DESIGNING SWITCHED OBSERVERS FOR SWITCHED SYSTEMS USING MULTIPLE LYAPUNOV FUNCTIONS AND DWELL-TIME SWITCHING

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Abstract: In this paper, observers are synthesized for switched linear systems, resulting in switched observers including state jumps. The synthesis problem involves multiple Lyapunov functions and is formulated as a linear matrix inequality problem. It is assumed that the current mode (active dynamics) of the switched linear is unknown, and it will be shown that the estimate of the continuous states will be bounded at worst, if the mode is wrongly estimated. If the active dynamics is estimated correctly within a certain time, and the dwell time of the switched linear system is lower bounded, it will be shown that the bound of the estimation error can be reduced significantly.

Keywords: Switched systems; Hybrid systems; Observer design; Linear matrix inequalities (LMI); Lyapunov stability.

1. INTRODUCTION

A large class of systems is reasonably modelled by a family of continuous-time subsystems and logic rules that govern the switchings between them. In this paper, we are interested in the estimation problem of such *switched system*, and switched observers including state jumps are synthesized. The synthesis problem how to design the observer gains, or showing stability for existing observer gains, will be formulated as a linear matrix inequality problem.

Existing synthesis results can be divided into two categories depending on whether the discrete state (active dynamics or mode) of the switched system is known or not. The estimation problem simplifies significantly if the active dynamics is known (since the mode of the observer can change correspondingly), and synthesis results are proposed guaranteeing that the estimation error converges to zero if certain conditions are satisfied, see for instance (Alessandri and Coletta, 2001; Feron, 1996). However, if the active dynamics is unknown and needs to be estimated together with the continuous state, there are quite a few synthesis results. If the discrete mode is estimated from the continuous part of the switched system, there are not yet any synthesis results that are applicable to a large class of switched (linear) systems. Results so far only guarantee that the estimation error of the continuous state is bounded, but it cannot be shown that it goes to zero since there are no guarantees that the active dynamics is correctly estimated, cf. (Juloski *et al.*, 2002).

It is common to use a quadratic Lyapunov function when showing estimation error (stability) properties of the underlying switched observer, see for instance (Alessandri and Coletta, 2001; Juloski *et al.*, 2002). By using a common quadratic Lyapunov function, stability is guaranteed regardless of the mode switches in the system (and observer). However, the existing results are conservative since the estimation error might converge or be bounded without the existence of a common Lyapunov function. By introducing multiple quadratic Lyapunov functions, one for each observer mode, the conservatism can be relaxed, implying that a larger class of switched (linear) systems can be handled, see (Pettersson, 2005*a*; Pettersson, 2005*b*).

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In this paper, we will study the estimation problem in the case when the active dynamics of the switched linear is unknown. A generic illustration of the observer is given in Figure 1, cf. (Balluchi *et al.*, 2002). The observer is divided into two parts:

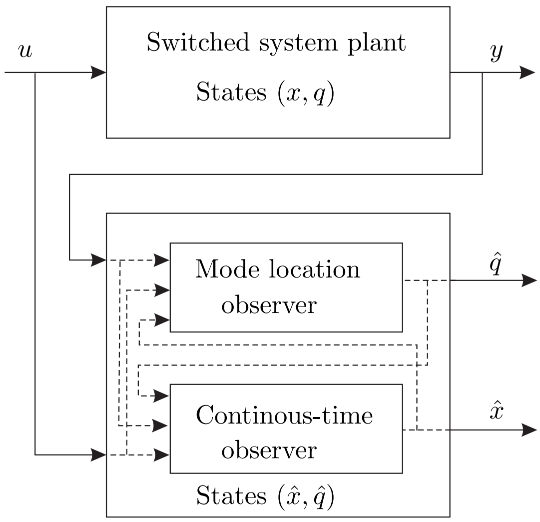

Fig. 1. Switched observer of a switched system.

