TRACKING CONTROL OF JOIN-FREE TIMED CONTINUOUS PETRI NET SYSTEMS

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Abstract: A new low-and-high gain algorithm is presented for tracking control of timed continuous Petri Net (contPN) systems working under infinite servers semantics. The inherent properties of timed contPN determine that the control signals must be non-negative and upper bounded by functions of system states. In the proposed control approach, LQ theory is first utilized to design a low-gain controller such that the control signals satisfy the input constraints. Based on the low-gain controller, a high-gain term is further added to improve the system transient performance. In order to guarantee global tracking convergence and smoothness of control signals, in our work, a new mixed tracking trajectory (state step and ramp) is utilized instead of a pure step reference signal.

1. INTRODUCTION

Petri Nets (PNs) are powerful mathematical tools with appealing graphical representations for the modeling, analysis and synthesis of Discrete Even Systems (DESs). However, like in all DESs, discrete PN systems suffer from the so called state explosion problem. One possible way to partially tackle this problem is to fluidify the discrete PN models. The resulting *continuous* Petri Net (contPN) systems have the potential for the applications of more analytical techniques which were originally developed for continuous and hybrid systems. Furthermore, analogous to discrete PN systems, time has been introduced to contPN systems, which leads to timed contPN systems. Steady state control is studied in (Mahulea *et al.*, 2005); assuming all transitions can be controlled, it can be solved by means of a LPP. Nevertheless, dynamic control of timed contPN systems needs further investigations and great improvements.

For control of timed contPN systems, we will begin from *Join-Free* (JF) timed contPN systems with step-tracking control target. For a JF timed contPN system, a linear differential equation describes its behavior, but it is subject to certain input constraints, i.e. the control signal must be non-negative and upper bounded by a function of system states. These constraints result in a *hybrid system*, more precisely a piecewise linear system. The main challenge in our work is to develop control laws under these special input constraints so that global tracking convergence can be ensured.

The input constraints can be treated as input saturations. As input saturation is one of the common phenomena encountered in control systems, hitherto lots of works have been done. In (Wredenhage and Bélanger, 1994), a kind of piecewise-linear control law was derived. However, such a design method will lead to a low-gain controller. To improve the control performance, a low-and-high gain approach was given in (Saberi *et al.*, 1996). Recently, several nonlinear control methods were further presented (Lin *et* *al.*, 1998), which mainly focus on the high-gain design in the low-and-high gain algorithms aiming to achieve better transient control performance. It should be pointed out that common assumptions in all these works are that the lower saturation bounds are negative constants and the upper saturation bounds are positive constants. However, in JF timed contPN systems, the lower saturation bound is zero and the upper saturation bound depends on system states. Therefore, the existing control strategies for systems with input saturations cannot be applied directly.

In this paper, a new low-and-high gain approach will be proposed for step-tracking control of JF timed contPN systems. The presented algorithm can ensure global asymptotical convergence in presence of the input constraints existing in JF timed contPN systems. According to initial marking, desired marking and net structure, a new reference trajectory is constructed. Based on the new tracking target, a low-gain controller is designed according to LQ theory so that the control signals are within the required regions. Since the upper bounds of the control inputs depend on the system states, the design method in (Wredenhage and Bélanger, 1994) fails to work. By combining the new tracking trajectory design with LQ method, a novel design scheme for the low-gain part is given in our work. Analogous to the works of (Saberi *et al.*, 1996), a high-gain part is further added to make better use of the available control authority, and consequently faster system response can be obtained. Note that in the high-gain term, the control vector of the low-gain part is maintained, which makes possible the analysis of system stability. Rigorous proof is provided to guarantee the global asymptotical convergence.

The paper is organized as follows. Section 2 introduces the required concepts of contPN systems and timed contPN systems. The control for JF timed contPN systems is formulated in Section 3. How to construct the new tracking trajectory is outlined in Section 4. Section 5 focuses on the development of control laws and the analysis of global asymptotical convergence property. An illustrative example is given in Section 6. Finally, Section 7 concludes the paper.

2. CONTINUOUS PETRI NET SYSTEMS

2.1 Untimed Continuous Petri Net Systems

A contPN system can be described as $\langle \mathcal{N}, \mathbf{m}_0 \rangle$, where $\mathcal{N} = \langle \mathbf{P}, \mathbf{T}, \mathbf{Pre}, \mathbf{Post} \rangle$ specifies the net structure ($\mathbf{P}$ and $\mathbf{T}$ are disjoint (finite) sets of places and transitions, and $\mathbf{Pre}$ and $\mathbf{Post}$ are incidence matrices), and $\mathbf{m}_0$ is the initial marking. $\mathcal{N}$ is assumed to be connected, while $\mathbf{P}$ and $\mathbf{T}$ have n and m elements, respectively. The marking $\mathbf{m}$ belongs to R^{+n}, where R^{+} is the set of non-negative real numbers, and both $\mathbf{Pre}$ and $\mathbf{Post}$ are of the size $n \times m$. For $w \in \mathbf{P} \cup \mathbf{T}$, the set of its input and output nodes are denoted as ${}^{\bullet}w$, and $w^{\bullet}$, respectively. $\mathcal{N}$ is JF (or rendez-vous free) iff $\forall t \in \mathbf{T}$, $|{}^{\bullet}t| = 1$. If $\mathbf{m}$ is reachable from $\mathbf{m}_0$ through a sequence $\boldsymbol{\sigma} \in \mathsf{R}^{+m}$, the state equation is $\mathbf{m} = \mathbf{m}_0 + \mathbf{C} \cdot \boldsymbol{\sigma}$, where $\mathbf{C} = \mathbf{Post} - \mathbf{Pre}$ is the token flow matrix and $\boldsymbol{\sigma}$ is the firing count vector.

