases 1 and 2. As expected, these two performance indices are higher in Case 1, exhibiting a higher level of congestion caused by the faster system production. Moreover, Fig. 8 reports the throughput of retailers R1 and R2 and the system throughput T. We remark that the throughput values in Case 1 are higher than the corresponding values in Case 2: this is expected, because manufacturers work in Case 1 with higher production rates.

CONCLUSIONS

The paper considers First-Order Hybrid Petri Nets (FOHPN) to model Distributed Manufacturing Systems (DMS), which are new emerging company networks, very complex to describe and manage. Combining continuous and discrete dynamics, FOHPN appears a promising formalism, able to capture the different properties of such discrete event systems, characterized by a large number of states.

Future research will apply the FOHPN conflict resolution policy in order to optimize a given DMS objective function under different management policies.

REFERENCES

Alla, H., and R. David, (1998). Continuous and Hybrid Petri Nets. J. *of Circuits, Systems and Computers*, **8** (1), 159-188.

Balduzzi, F, A. Giua, and G. Menga, (2000). Modelling and Control with First order Hybrid Petri Nets. *IEEE Trans. on Robotics and Automation,* **4** (16), 382 – 399.

Desrochers, A., T.J. Deal and M.P. Fanti, (2005) Complex-Valued Token Petri nets. *IEEE Transactions on Automation Science and Engineering*, **2**(4), 309-318.

Dotoli, M. and M.P. Fanti, (2005). A Generalized Stochastic Petri Net Model for Distributed Manufacturing System Management. *44th IEEE Conf. on Decision and Control and European Control Conference*, Seville, (Spain), 12-15.

Viswanadham, N. and S. Raghavan, (2000). Performance analysis and design of supply chain: a Petri net approach. *Journal of the Operational Research Society,* **51** (10), 1158-1169.

Zhou, M.C. and K. Venkatesh (1998). *Modeling, Simulation and Control of Flexible Manufacturing Systems. A Petri Net Approach.* World Scientific, Singapore.

Table 2: Initial marking of continuous places, capacities and edge weights for Cases 1 and 2.

Initial marking	Product units	Capacities
m_1 m_2	20	C_1, C_2=200
m_3 m_5 m_9 m_{11}	20	C_4, C_6, C_{10}, C_{12}=100
m_7 m_{13}	20	C_8, C_{14}=80
m_{15} m_{21} m_{17} m_{23}	20	$C_{16}, C_{22}, C_{18}, C_{24}$=100
m_{19} m_{25}	20	C_{20}, C_{26}=80
m_{27} m_{28} m_{29}	0	
m_{30} m_{31} m_{32}	0	
Edge weights		
max_M=190		max_A=190
med_M=50		med_A=50

Table 3: Overall lead time and system inventory for Cases 1 and 2.

	Case 1	Case 2
LT (hours)	190.60	167.87
SI (product units)	445.80	218.38

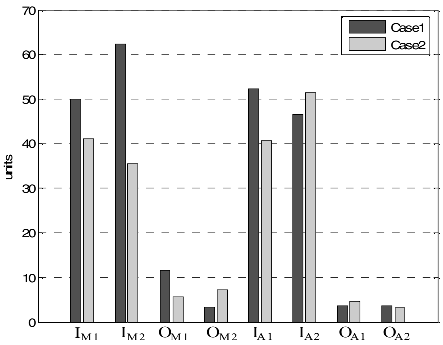

Fig. 7. The average inventory in manufacturers and in assemblers in Cases 1 and 2.

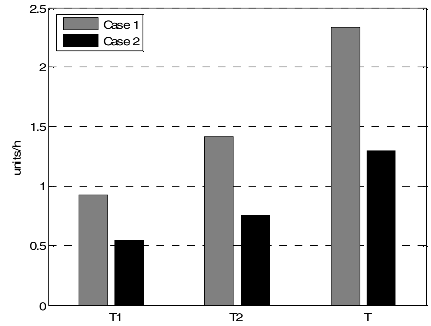

Fig. 8. Throughputs (units/hours) in Cases 1 and 2.