the *Mode location observer* estimating the active dynamics and the *Continuous-time observer* estimating the continuous state of the switched system. We will focus on the continuous-time estimation problem in this paper, and not give any details of the *Mode location observer*. If the active dynamics needs to be estimated, we cannot show that the estimation error of the continuous-time state converges to zero since there is a possibility that a wrong location mode is estimated during a certain time which means that also the continuous state is estimated wrongly. However, it will be shown that the estimation error of the continuous state will be bounded, similar to the results in (Juloski *et al.*, 2002; Pettersson, 2005*b*). In this paper, we will improve the results regarding the precision of the bound of the estimation error, which is possible if it is assumed that the active dynamics is estimated correctly within a certain time, and the dwell time of the switched linear system is lower bounded. The first assumption is reasonable since otherwise the design of the *Mode location observer* is not very good. The second assumption is of no practical importance; see the comments in the next section.

The outline of this paper is: we start by defining the switched linear system model in the next section, followed by a detailed description of the switched observer with state jumps. In Section 4, the observer synthesis problem is formulated, followed by a section explaining how to solve the problem using linear matrix inequalities. Finally, the method is applied to an example.

2. SWITCHED LINEAR SYSTEM

The switched linear systems considered in this paper are described by the equations

$$\dot{x} = A_{q(t)}x + Bu, \quad y = Cx, \tag{1}$$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the input vector, $y \in \Re^p$ is the measurement (output) vector and $q(t)$ is an index function (discrete state) $q : [0\ \infty) \rightarrow I_N = \{1, \ldots, N\}$ deciding which one of the linear vector fields that is active at a certain time instant. Each of the indexes corresponds to a different model description and is referred to as a *mode* of the switched linear system. By *active dynamics* we mean the active subsystem, or model description, of (1).

The change of value of the index function occurs at certain times, which are defined by the set $\mathcal{T}$. One possibility is to define switch sets $S_{i,j} \subset \Re^n$, $(i, j) \in I_s$, where I_s is a set of tuples indicating which mode changes that might occur in the switched system. If $q(t) = i$ and the trajectory reach a state in $S_{i,j}$ at time t^+, then $q(t^+) = j$.

We will assume that there are only a finite number of mode changes in finite time. This does not exclude sliding motions, since if sliding motions occur in the switched system, new modes corresponding to the sliding modes are additionally introduced. The dynamics associated with the sliding mode is given by a (unique) vector field specified for instance by Fillipov's convex combination (Filippov, 1988). Then, a switched system with an equivalent dynamics is obtained, where there is a finite number of switches of the modes in finite time. The observer is designed for this equivalent switched system dynamics. To improve the results regarding the precision of the bound of the estimation error, we will later give results where the dwell time of the equivalent dynamics of a switched linear system is at least a time T.

3. SWITCHED OBSERVER WITH STATE JUMPS

The dynamics of the *Continuous-time observer* is defined as follows:

$$\dot{\hat{x}} = A_{\hat{q}(t)}\hat{x} + Bu + K_{\hat{q}(t)}(y - \hat{y}), \quad \hat{y} = C\hat{x}, \tag{2}$$

where $\hat{x} \in \Re^n$ is the estimate of the state vector x and $K_j \in \Re^{n\times p}$, $j \in I_N$, are the observer gains. The index function $\hat{q} : [0\ \infty) \rightarrow I_N = \{1, \ldots, N\}$ decides which one of the observer modes that is active at a certain time instant, and is the output of the *Mode location observer*, see Figure 1.

The purpose of the *Mode location observer* is to estimate the current mode q of the switched system, but we will not specify the details since we will focus on properties of the continuous state estimate in this paper. We merely indicate by $\hat{\mathcal{T}}$ the set of times when the *Mode location observer* switches mode, which are the times when $\hat{q}$ changes value. If the *Mode location observer* never estimates the correct discrete mode, the estimation error bound will be very conservative (related to the worst combination of system mode dynamics and observer mode dynamics), see (Pettersson, 2005*b*). However, if the *Mode location observer* estimates the correct discrete states part of the time, say within Δ time units, the bound of the estimation error can be reduced significantly, shown later on.