2.2 Timed Continuous Petri Net Systems

A timed contPN system can be represented as $\langle \mathcal{N}, \boldsymbol{\lambda}, \mathbf{m}_0 \rangle$, where $\boldsymbol{\lambda}[t] > 0$ is the internal firing rate of transition t. The state equation has an explicit dependence on time $\mathbf{m}(\tau) = \mathbf{m}_0 + \mathbf{C}\boldsymbol{\sigma}(\tau)$, where τ is time. Deriving with respect to it, $\dot{\mathbf{m}}(\tau) = \mathbf{C}\dot{\boldsymbol{\sigma}}(\tau)$ is obtained. Define $\mathbf{f}(\tau) = \dot{\boldsymbol{\sigma}}(\tau)$, which denote flows of transitions. The state equation is $\dot{\mathbf{m}}(\tau) = \mathbf{C}\mathbf{f}(\tau)$. For notation simplicity, τ will be omitted in the rest of the paper. For the definition of flow $\mathbf{f}$, different semantics have been introduced and the most important ones are *infinite servers* and *finite servers*. Infinite servers semantics will be considered in this work, which usually provides a much better approximation of discrete behaviors. Under infinite server semantics, $\mathbf{f}$ is the product of $\boldsymbol{\lambda}[t]$ and the instantaneous enabling of the transition, i.e. $\mathbf{f}[t] = \boldsymbol{\lambda}[t] \cdot enab(t, \mathbf{m}) = \boldsymbol{\lambda}[t] \cdot \min_{p \in {}^{\bullet}t}\{\mathbf{m}[p]/\mathbf{Pre}[p,t]\}$. As in JF nets any transition has only one input place, the flow can be expressed as $\mathbf{f} = \boldsymbol{\Phi} \cdot \mathbf{m}$, where $\boldsymbol{\Phi} \in \mathsf{R}^{+m \times n}$ and $\boldsymbol{\Phi}[t,p] = \boldsymbol{\lambda}[t]/\mathbf{Pre}[p,t]$ if $p = {}^{\bullet}t$, $\boldsymbol{\Phi}[t,p] = 0$ otherwise. Moreover, each row of $\boldsymbol{\Phi}$ has only one non-zero element. $\forall j \in M \triangleq \{1, 2, \cdots, m\}$, the non-zero element for the j-th row of $\boldsymbol{\Phi}$ is denoted as $\phi_{j,i}$ where $i \in N \triangleq \{1, 2, \cdots, n\}$. Therefore, $\forall j \in M$, we have $f_j = \phi_{j,i} m_i$, where f_j is the j-th element of $\mathbf{f}$ and m_i is the i-th element of $\mathbf{m}$. The evolution of the marking can be written as follows:

$$\dot{\mathbf{m}} = \mathbf{C} \cdot \mathbf{f} = \mathbf{A} \cdot \mathbf{m}, \tag{1}$$

where $\mathbf{A} = \mathbf{C} \cdot \boldsymbol{\Phi}$. Finally the definition of conservativeness and some related properties for a JF timed contPN are listed as follows.

Definition 1. A PN is *conservative* iff $\exists \mathbf{y} > 0$, such that $\mathbf{y} \cdot \mathbf{C} = \mathbf{0}$. Any left non-negative annuller of matrix $\mathbf{C}$, i.e. $\mathbf{y}$, is called P-semiflow. A P-semiflow is *minimal* if 1 is the greatest common divisor of its elements, and its support does not strictly contain the support of other P-semiflow.

Property 1. (Teruel *et al.*, 1997) Let $\mathcal{N}$ be a JF net.

1.1 If $\mathcal{N}$ is strongly connected,

a) $\mathcal{N}$ has at most one minimal P-semiflow;

b) $\mathcal{N}$ is conservative iff it has one P-semiflow.

1.2 If $\mathcal{N}$ is conservative, it is consistent iff it is strongly connected.

Property 2. Let $\mathcal{N}$ be a conservative and strongly connected JF timed contPN. The matrix $\mathbf{A}$ for $\mathcal{N}$ defined in (1) has only one zero-eigenvalue. Moreover, the remaining eigenvalues of $\mathbf{A}$ are with negative real parts.

Proof: As $\mathcal{N}$ is conservative and strongly connected, from *Property 1.2*, $\mathcal{N}$ is consistent. From the *Proposition 1* in (Jiménez *et al.*, 2005), for the conservative and consistent JF PN $\mathcal{N}$, the matrix $\mathbf{A}$ has only one zero-valued eigenvalue. On the other hand, as $\mathbf{A}$ is a Metzler matrix and the eigenvalue of zero is the (Frobenius) dominant eigenvalue (Jiménez *et al.*, 2005), the remaining eigenvalues of $\mathbf{A}$ are with negative real parts. ■

3. PROBLEM FORMULATION

We will restrict our research to conservative and strongly connected JF timed contPN systems and assume all the markings are observable. For concise expression, "timed contPN" will be written as "contPN". Like all the other systems, control actions can also be introduced to modify autonomous evolution of PN systems. The possible control action that can be applied to PN systems is to slow down their unforced firing flows. Hence, the forced flows of controlled transitions become $\mathbf{f}-\mathbf{u}$, where $\mathbf{u}$ is the control signal and must satisfy $\mathbf{0} \le \mathbf{u} \le \mathbf{f}$. Considering (1), a JF contPN system with a control action can be described as follows:

$$\dot{\mathbf{m}} = \mathbf{C}(\mathbf{\Phi}\mathbf{m} - \mathbf{u}) \triangleq \mathbf{A}\mathbf{m} - \mathbf{B}\mathbf{u}, \qquad (2)$$

where $\mathbf{m} \in \mathsf{R}^{+n}$, $\mathbf{A} \in \mathsf{R}^{n\times n}$, $\mathbf{B} \triangleq \mathbf{C} \in \mathsf{R}^{n\times m}$ and $\mathbf{u} \in \mathsf{R}^{+m}$. It should be noted that the constraints on input $\mathbf{u}$ lead the closed system to be a piecewise linear system.

Our control objective is to construct control laws such that $\mathbf{m}$ and $\mathbf{u}$ respectively converge to desired values: $\mathbf{m}_d$ and $\mathbf{u}_d$. To satisfy reachability, $\mathbf{m}_d$ must fulfill that $\mathbf{y}\cdot\mathbf{m}_d = \mathbf{y}\cdot\mathbf{m}_0$ where $\mathbf{y} \in \mathsf{R}^n$ is the basis of P-semiflows. For $\mathbf{u}_d$, $\mathbf{0} \le \mathbf{u}_d \le \mathbf{\Phi}\mathbf{m}_d$ must be satisfied, i.e. $\forall j \in M$, $0 \le u_{d,j} \le \phi_{j,i} m_{d,i}$ where $u_{d,j}$ and $m_{d,i}$ are the j-th and i-th elements of $\mathbf{u}_d$ and $\mathbf{m}_d$ respectively. Moreover, as $\mathbf{m}_d$ is constant, according to (2), the desired control input must be a solution of the following equation.

$$\mathbf{0} = \mathbf{A}\mathbf{m}_d - \mathbf{B}\mathbf{u}_d. \qquad (3)$$

In our work, the following assumption is made for $\mathbf{m}_0$ and $\mathbf{m}_d$.

Assumption 1. $\forall i \in N$, $m_{0,i} > 0$ and $m_{d,i} > 0$.