There is no guarantee that the estimation error will converge even if the active mode of the switched system is known and the observer gains K_i is designed such that the estimation error of each subsystem converges. What is further needed in the observer design is to properly update the estimated states of the observer, at the times in $\hat{\mathcal{T}}$ when the observer mode changes occur. If observer mode i is active and a mode change occur, the estimate $\hat{x}$ will abruptly be changed (jump) to $\hat{x}^+$, where $\hat{x}^+$ indicates the updated value of $\hat{x}$. More specifically, the estimated state jumps will be updated according to

$$\hat{x}^+ = T_1\hat{x} + T_2 y, \quad t \in \hat{\mathcal{T}},$$

which only depends on the observer states $\hat{x}$ and the measured value y. In the next section, we will show how to calculate T_1 and T_2, guaranteeing that the error between the estimated states and the states of the switched system is bounded.

4. OBSERVER SYNTHESIS

The estimation error dynamics obeys the equation

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = (A_{\hat{q}} - K_{\hat{q}}C_{\hat{q}})\tilde{x} + [A_q - A_{\hat{q}}]x.$$

Let us introduce multiple Lyapunov functions, one for each observer mode i,

$$V_i(\tilde{x}) = \tilde{x}^T P_i \tilde{x}, \quad i \in I_N,$$

where each $P_i \in \Re^{n\times n}$ is a symmetric matrix. The time derivative for the observer mode i, when the system state evolves according to mode j, becomes

$$\begin{aligned}\dot{V}_i(\tilde{x}) &= \tilde{x}^T([A_i - K_iC]^T P_i + P_i[A_i - K_iC])\tilde{x} \\ &\quad + \tilde{x}^T P_i(A_j - A_i)x + x^T(A_j - A_i)^T P_i\tilde{x}.\end{aligned} \tag{3}$$

We are now ready for the main theorem. If desirable, we can associate regions $x^TQ_ix \geq 0$ to the switched system (1) where mode i is possible, see (Pettersson and Lennartson, 2002). If not desirable, the $\mu_{i,j}$'s in the theorem is put to zero. The advantage of specifying regions where mode i is possible is to improve the bound given in the theorem. This is one form of relaxation which is similar to the one in (Juloski *et al.*, 2002).

Theorem 1. If there exist a solution to $(\epsilon \geq 0,\ \alpha > 0,\ \mu_{i,j} \geq 0,\ \gamma \geq 0)$

1. $\alpha I \leq P_i \leq \beta I, \qquad i \in I_N$
2. $\Gamma_{i,j} = \begin{bmatrix} \Gamma_{i,j}^{11} & \Gamma_{i,j}^{12} \\ (\Gamma_{i,j}^{12})^T & \Gamma_{i,j}^{22} \end{bmatrix} \leq 0, \qquad (i,j) \in I_s$
3. $P_j = P_i + d_{i,j}^T C + C^T d_{i,j}, \qquad (i,j) \in I_s$

where

$$\begin{aligned}\Gamma_{i,j}^{11} &= (A_i - K_iC)^T P_i + P_i(A_i - K_iC) + \gamma I \\ \Gamma_{i,j}^{12} &= P_i(A_j - A_i) \\ \Gamma_{i,j}^{22} &= \mu_{i,j}Q_j - \gamma\epsilon^2 I\end{aligned}$$

and the states of the hybrid observer is updated according to [2]

$$\begin{aligned}&\hat{x}^+ = (I - R_i^{-1}(CR_i^{-1})^\dagger C)\hat{x} + R_i^{-1}(CR_i^{-1})^\dagger y \\ &\forall t \in \hat{\mathcal{T}}\end{aligned} \tag{4}$$

then if for some $T_0 > 0$

$$\sup_{t>T_0} ||x(t)|| \leq x_{max}, \tag{5}$$

we have

$$\lim_{t\to\infty} \sup ||\tilde{x}(t)|| \leq \sqrt{\frac{\beta}{\alpha}}\epsilon x_{max}. \tag{6}$$

Furthermore, if the switched system (1) is in every mode at least a time T and it takes $\Delta \leq T$ to identify correct mode of the switched system, then we have

$$\lim_{t\to\infty} \sup ||\tilde{x}(t)|| \leq \left(\frac{e^{-\frac{\gamma}{\beta}(T-\Delta)} - e^{-\frac{\gamma}{\beta}T}}{1 - e^{-\frac{\gamma}{\beta}T}}\right)\sqrt{\frac{\beta}{\alpha}}\epsilon x_{max}. \tag{7}$$

Proof: We need to prove that the overall energy function $V(\tilde{x}(t))$ eventually is upper bounded by a constant. To do this, we will show that the energy decreases at the switching instants in $\hat{\mathcal{T}}$ when changing observer modes and that the energy in every observer mode eventually is upper bounded by a constant (regardless of the system mode). We begin by the first part and have to show that

$$(x - \hat{x}^+)^T P_j(x - \hat{x}^+) \leq (x - \hat{x})^T P_i(x - \hat{x}). \tag{8}$$

Let $\hat{x}^+$ be an arbitrary estimated state satisfying $y = C\hat{x}^+$. Since also $y = Cx$, we have

$$C(x - \hat{x}^+) = y - y = 0,$$

implying that $(x-\hat{x}^+)^T(d_{i,j}^TC + C^Td_{i,j})(x - \hat{x}^+) = 0$. Due to the relation in Condition 3, it means that (8) becomes

$$(x - \hat{x}^+)^T P_i(x - \hat{x}^+) \leq (x - \hat{x})^T P_i(x - \hat{x}), \tag{9}$$

and it remains to choose $\hat{x}^+$ satisfying $y = C\hat{x}^+$ such that this inequality is satisfied.

[2] $(*)^\dagger$ is the *pseudoinverse* of $(*)$, see (Strang, 1988).

Factorize P_i as $P_i = R_i^T R_i$, where $R_i \in \Re^{n\times n}$ is a symmetric positive definite matrix. This is always possible since P_i is a real symmetric positive definite (imposed by Condition 1) matrix, see (Strang, 1988). One choice is for instance,

$$R_i = V_i\sqrt{\Lambda_i}V_i^T,$$

where $V_i \in \Re^{n\times n}$ is the orthonormal eigenvectors of P_i and $\sqrt{\Lambda_i}$ is a diagonal matrix consisting of the square root of the (positive) eigenvalues of P_i. Now, Condition (9) is equivalent to show that

$$||R_i(x - \hat{x}^+)|| \le ||R_i(x - \hat{x})||, \qquad (10)$$

is fulfilled where $\hat{x}^+$ satisfies $y = C\hat{x}^+$.

We are now interested to find the updated value $\hat{x}^+$, lying on the hyper plane $y = C\hat{x}^+$, that minimizes the distance $||R_i(\hat{x}^+ - \hat{x})||$. This optimization problem can formally be defined as

$$\begin{aligned}&\min_{\hat{x}^+} ||R_i(\hat{x}^+ - \hat{x})||\\ &\text{subject to: } C\hat{x}^+ = y\end{aligned} \qquad (11)$$

which is geometrically illustrated in Figure 2.

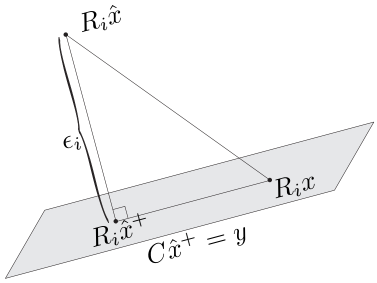

Fig. 2. The projection of $R_i\hat{x}$ onto the plane $C\hat{x}^+ = y$, resulting in the point $R_i\hat{x}^+$.