Remark 1. If we consider optimal steady states in practical manufacture systems, generally $\mathbf{m}_d > \mathbf{0}$ is valid. On the other hand, if some elements of $\mathbf{m}_0$ are zeros, as $\mathbf{m}_d > \mathbf{0}$, a firing sequence can always be found such that $\mathbf{m}_0[\boldsymbol{\sigma} > \mathbf{m}$ and $\mathbf{m} > \mathbf{0}$ (Recalde *et al.*, 1999). Then the control algorithm proposed in our work can be further applied.

4. DESIGN OF NEW TRACKING REFERENCE

To ensure global convergence and smoothness of the control signal, a step target $\mathbf{m}_d$ is replaced by the following reference trajectory $\mathbf{m}_r(\tau)$.

$$\mathbf{m}_r(\tau) = \begin{cases} \mathbf{m}_{r0} + \dfrac{\mathbf{m}_d - \mathbf{m}_{r0}}{h}\tau, & \tau \in [0, h] \\ \mathbf{m}_d, & \tau \in [h, \infty) \end{cases} \qquad (4)$$

where $\mathbf{m}_r(\tau) \in \mathsf{R}^{+n}$, $\mathbf{m}_{r0} \triangleq \mathbf{m}_r(0)$ and $h > 0$ determines the time when $\mathbf{m}_r(\tau) = \mathbf{m}_d$. Here we choose $\mathbf{m}_{r0} = \mathbf{m}_0 + \delta(\mathbf{m}_d - \mathbf{m}_0)$, where $0 \le \delta < 1$ is a parameter to be designed. The parameter h is chosen such that, for given $\mathbf{m}_{r0}$ and $\mathbf{m}_d$, valid control actions $\mathbf{u}_{r0} \in \mathsf{R}^{+m}$ and $\mathbf{u}_{rh^-} \in \mathsf{R}^{+m}$ exist for the following equations.

$$\mathbf{A}\mathbf{m}_{r0} - \mathbf{B}\mathbf{u}_{r0} = \frac{\mathbf{m}_d - \mathbf{m}_{r0}}{h} \qquad (5)$$

$$\mathbf{A}\mathbf{m}_d - \mathbf{B}\mathbf{u}_{rh^-} = \frac{\mathbf{m}_d - \mathbf{m}_{r0}}{h}, \qquad (6)$$

where $0 \le \mathbf{u}_{r0} \le \mathbf{\Phi}\mathbf{m}_{r0}$ and $0 \le \mathbf{u}_{rh^-} \le \mathbf{\Phi}\mathbf{m}_d$.

Proposition 1. For given $\mathbf{m}_0$ and reachable $\mathbf{m}_d$, δ and h can always be found such that (5) and (6) are valid. Moreover, as δ increase, smaller h can be chosen.

Proof: Given $\mathbf{m}_0$ and reachable $\mathbf{m}_d$, $\boldsymbol{\sigma} \ge \mathbf{0}$ can always be found so that $\mathbf{m}_d = \mathbf{m}_0 + \mathbf{B}\boldsymbol{\sigma}$. Thus,

$$\mathbf{m}_d - \mathbf{m}_{r0} = \mathbf{B}(1-\delta)\boldsymbol{\sigma}. \qquad (7)$$

Substituting (7) into (5) and (6) yields

$$\mathbf{B}\mathbf{\Phi}\mathbf{m}_{r0} - \mathbf{B}\mathbf{u}_{r0} = \mathbf{B}(1-\delta)\boldsymbol{\sigma}\frac{1}{h},$$

$$\mathbf{B}\mathbf{\Phi}\mathbf{m}_d - \mathbf{B}\mathbf{u}_{rh^-} = \mathbf{B}(1-\delta)\boldsymbol{\sigma}\frac{1}{h}.$$

Obviously, $\mathbf{u}_{r0} = \mathbf{\Phi}\mathbf{m}_{r0} - (1-\delta)\boldsymbol{\sigma}\frac{1}{h}$ and $\mathbf{u}_{rh^-} = \mathbf{\Phi}\mathbf{m}_d - (1-\delta)\boldsymbol{\sigma}\frac{1}{h}$ are solutions for the above equations. From $\mathbf{0} \le \mathbf{u}_{r0} \le \mathbf{\Phi}\mathbf{m}_{r0}$, we have:

$$\mathbf{0} \le \boldsymbol{\sigma}\frac{1}{h} \le \frac{1}{1-\delta}\mathbf{\Phi}\mathbf{m}_{r0} \qquad (8)$$

$$\Rightarrow \mathbf{0} \le \boldsymbol{\sigma}\frac{1}{h} \le \mathbf{\Phi}\mathbf{m}_0 + \frac{\delta}{1-\delta}\mathbf{m}_d. \qquad (9)$$

From $\mathbf{0} \leq \mathbf{u}_{rh^-} \leq \mathbf{\Phi m}_d$, we can obtain that

$$\mathbf{0} \leq \boldsymbol{\sigma}\frac{1}{h} \leq \frac{1}{1-\delta}\mathbf{\Phi m}_d. \quad (10)$$

Therefore, the final constraints for h are:

$$\mathbf{0} \leq \frac{\boldsymbol{\sigma}}{h} \leq \min\{\mathbf{\Phi m}_0 + \frac{\delta \mathbf{m}_d}{1-\delta}, \frac{1}{1-\delta}\mathbf{\Phi m}_d\} \quad (11)$$

where, for $\mathbf{a}_1 \in \mathsf{R}^{+n}$ and $\mathbf{a}_2 \in \mathsf{R}^{+n}$, the i-th element of $\min\{\mathbf{a}_1, \mathbf{a}_2\}$ is defined as $\min\{a_{1,i}, a_{2,i}\}$. From *Assumption 1*, all the elements of $\mathbf{m}_0$ and $\mathbf{m}_d$ are strictly positive. Hence, $\min\{\mathbf{\Phi m}_0 + \frac{\delta}{1-\delta}\mathbf{m}_d, \frac{1}{1-\delta}\mathbf{\Phi m}_d\}$ is positive. Considering $\boldsymbol{\sigma} \geq \mathbf{0}$, for a given $0 \leq \delta < 1$, a sufficiently large $h > 0$ can always be found such that (11) is valid. Moreover, for a chosen $\boldsymbol{\sigma}$, as δ increase, smaller h can be obtained. ■

Proposition 1 clearly shows that the input constraints result in the constraints for h and, for a chosen $\boldsymbol{\sigma}$, smaller h can be realized by increasing δ. Hence, to obtain faster response, in (4), $\mathbf{m}_{r0}$ is designed instead of using $\mathbf{m}_0$ directly. However, larger δ will lead to larger initial error, which may destroy the tracking convergence. Hence, δ and h should be properly chosen such that both tracking convergence and possible fast response can be obtained.

To simplify the design procedure, in the proposed algorithm, δ will be designed first. Its design will be given in next Section. Now let us discuss how to calculate h for a given δ. According to (8) and (10), the constraints for h can be rewritten as $\mathbf{0} \leq \frac{\boldsymbol{\sigma}}{h} \leq \frac{\mathbf{\Phi}\min\{\mathbf{m}_{r0}, \mathbf{m}_d\}}{1-\delta}$. From the definition of $\mathbf{m}_{r0}$, we have $\mathbf{0} < \frac{\mathbf{\Phi}\min\{\mathbf{m}_0, \mathbf{m}_d\}}{1-\delta} \leq \frac{\mathbf{\Phi}\min\{\mathbf{m}_{r0}, \mathbf{m}_d\}}{1-\delta}$. Hence, h can be designed to satisfy $\mathbf{0} \leq \frac{\boldsymbol{\sigma}}{h} \leq \frac{\mathbf{\Phi}\min\{\mathbf{m}_0, \mathbf{m}_d\}}{1-\delta}$. As $\min\{\mathbf{m}_0, \mathbf{m}_d\}$ is strictly positive, the solution of h always exists. Furthermore, h can be chosen as $h = \beta(1-\delta)$, where $\beta > 0$ and $\frac{\boldsymbol{\sigma}}{\beta} \leq \mathbf{\Phi}\min\{\mathbf{m}_0, \mathbf{m}_d\}$. Consequently, $\mathbf{u}_{r0}$ and $\mathbf{u}_{rh^-}$ can be rewritten as follows:

$$\mathbf{u}_{r0} = \mathbf{\Phi m}_{r0} - \frac{\boldsymbol{\sigma}}{\beta} \quad (12)$$

$$\mathbf{u}_{rh^-} = \mathbf{\Phi m}_d - \frac{\boldsymbol{\sigma}}{\beta}. \quad (13)$$

Remark 2. For given $\mathbf{m}_0$ and $\mathbf{m}_d$, $\boldsymbol{\sigma}$ is designed first. Based on the chosen $\boldsymbol{\sigma}$, the minimum β such that $\frac{\boldsymbol{\sigma}}{\beta} \leq \mathbf{\Phi}\min\{\mathbf{m}_0, \mathbf{m}_d\}$ is chosen.

5. TRACKING CONTROL OF JF CONTPN SYSTEMS

The control signal $\mathbf{u}$ is designed as follows:

$$\mathbf{u} = sat(\mathbf{u}_{lg} + \mathbf{u}_{hg}) + \mathbf{u}_r(\tau), \quad (14)$$

where $\mathbf{u}_{lg}$ is the low-gain part, $\mathbf{u}_{hg}$ is the high-gain term and $\mathbf{u}_r(\tau)$ is the reference control input defined as follows:

$$\mathbf{u}_r(\tau) = \begin{cases} \mathbf{u}_{r0} + \dfrac{\mathbf{u}_{rh^-} - \mathbf{u}_{r0}}{h}\tau, & \tau \in [0, h^-] \\ \mathbf{u}_d, & \tau \in [h^+, \infty) \end{cases},$$

where $\mathbf{u}_{r0}$ and $\mathbf{u}_{rh^-}$ are given in (12) and (13) respectively. Moreover, $\forall\ \mathbf{d} \in \mathsf{R}^m$, $sat(\mathbf{d}) \triangleq [sat(d_1), \cdots, sat(d_m)]^T$ and $\forall j \in M$, $sat(d_j)$ is

$$sat(d_j) = \begin{cases} \phi_{j,i}m_i - u_{r,j}, & \textbf{if} \;\; d_j \geq \phi_{j,i}m_i - u_{r,i} \\ d_j, & \textbf{if} \;\; -u_{r,j} < d_j \\ & \quad\; < \phi_{j,i}m_i - u_{r,j} \\ -u_{r,j}, & \textbf{if} \;\; d_j \leq -u_{r,j} \end{cases}.$$

Define $\mathbf{e} = \mathbf{m}_r(\tau) - \mathbf{m}$. From (2) and (4), we have

$$\dot{\mathbf{e}} = \begin{cases} \dfrac{\mathbf{m}_d - \mathbf{m}_{r0}}{h} - \mathbf{A m}_r(\tau) \\ \quad +\mathbf{Ae} + \mathbf{Bu} & \tau \in [0, h^-] \\ -\mathbf{Am}_d + \mathbf{Ae} + \mathbf{Bu} & \tau \in [h^+, \infty) \end{cases} \quad (15)$$

From the definition of $\mathbf{u}_r(\tau)$ and considering (5) and (6), for $\tau \in [0, h^-]$, it can be derived that

$$\begin{aligned} \mathbf{Bu}_r(\tau) &= \mathbf{Am}_{r0} - (\mathbf{m}_d - \mathbf{m}_{r0})\frac{1}{h} + [\mathbf{Am}_d - (\mathbf{m}_d \\ &\quad -\mathbf{m}_{r0})\frac{1}{h} - \mathbf{Am}_{r0} + (\mathbf{m}_d - \mathbf{m}_{r0})\frac{1}{h}]\frac{\tau}{h} \\ &= -(\mathbf{m}_d - \mathbf{m}_{r0})\frac{1}{h} + \mathbf{Am}_r(\tau). \end{aligned} \quad (16)$$

According to (3) and (16), the following is valid:

$$\mathbf{Bu}_r(\tau) = \begin{cases} -\dfrac{\mathbf{m}_d - \mathbf{m}_{r0}}{h} + \mathbf{Am}_r(\tau), & \tau \in [0, h^-] \\ \mathbf{Am}_d, & \tau \in [h^+, \infty) \end{cases}.$$

Substituting (14) into (15) and considering the above results, (15) can be rewritten as

$$\dot{\mathbf{e}} = \mathbf{Ae} + \mathbf{B}sat(\mathbf{u}_{lg} + \mathbf{u}_{hg}). \quad (17)$$

ContPNs with at least one P-semiflow are non-controllable, according to the classical linear control theory (Mahulea *et al.*, 2005). Hence, a transformation matrix $\mathbf{H} \in \mathsf{R}^{n\times n}$ is constructed to separate the system states into controllable and non-controllable parts. The first row of $\mathbf{H}$ is a basis of P-semiflow (here only one vector; *Property 1.1*) and the remaining rows are completed with elementary vectors such that $\mathbf{H}$ is full rank. Define $\bar{\mathbf{e}} = \mathbf{He}$. Then (17) becomes

$$\dot{\bar{\mathbf{e}}} = \bar{\mathbf{A}}\bar{\mathbf{e}} + \bar{\mathbf{B}}sat(\mathbf{u}_{lg} + \mathbf{u}_{hg}) \quad (18)$$

where $\bar{\mathbf{A}} = \mathbf{HAH}^{-1}$ and $\bar{\mathbf{B}} = \mathbf{HB}$. According to the definitions of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{H}$ and considering $\mathbf{y} \cdot \mathbf{C} = \mathbf{0}$, it can be derived that the first rows of both $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$ are zeros, which leads to $\dot{\bar{e}}_1 = 0$.