By introducing $\epsilon_i = R_i(\hat{x}^+ - \hat{x})$, we have $R_i\hat{x}^+ = \epsilon_i + R_i\hat{x}$, leading to the optimization problem

$$\begin{aligned}&\min ||\epsilon_i||\\ &\text{subject to: } CR_i^{-1}\epsilon_i = y - C\hat{x}\end{aligned}$$

The solution to this problem, the minimum length least squares solution to $y - C\hat{x}$, is

$$\epsilon_i = (CR_i^{-1})^\dagger(y - C\hat{x}).$$

Hence, $R_i\hat{x}^+ = R_i\hat{x} + (CR_i^{-1})^\dagger(y - C\hat{x})$, which is equivalent to (4) after a multiplication of R_i^{-1} from the left.

It remains to show that the condition in (10) is satisfied for the state jump update (4). By construction, the vectors $R_i(\hat{x}^+ - \hat{x})$ and $R_i(x - \hat{x}^+)$ are orthogonal; otherwise ϵ_i would not be optimal. Hence, by Pythagoras' law

$$\begin{aligned}&||R_i(x-\hat{x})||^2 = ||R_i(x-\hat{x}^+) + R_i(\hat{x}^+ - \hat{x})||^2 =\\ &= ||R_i(x-\hat{x}^+)||^2 + \underbrace{2(x-\hat{x}^+)^T R_i^T R_i(\hat{x}^+ - \hat{x})}_{0} +\\ &+ ||R_i(\hat{x}^+ - \hat{x})||^2 \ge ||R_i(x - \hat{x}^+)||^2,\end{aligned}$$

where the inequality is true since $||R_i(\hat{x}^+ - \hat{x})|| \ge 0$. Hence, we have shown that (10) and consequently (8) is satisfied, ending the first part of the proof.

We now need to prove that the energy in every observer mode eventually is upper bounded by a constant (regardless of the system mode). By adding and subtracting $\gamma\tilde{x}^T\tilde{x}$, $-\gamma\epsilon^2x^Tx$, and $\mu_{i,j}x^TQ_ix$ (where $\mu_{i,j} \ge 0$), $\dot{V}_i$ in (3) becomes

$$\begin{aligned}\dot{V}_i(\tilde{x}) &= [\tilde{x}^Tx^T]\Gamma_{i,j}[\tilde{x}^Tx^T]^T - \mu_{i,j}x^TQ_ix\\ &\quad -\gamma\tilde{x}^T\tilde{x} + \gamma\epsilon^2x^Tx\\ &\le -\gamma\tilde{x}^T\tilde{x} + \gamma\epsilon^2x^Tx \le -\frac{\gamma}{\beta}V_i(\tilde{x}) + \gamma\epsilon^2x_{max}^2,\end{aligned}$$

where the first and second inequality is due to Condition 2 and the fact that $-\mu_{i,j}x^TQ_ix \le 0$ (since $\mu_{i,j} \ge 0$ and $x^TQ_ix \ge 0$ in regions where mode i of the switched system (1) is possible), and Condition 1 and (5) respectively. This differential inequality implies that

$$\begin{aligned}V_i(\tilde{x}(t)) &\le e^{-\frac{\gamma}{\beta}(t-t_0)}V_i(\tilde{x}(t_0))\\ &\quad + \beta\epsilon^2x_{max}^2(1 - e^{-\frac{\gamma}{\beta}(t-t_0)})\\ &\le e^{-\frac{\gamma}{\beta}(t-t_0)}V_i(\tilde{x}(t_0))\\ &\quad + \beta\epsilon^2x_{max}^2(1 - e^{-\frac{\gamma}{\beta}(t-t_0)}),\end{aligned} \qquad (12)$$

where $t_0 \ge T_0$. Consequently, the overall energy $V(\tilde{x}(t))$ decreases at the switching instants and is upper bounded by a constant. Due to Condition 1, we then have

$$\begin{aligned}||\tilde{x}(t)|| &\le \Big(e^{-\frac{\gamma}{\beta}(t-t_0)}V(\tilde{x}(t_0))/\alpha\\ &\quad + \frac{\beta}{\alpha}\epsilon^2x_{max}^2(1 - e^{-\frac{\gamma}{\beta}(t-t_0)})\Big)^{\frac{1}{2}}.\end{aligned} \qquad (13)$$

Hence, when $t \to \infty$ the exponential functions converge to zero implying that (6) is satisfied.