From the definition of $\bar{\mathbf{e}}$ and $\mathbf{H}$, we have $\bar{e}_1(0) = \mathbf{y}(m_{d,1} - m_1(0))$. As $\mathbf{y}\mathbf{m}_d = \mathbf{y}\mathbf{m}(0)$, $\bar{e}_1(0) = 0$ can be obtained. Therefore, $\forall\tau \in [0, \infty)$, $\bar{e}_1(\tau) = 0$. Moreover, the controllable part of (18) is

$$\dot{\bar{\mathbf{e}}}_c = \bar{\mathbf{A}}_c\bar{\mathbf{e}}_c + \bar{\mathbf{B}}_c sat(\mathbf{u}_{lg} + \mathbf{u}_{hg}), \qquad (19)$$

where $\bar{\mathbf{e}}_c \triangleq [\bar{e}_2, \cdots, \bar{e}_n]^T \in \mathsf{R}^{n-1}$, $\bar{\mathbf{A}}_c \in \mathsf{R}^{(n-1)\times(n-1)}$ and $\bar{\mathbf{B}}_c \in \mathsf{R}^{(n-1)\times m}$. From the definition of $\bar{\mathbf{e}}$, we have $\mathbf{e} = \mathbf{H}^{-1}\bar{\mathbf{e}}$. As $\bar{e}_1 = 0$, $\mathbf{e}$ can be rewritten as $\mathbf{S}\bar{\mathbf{e}}_c$ where $\mathbf{S} \in \mathsf{R}^{n\times(n-1)}$ is $\mathbf{H}^{-1}$ without the first column.

5.1 Design of $\mathbf{u}_{lg}$ *and* $\mathbf{u}_{hg}$

$\mathbf{u}_{lg}$ is designed to minimize the following quadratic performance criterion

$$J(\bar{\mathbf{e}}_c(0)) = \int_0^\infty (\bar{\mathbf{e}}_c^T\mathbf{Q}\bar{\mathbf{e}}_c + \gamma\mathbf{u}^T\mathbf{R}\mathbf{u})d\tau, \qquad (20)$$

where $\mathbf{Q} \in \mathsf{R}^{(n-1)\times(n-1)}$ is a diagonal positive definite matrix, $\mathbf{R} = diag(r_1, \cdots, r_m)$ is positive definite and the $\gamma > 0$ is a parameter to be designed.

Define $\mathbf{c}_1 = \min\{\mathbf{u}_{r0}, \mathbf{u}_{rh^-}, \mathbf{u}_d\}$. Obviously, $\mathbf{c}_1 \geq \mathbf{0}$. The design of $\mathbf{u}_{lg}$ is classified into two cases.

Case 1. $\mathbf{c}_1 > \mathbf{0}$

The low gain controller is $\mathbf{u}_{lg} = -\mathbf{K}\bar{\mathbf{e}}_c$, where $\mathbf{K} = \frac{1}{\gamma}\mathbf{R}^{-1}\bar{\mathbf{B}}_c^T\mathbf{W}$ and $\mathbf{W}$ can be found from the following Riccati equation,

$$\mathbf{W}\bar{\mathbf{A}}_c + \bar{\mathbf{A}}_c^T\mathbf{W} - \frac{1}{\gamma}\mathbf{W}\bar{\mathbf{B}}_c\mathbf{R}^{-1}\bar{\mathbf{B}}_c^T\mathbf{W} + \mathbf{Q} = 0. \qquad (21)$$

Case 2. $\mathbf{c}_1$ have zero-elements

To clearly explain the basic idea, assume only one element of $\mathbf{c}_1$, i.e. $\mathbf{c}_{1,z}$ $(z \in M)$, is zero. However, if $\mathbf{c}_1$ have several zero-elements, the design of $\mathbf{u}_{lg}$ can be derived analogously. In this Case, $\mathbf{W}$ is calculated from the following Riccati equation,

$$\mathbf{W}\bar{\mathbf{A}}_c + \bar{\mathbf{A}}_c^T\mathbf{W} - \frac{1}{\gamma}\mathbf{W}(\bar{\mathbf{B}}_c - \Delta\bar{\mathbf{B}}_c) \mathbf{R}^{-1}(\bar{\mathbf{B}}_c - \Delta\bar{\mathbf{B}}_c)^T\mathbf{W} + \mathbf{Q} = 0, \qquad (22)$$

where $\Delta\bar{\mathbf{B}}_c \in \mathsf{R}^{(n-1)\times m}$, the z-th column of $\Delta\bar{\mathbf{B}}_c$ is same as the z-th column of $\bar{\mathbf{B}}_c$ and all the remaining columns of $\Delta\bar{\mathbf{B}}_c$ are $\mathbf{0}$. As $\mathbf{A}_c$ is stable (from *Property 2*), the solution of $\mathbf{W}$ always exists. The low gain controller is $\mathbf{u}_{lg} = -\mathbf{K}\bar{\mathbf{e}}_c$, where $\mathbf{K} = \frac{1}{\gamma}\mathbf{R}^{-1}(\bar{\mathbf{B}}_c - \Delta\bar{\mathbf{B}}_c)^T\mathbf{W}$. From the definition of $\Delta\bar{\mathbf{B}}_c$, it can be derived that the z-th row of $\mathbf{K}$, i.e. $\mathbf{k}_z$, is $\mathbf{0}$. Hence, $\Delta\bar{\mathbf{B}}_c\mathbf{K} = \mathbf{0}$.

For the high-gain term, in both **Case 1** and **Case 2**, $\mathbf{u}_{hg} = -l\bar{\mathbf{B}}_c^T\mathbf{W}\bar{\mathbf{e}}_c$ where l is a positive constant.

5.2 Design of the parameters δ and γ

Define $\epsilon(\mathbf{W}, \rho) \triangleq \{\bar{\mathbf{e}}_c : \bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{e}}_c \leq \rho\}$, where $\rho = \bar{\mathbf{e}}_c^T(0)\mathbf{W}\bar{\mathbf{e}}_c(0)$. δ and γ are designed off-line such that $\forall\bar{\mathbf{e}}_c \in \epsilon(\mathbf{W}, \rho)$ and $\forall j \in M$, $-\mathbf{k}_j\bar{\mathbf{e}}_c \geq -c_{1,j}$ and $-\mathbf{k}'_j\bar{\mathbf{e}}_c \leq c_{2,j}$, where $c_{1,j}$ is the j-th element of $\mathbf{c}_1$, $c_{2,j} \triangleq \min\{\frac{\sigma_j}{\beta}, \phi_{j,i}m_{d,j} - u_{d,j}\}$, $\mathbf{k}_j$ is the j-th row of $\mathbf{K}$, $\mathbf{k}'_j \triangleq \mathbf{k}_j - \phi_{j,i}\mathbf{s}_i$ and $\mathbf{s}_i$ is the i-th row of $\mathbf{S}$. Note that $c_{2,j} \geq 0$.

The following proposition implies the existence of the solutions of δ and γ.

Proposition 2. Let $\langle\mathcal{N}, \boldsymbol{\lambda}, \mathbf{m}_0\rangle$ be a conservative and strongly connected JF contPN system. For given $\mathbf{Q}$ and $\mathbf{R}$, δ and γ can always be found such that $\forall\bar{\mathbf{e}}_c \in \epsilon(\mathbf{W}, \rho)$ and $\forall j \in M$, $-\mathbf{k}_j\bar{\mathbf{e}}_c \geq -c_{1,j}$ and $-\mathbf{k}'_j\bar{\mathbf{e}}_c \leq c_{2,j}$.

Proof: Same as the design of $\mathbf{u}_{lg}$, the proof also contains two cases.

Case 1. $\mathbf{c}_1 > \mathbf{0}$

$\forall j \in M$, the maximum values of $|-\mathbf{k}_j\bar{\mathbf{e}}_c|$ and $|-\mathbf{k}'_j\bar{\mathbf{e}}_c|$ subjected to $\bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{e}}_c \leq \rho$ are as follows (Wredenhage and Bélanger, 1994):

$$\max_{\bar{\mathbf{e}}_c\in\epsilon(\mathbf{W},\rho)} |-\mathbf{k}_j\bar{\mathbf{e}}_c| = \sqrt{\rho}(\mathbf{k}_j\mathbf{W}^{-1}\mathbf{k}_j^T)^{1/2}, \qquad (23)$$

$$\max_{\bar{\mathbf{e}}_c\in\epsilon(\mathbf{W},\rho)} |-\mathbf{k}'_j\bar{\mathbf{e}}_c| = \sqrt{\rho}(\mathbf{k}'_j\mathbf{W}^{-1}\mathbf{k}_j'^T)^{1/2}. \qquad (24)$$

To satisfy the design requirements, we need to prove that, $\forall j \in M$,

$$\sqrt{\rho}(\mathbf{k}_j\mathbf{W}^{-1}\mathbf{k}_j^T)^{1/2} \leq c_{1,j}, \qquad (25)$$

$$\sqrt{\rho}(\mathbf{k}'_j\mathbf{W}^{-1}\mathbf{k}_j'^T)^{1/2} \leq c_{2,j}. \qquad (26)$$

Based on (25) and (26) and considering the definitions of ρ, $\mathbf{k}_j$ and $\mathbf{k}'_j$, δ_j and γ_j can be calculated for every $j \in M$. Therefore, $\delta = \min_{j\in M}\{\delta_j\}$ and $\gamma = \max_{j\in M}\{\gamma_j\}$. $\forall j \in M$, the existence of γ_j and δ_j can be disscused according to two cases:

A. $c_{2,j} = 0$

In this case, let $\delta_j = 0$ and γ_j can be any positive value.

B. $c_{2,j} > 0$

From the definition of $\mathbf{k}_j$ and $\mathbf{k}'_j$, it can be derived that any given γ, $\mathbf{k}_j$ and $\mathbf{k}'_j$ are finite constant matrices, i.e. irrelated with δ. Therefore, $\forall j \in M$, both $\mathbf{k}_j\mathbf{W}^{-1}\mathbf{k}_j^T$ and $\mathbf{k}'_j\mathbf{W}^{-1}\mathbf{k}_j'^T$ are constants. On the other hand, smaller δ_j will lead to smaller initial error which further results in smaller $\bar{\mathbf{e}}_c(0)$ and ρ. Therefore, as both $c_{1,j}$ and $c_{2,j}$ are strictly positive constants, a positive δ_j, which is small enough, can always be found such that (25) and (26) are valid.

Case 2. $\mathbf{c}_1$ have zero-elements

Assume $\mathbf{c}_{1,z} = 0$. δ and γ are designed off-line such that $\forall \bar{\mathbf{e}}_c \in \epsilon(\mathbf{W}, \rho)$, $-\mathbf{k}_j\bar{\mathbf{e}}_c \geq -c_{1,j}$ ($\forall j \in \{1, \cdots, z-1, z+1, \cdots, m\}$) and $-\mathbf{k}'_j\bar{\mathbf{e}}_c \leq c_{2,j}$ ($\forall j \in M$). According to the result in **Case 1** and considering the strictly positiveness of $c_{1,j}$ ($\forall j \in \{1, \cdots, z-1, z+1, \cdots, m\}$), the existence of δ and γ can also be guaranteed. On the other hand, as $\mathbf{k}_z = \mathbf{0}$, $-\mathbf{k}_j\bar{\mathbf{e}}_c = 0$. Consequently, $-\mathbf{k}_z\bar{\mathbf{e}}_c \geq -c_{1,z}$ is always valid. Therefore, δ and γ can always be found such that $\forall \bar{\mathbf{e}}_c \in \epsilon(\mathbf{W}, \rho)$ and $\forall j \in M$, $-\mathbf{k}_j\bar{\mathbf{e}}_c \geq -c_{1,j}$ and $-\mathbf{k}'_j\bar{\mathbf{e}}_c \leq c_{2,j}$. ■

5.3 Asymptotical convergence analysis

Theorem 1. Let $\langle \mathcal{N}, \boldsymbol{\lambda}, \mathbf{m}_0 \rangle$ be a conservative and strongly connected JF contPN system. For any $\mathbf{m}_0 > \mathbf{0}$ and any reachable $\mathbf{m}_d > \mathbf{0}$ (*Assumption 1*) and $\mathbf{u}_d$, control law (14) with the parameters δ and γ designed in *Proposition 2* can ensure the global asymptotical convergence of both the system markings and the control signals.