To prove that (7) is satisfied, we assume that the switched system (1) is in every mode at least a time T and it takes $\Delta \le T$ to identify correct mode of the switched system. When the switched system changes mode at time, say, $t_0 \in \mathcal{T}$, to mode i, the observer is still in mode j during a time Δ. Hence, from (12) we have that

$$\begin{aligned}&V(\tilde{x}(t)) = V_j(\tilde{x}(t)) \le e^{-\frac{\gamma}{\beta}(t-t_0)}V_j(\tilde{x}(t_0)) +\\ &\beta\epsilon^2x_{max}^2(1 - e^{-\frac{\gamma}{\beta}(t-t_0)}),\ t_0 \le t \le t_0 + \Delta.\end{aligned}$$

At time $t_0 + \Delta$ ($\in \hat{\mathcal{T}}$), we identify correct mode of the system, and the observer changes correspondingly. The mode is correctly estimated at least in the time interval $t_0 + \Delta$ to $t_0 + T$ according to the assumptions. During this time interval,

$$\begin{aligned}&V(\tilde{x}(t)) = V_i(\tilde{x}(t)) \le\\ &e^{-\frac{\gamma}{\beta}(t-t_0-\Delta)}V_i(\tilde{x}(t_0+\Delta)),\ t_0 + \Delta \le t \le t_0 + T,\end{aligned}$$

since Condition 2 implies that $\Gamma_{i,i}^{11} < 0$ with $\epsilon = 0$. At the switch time $t_0 + \Delta$, the state $\hat{x}$ is updated according to (4), implying that $V_i(\tilde{x}(t_0 + \Delta)) \le V_j(\tilde{x}(t_0+\Delta))$. Combining this with the two inequalities, implies that we at time $t_0 + T$ have

$$\begin{aligned}
&V_i(\tilde{x}(t_0+T)) \leq e^{-\frac{\gamma}{\beta}(T-\Delta)} V_i(\tilde{x}(t_0+\Delta)) \\
&\leq e^{-\frac{\gamma}{\beta}(T-\Delta)} V_j(\tilde{x}(t_0+\Delta)) \\
&\leq e^{-\frac{\gamma}{\beta}(T-\Delta)} \Big(e^{-\frac{\gamma}{\beta}\Delta} V_j(\tilde{x}(t_0)) \\
&+\beta\epsilon^2 x_{max}^2 (1-e^{-\frac{\gamma}{\beta}\Delta}) \Big) \\
&\leq e^{-\frac{\gamma}{\beta}T} V_j(\tilde{x}(t_0)) \\
&+\beta\epsilon^2 x_{max}^2 (e^{-\frac{\gamma}{\beta}(T-\Delta)} - e^{-\frac{\gamma}{\beta}T}) \Big) .
\end{aligned}$$

By repeatedly switchings, the energy converges (at worst) to the bound

$$V_{max} = \beta\epsilon^2 x_{max}^2 \left(\frac{e^{-\frac{\gamma}{\beta}(T-\Delta)} - e^{-\frac{\gamma}{\beta}T}}{1-e^{-\frac{\gamma}{\beta}T}} \right),$$

which gives (7) due to Condition 1, ending the proof. ■

A sufficient condition for the existence of a solution to the inequalities in the theorem is that $\Gamma_{i,j}^{11} < 0$ in Condition 2. This is the formulation of the estimation problem assuming that the system mode is known. In this case, the estimation error obeys (cf. (13) in case when $\epsilon = 0$ and $t_0 = 0$)

$$||\tilde{x}(t)|| \leq \sqrt{\frac{\beta}{\alpha}} e^{-\frac{\gamma}{2\beta}t} ||\tilde{x}_0||,$$

implying that $||\tilde{x}(t)||$ goes to zero as time goes to infinity regardless of the value of $x(t)$. When we do not know the mode, which is handled in the theorem, we cannot say that the estimation error goes to zero but is upper bounded according to (6), which depends on the largest value of $||x(t)||$. This bound is usually very conservative, indicated by the example later on, since it is obtained having the worst possible combination of observer mode and system mode. However, the result shows that if the active dynamics is estimated correctly within a certain time (Δ), and the dwell time of the switched linear system is lower bounded (by T), the bound of the estimation error can be reduced significantly according to (7). If $\Delta \to 0$ the bound becomes zero, but if we do not succeed to estimate the discrete state correctly until the next mode change of the switched system, implying that $\Delta \to T$, then we again get (6). Note that if the switched system stops to switch, and the correct mode is identified, the estimation error will converge to zero.

Except the properly updates according to (4), the theorem uses multiple Lyapunov functions, which increases the possibility to find the unknown variables satisfying the conditions in the theorem. Using a common Lyapunov function (corresponds to $d_{i,j} = 0$ in the theorem) to prove convergence, the energy decrease condition (9) is trivially satisfied by letting $\hat{x}^+ = \hat{x}$, i.e. no updates of the estimated states are necessary. However, also in this case, the updates of the estimated states according to (4) will improve the real convergence rate and should be used also in case when a common Lyapunov function is searched for.

5. SOLUTION USING LINEAR MATRIX INEQUALITIES

Theorem 1 has to be valid whether the observer gains K_i are decided *a priori* or not. The unknowns in Theorem 1 will be found by iteratively fixing ϵ to a value and search for the smallest β satisfying the conditions to find a low bound on the right-hand side of (6). If there is no solution for the fixed value of ϵ, the value is increased. Furthermore, without loss of generality, α is scaled to 1 to prevent the P_i's to be positive semi-definite.

Whether the observer gains K_i are decided *a priori* or not, Theorem 1 can be reformulated as a linear matrix inequality (LMI) problem in the unknown variables P_i, $d_{i,j}$, $\mu_{i,j}$, γ and possible K_i. In the case of unknown $K_i's$ they have to be constrained in some way to prevent them from being too large. One possibility is to introduce the condition

$$\begin{bmatrix} \lambda_i^2 I_{p\times p} & W_i^T \\ W_i & I_{n\times n} \end{bmatrix} \geq 0, \quad i \in I_N$$

implying that $K_i^T K_i \leq \lambda_i^2 I_{p\times p}$, cf. (Pettersson, 2005*a*; Pettersson, 2005*b*).

6. EXAMPLE

We now illustrate the observer synthesis in this paper in case of two modes of the (autonomous) switched linear system (1) given by

$$A_1 = \begin{bmatrix} 1 & -5 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 \\ -2.4 \end{bmatrix}^T.$$

The system and observer time switchings are indirectly given by specifying switch sets defined by linear hyper planes according to

$$S_{i,j} = \{x \in \Re^n \mid s_{i,j} x = 0\},\ (i,j) \in \{(1,2),(2,1)\},$$

$$\hat{S}_{i,j} = \{\hat{x} \in \Re^n \mid \hat{s}_{i,j} \hat{x} = 0\},\ (i,j) \in \{(1,2),(2,1)\},$$

where the switch planes of the observer are put equal to the switch planes of the system to mimic the switching behavior, according to

$$s_{1,2} = \hat{s}_{1,2} = [1.56 \ \ 1], \quad s_{2,1} = \hat{s}_{2,1} = [1 \ \ -1.56].$$

We assume that the design of the the observer gains is not known *a priori* but is a part of the synthesis problem. We will study the solution in case when $\lambda = \lambda_1 = \lambda_2 = 5$.