Proof:

Case 1. $\mathbf{c}_1 > \mathbf{0}$

Assume $\bar{\mathbf{e}}_c \in \epsilon(\mathbf{W}, \rho)$, where $\epsilon(\mathbf{W}, \rho)$ is defined in *Proposition 2*. *Proposition 2* implies that δ and γ can always be found so that, $\forall \bar{\mathbf{e}}_c \in \epsilon(\mathbf{W}, \rho)$ and $\forall j \in M$, $-\mathbf{k}_j\bar{\mathbf{e}}_c \geq -c_{1,j}$ and $-\mathbf{k}'_j\bar{\mathbf{e}}_c \leq c_{2,j}$. Define $V = \bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{e}}_c$, where $\mathbf{W}$ is obtained from (21). Hence,

$$\dot{V} = \dot{\bar{\mathbf{e}}}_c^T\mathbf{W}\bar{\mathbf{e}}_c + \bar{\mathbf{e}}_c^T\mathbf{W}\dot{\bar{\mathbf{e}}}_c. \quad (27)$$

On the other hand, (19) can be rewritten as

$$\dot{\bar{\mathbf{e}}}_c = (\bar{\mathbf{A}}_c - \bar{\mathbf{B}}_c\mathbf{K})\bar{\mathbf{e}}_c + \bar{\mathbf{B}}_c\mathbf{v}, \quad (28)$$

where $\mathbf{v} = sat(\mathbf{u}_{lg} + \mathbf{u}_{hg}) + \mathbf{K}\bar{\mathbf{e}}_c$. Substituting (28) into (27) and considering (21), we have

$$\begin{aligned}\dot{V} &= \bar{\mathbf{e}}_c^T(\mathbf{W}\bar{\mathbf{A}}_c + \bar{\mathbf{A}}_c^T\mathbf{W} - \frac{1}{\gamma}\mathbf{W}\bar{\mathbf{B}}_c\mathbf{R}^{-1}\bar{\mathbf{B}}_c^T\mathbf{W})\bar{\mathbf{e}}_c \\ &\quad -\frac{1}{\gamma}\bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{B}}_c\mathbf{R}^{-1}\bar{\mathbf{B}}_c^T\mathbf{W}\bar{\mathbf{e}}_c + 2\bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{B}}_c\mathbf{v} \\ &\leq -\bar{\mathbf{e}}_c^T\mathbf{Q}\bar{\mathbf{e}}_c + 2\sum_{j=1}^{m}\bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{b}}_{c,j}v_j \quad (29)\end{aligned}$$

where $v_j = sat(u_{lg,j} + u_{hg,j}) + \mathbf{k}_j\bar{\mathbf{e}}_c$ is the j-th element of $\mathbf{v}$. $\forall j \in M$, let us discuss $\bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{b}}_{c,j}v_j$ in (29) according to the following three cases.

I. $-u_{r,i} < u_{lg,j} + u_{hg,j} < \phi_{j,i}m_i - u_{r,j}$

$u_{lg,j} + u_{hg,j}$ is not saturated. Hence,

$$\begin{aligned}\bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{b}}_{c,j}v_j &= \bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{b}}_{c,j}(-\mathbf{k}_j\bar{\mathbf{e}}_c - l\bar{\mathbf{b}}_{c,j}^T\mathbf{W}\bar{\mathbf{e}}_c + \mathbf{k}_j\bar{\mathbf{e}}_c) \\ &= -l(\bar{\mathbf{e}}_c{}^T\mathbf{W}\bar{\mathbf{b}}_{c,j})^2 \leq 0. \quad (30)\end{aligned}$$

II. $u_{lg,j} + u_{hg,j} \leq -u_{r,j}$

$$\bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{b}}_{c,j}v_j = \bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{b}}_{c,j}(-u_{r,j} + \mathbf{k}_j\bar{\mathbf{e}}_c). \quad (31)$$

As $-\mathbf{k}_j\bar{\mathbf{e}}_c \geq -c_{1,j}$, considering the definitions of $c_{1,j}$ and $\mathbf{u}_r(\tau)$, we have

$$-\mathbf{k}_j\bar{\mathbf{e}}_c \geq -u_{r,j} \Rightarrow -u_{r,j} + \mathbf{k}_j\bar{\mathbf{e}}_c \leq 0. \quad (32)$$

On the other hand,

$$\begin{aligned}&u_{lg,j} + u_{hg,j} \leq -u_{r,j} \Rightarrow u_{hg,j} \leq -u_{r,j} - u_{lg,j} \\ &\Rightarrow -l\bar{\mathbf{b}}_{c,j}^T\mathbf{W}\bar{\mathbf{e}}_c \leq -u_{r,j} + \mathbf{k}_j\bar{\mathbf{e}}_c. \quad (33)\end{aligned}$$

From (32) and (33), it can be derived that $\bar{\mathbf{b}}_{c,j}^T\mathbf{W}\bar{\mathbf{e}}_c > 0$. Hence, from (31), $\bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{b}}_{c,j}v_j < 0$.

III. $u_{lg,j} + u_{hg,j} \geq \phi_{j,i}m_i - u_{r,j}$

Similarly to the proof in *II*, $\bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{b}}_{c,j}v_j < 0$ can be derived.

From *I*, *II* and *III*, $\dot{V} < -\bar{\mathbf{e}}_c^T\mathbf{Q}\bar{\mathbf{e}}_c$. Hence, $\epsilon(\mathbf{W}, \rho)$ is a positively invariant region. As $\bar{\mathbf{e}}_c(0) \in \epsilon(\mathbf{W}, \rho)$, $\bar{\mathbf{e}}_c(\tau) \in \epsilon(\mathbf{W}, \rho)$ for all $\tau \geq 0$. Therefore, since $\dot{V} < -\bar{\mathbf{e}}_c^T\mathbf{Q}\bar{\mathbf{e}}_c$, $\bar{\mathbf{e}}_c$, $\bar{\mathbf{e}}$ and $\mathbf{e}$ asymptotically converge to zero. Furthermore, the convergence of $\bar{\mathbf{e}}_c$ leads to the convergence of $\mathbf{u}$ to $\mathbf{u}_r(\tau)$.