It should be noted that there does not exist a solution to Theorem 1 with a common P; hence, the suggested observer synthesis in this paper is less conservative than existing results using a common quadratic Lyapunov function. Solving the corresponding LMI problem of Theorem 1 with multiple Lyapunov functions, results in a solution

$$K_1 = \begin{bmatrix} 1.79 - 1.02 \end{bmatrix}^T,\ K_2 = \begin{bmatrix} -3.44 - 3.54 \end{bmatrix}^T,$$

with $\alpha = 1$, $\beta = 5.70$, $\epsilon = 4.98$, $\gamma = 1.88$. According to (6) in the theorem, we therefore have the bound $||\tilde{x}(t)|| \leq 11.87x_{max}$. The switched system changes mode every 0.45 time units; hence $T = 0.45$. The bound can be improved according to (7) if we can identify the correct mode within a time $\Delta < T$.

Figure 3 shows a trajectory simulation x of the switched linear system, in the case when $x_{max} = 1$, together with the estimated states $\hat{x}$ updating the estimator states according to (4) at the switching instants. As can be seen from the figure, the estimated states converge to the switched linear states exactly. Consequently, the times $\Delta \to 0$.

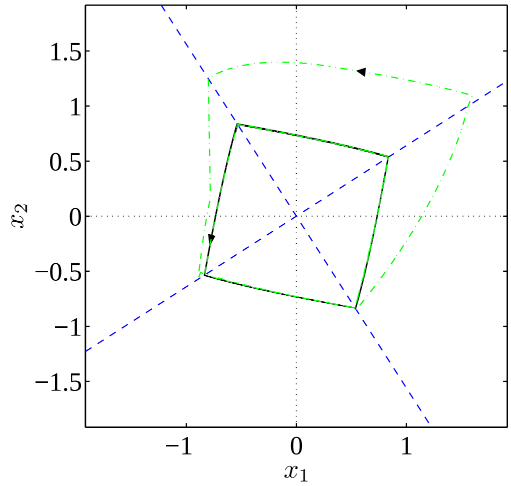

Fig. 3. The estimated states $\hat{x}$ (dash-dotted) converges to the switched linear system states x (solid line) using the projection.

To compare, a trajectory simulation of the estimated states $\hat{x}$ when not updating the estimator states according to (4) at the switching instants is shown in Figure 4. In this case, it can be seen that the estimated states converge to a limit cycle. Hence, it is advantageous to update the continuous estimator states at the switching instants.

7. CONCLUSIONS

In this paper, it has been shown how to estimate the continuous states of a switched linear systems by designing a switched observer including state jumps. By using multiple Lyapunov functions and properly update the continuous estimated states when the mode changes occur, an observer is synthesized by solving a linear matrix inequality problem. The bound of the estimation error is reduced compared to earlier result, if it can be shown that the active dynamics is estimated correctly within a certain time, and if the dwell time of the switched linear system is lower bounded. Future research will deal with the question how to guarantee that the estimation error actually goes to zero, as in the example.

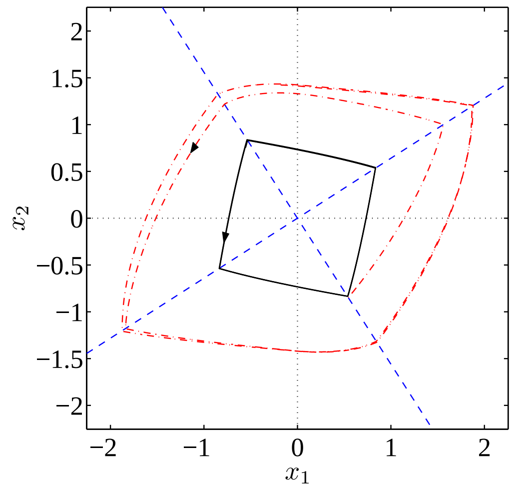

Fig. 4. The estimated states $\hat{x}$ (dash-dotted) converges not to the switched linear system states x (solid line) since projection is not used.

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