Case 2. $\mathbf{c}_1$ have zero-elements

The convergence analysis is quite similarly to that in **Case 1**. From (19), the error dynamics can be rewritten as follows:

$$\begin{aligned}\dot{\bar{\mathbf{e}}}_c &= [\bar{\mathbf{A}}_c - (\bar{\mathbf{B}}_c - \Delta\bar{\mathbf{B}}_c)\mathbf{K}]\bar{\mathbf{e}}_c + \bar{\mathbf{B}}_c[sat(\mathbf{u}_{lg} + \mathbf{u}_{hg}) \\ &\quad + \mathbf{K}\bar{\mathbf{e}}_c] - \Delta\bar{\mathbf{B}}_c\mathbf{K}\bar{\mathbf{e}}_c. \quad (34)\end{aligned}$$

As $\Delta\bar{\mathbf{B}}_c\mathbf{K} = \mathbf{0}$, (34) becomes

$$\dot{\bar{\mathbf{e}}}_c = [\bar{\mathbf{A}}_c - (\bar{\mathbf{B}}_c - \Delta\bar{\mathbf{B}}_c)\mathbf{K}]\bar{\mathbf{e}}_c + \bar{\mathbf{B}}_c\mathbf{v}. \quad (35)$$

Define the same Lyapunov function, considering the relationship (22), (29) can also be obtained here.

Since $\forall \bar{\mathbf{e}}_c \in \epsilon(\mathbf{W}, \rho)$ and $\forall j \in M$, $-\mathbf{k}_j\bar{\mathbf{e}}_c \geq -c_{1,j}$ and $-\mathbf{k}'_j\bar{\mathbf{e}}_c \leq c_{2,j}$, analogously to **Case 1**, $\bar{\mathbf{e}}_c^T\mathbf{W}\bar{\mathbf{b}}_{c,j}v_j < 0$ ($\forall j \in M$) can be derived. Therefore, $\dot{V} < -\bar{\mathbf{e}}_c^T\mathbf{Q}\bar{\mathbf{e}}_c$. The global asymptotical convergence of $\mathbf{e}$ and $\mathbf{u}$ can be obtained. ■

6. ILLUSTRATIVE EXAMPLE

Consider the JF net in Figure 1 with $\boldsymbol{\lambda} = [1, 1, 2, 1, 2]^T$. Hence, $\boldsymbol{\Phi} = diag(1, 1, 1, 1, 1)$. The minimal P-semiflow is $\mathbf{y} = [1, 1, 1, 2, 2]^T$. Adding elementary vectors, $\mathbf{H}$ is chosen as $\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

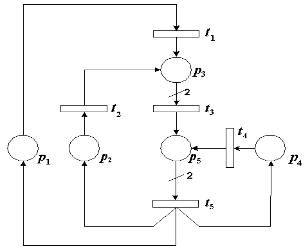

Fig. 1. Timed Join-Free Net System.

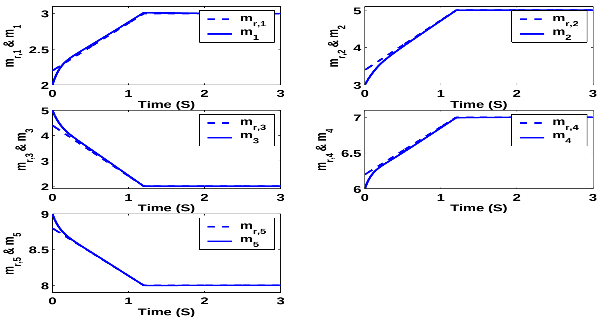

Fig. 2. Convergence of markings.

Assume an initial marking as $\mathbf{m}_0 = [2, 3, 5, 6, 9]^T$ and a desired marking $\mathbf{m}_d = [3, 5, 2, 7, 8]^T$. To maximize the flows of the steady state, the desired final control input $\mathbf{u}_d = [1, 3, 0, 5, 6]^T$.

Based on $\mathbf{m}_0$ and $\mathbf{m}_d$, $\boldsymbol{\sigma} = [1, 0, 2, 1, 2]^T + \alpha[1, 1, 1, 1, 1]^T$ where $\alpha \geq 0$. Here we randomly choose $\alpha = 1$, hence $\boldsymbol{\sigma} = [2, 1, 3, 2, 3]^T$. Considering $\boldsymbol{\sigma}$, $\mathbf{m}_0$ and $\mathbf{m}_d$, $\beta = 1.5$ is the minimum value such that $\frac{\boldsymbol{\sigma}}{\beta} \leq \boldsymbol{\Phi} \min\{\mathbf{m}_0, \mathbf{m}_d\}$. According to (12) and (13), it can be derived that $\mathbf{u}_{r0} = [0.6667, 2.3333, 3, 4.6667, 7]^T$ and $\mathbf{u}_{rh^-} = [1.6667, 4.3333, 0, 5.6667, 6]^T$. Hence, $\mathbf{c}_1 = [0.6667, 2.3333, 0, 4.6667, 6]^T$. As $\mathbf{c}_1$ has one zero-element, $\mathbf{u}_{lg}$ is calculated based on (22). Let $\delta = 0.2$ and $\gamma = 5$. It is easy to verify that $\forall j \in \{1, 2, 3, 4, 5\}$, both (25) and (26) are valid. Then, $h = \beta(1 - \delta) = 1.2$. For simplicity, choose $\mathbf{Q} = \mathbf{I}_{4\times 4}$ and $\mathbf{R} = \mathbf{I}_{5\times 5}$ for the low-gain design. For the high-gain term, any positive l can guarantee the tracking convergence. Generally, smaller l will lead to slower system response. However, due to the existence of input saturation, when l is sufficiently large, the system responses have little difference by further increasing l. In our simulation, we choose $l = 10$.

The simulation results are shown in Figures 2 and 3. Figure 2 illustrates the convergence of the markings under the designed control law. Figure 3 shows the control signals $\mathbf{u}$ and the state-related upper bound, i.e. $\phi_{j,i} m_i$. (Note the net structure determines that $\forall j \in \{1, 2, 3, 4, 5\}$, $i = j$.) It can be seen that $0 \leq u_j \leq \phi_{j,i} m_i$ and the final control signals converge to the desired ones.

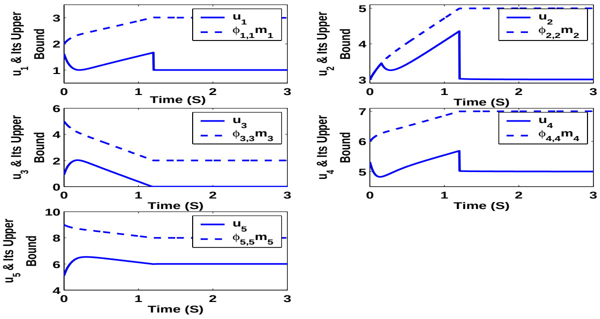

Fig. 3. Control signals.

7. CONCLUSION

The main concern of our work is to construct proper control laws for step-tracking of timed contPN systems in presence of the existing state-related input constraints. To guarantee global convergence and smoothness of states and control signals, the design method for a step-ramp tracking trajectory has been outlined. With the new tracking target, a novel low-and-high gain control method has been further proposed.

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