OBSERVER DESIGN FOR A CLASS OF DISCRETE TIME PIECEWISE-LINEAR SYSTEMS

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Abstract: In this paper we consider a class of discrete time piecewise-linear systems composed by linear discrete time LTI subsystems with autonomous switching. The aim is to propose a method for the synthesis of a hybrid observer. The approach proposed here consists on the combination of a discrete observer and a continuous observer. It is shown that under conditions related to minimum dwell time of each mode, one can express the switching law as a linear combination of the system input/output samples. The continuous observer is a piecewise-linear observer whose dynamics depend on the current active mode. Two estimation schemes are analyzed : on−line estimation and off−line estimation. *Copyright© 2006 IFAC.*

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1. INTRODUCTION

Observer design for hybrid systems is an important and challenging problem. Applications for control or fault detection purposes is of first importance. Recently, many papers have considered such a problem. A discussion on observability conditions for switched linear systems is proposed in (Vidal *et al.*, 2004). Moving horizon estimation strategy is discussed in (Ferrari-Trecate *et al.*, 2002) for piecewise-linear systems. Piecewise-linear systems are an important class of hybrid dynamic systems which has attracted a growing interest (Bemporad *et al.*, 2000),(Johansson, 2003),(Liberzon, 2003). In (Balluchi *et al.*, 2002), a hybrid observer is proposed for systems with a hybrid automaton description. The scheme of the proposed observer consists of two blocs: a discrete observer, based on the discrete event dynamic framework, and a continuous observer, based on the classical state observation theory. The former identifies the current mode, while the latter produces an estimate of the evolution of the continuous state of the hybrid system. In this paper, we study the observation problem for a class of piecewise-affine hybrid systems where the switching law depends on the continuous state vector. The observation scheme proposed here consists on the combination of an active mode detection and continuous observer. This paper is organized as follows. First, we present a solution for detecting the switching time instant and the corresponding active mode. This method is the transposition in discrete time of the idea introduced in (Benali *et al.*, 2004) for the continuous time case. The adaptation to the discrete time case is not immediate and introduces some specificities. The continuous observer is based on the switching observer developed in (Daafouz *et al.*, 2002), whose dynamics depend on the active mode provided by the discrete observer. Two configurations are considered

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: on−line gain attribution, where the two parts of the observer work simultaneously, off−line gain attribution, where the hybrid observer works with a delay compared to the real system. We finish by an illustrative example and a conclusion.

2. PROBLEM STATEMENT

We consider the class of discrete time piecewise-linear systems given by :

$$\begin{cases} x_{k+1} = A_i x_k + B_i u_k & \text{if } Hx_k \in [a_i, a_{i+1}] \\ y_k = C_i x_k & \text{for } \; i = 1, 2, \cdots, s \end{cases} \quad (1)$$

where $x_k \in \mathbb{R}^n$ is the state vector, y_k is the output vector. k refers to the sample index. q_k is the active mode index, and the discrete state of the hybrid system, it takes its values in the finite set $Q = \{1, 2, \cdots, s\}$. Each triplet (A_i, B_i, C_i) characterizes the dynamics of the system in a region of the state space. s is the number of subsystems (also the number of regions). The switching strategy is specified by the linear form $Hx_k = \sum_{i=1}^{n} h_i x_k^i, H = [h_1, ..., h_n]$ which indicates the active subsystem and defines a partition of the state-space where each region is delimited by $Hx_k = a_i$, with $a_i \in \mathbb{R}$ and $a_1 < a_2 < ... < a_s$.

The hybrid observer has to provide an evaluation of the active mode (discrete state) $\widehat{q}_k \in Q$, and an estimate of the state vector $\widehat{x}_k$. The hybrid observer proposed here consists of two parts: *discrete observer* and *continuous observer* (see figure 1). Knowing the system input and output $(u_k, y_k, k = 1, ..., N)$ on a time horizon N, an evaluation $\widehat{q}$ of the active mode index $\hat{q}_k \in \{1, 2..., s\}$ is calculated. The result is used by the piecewise-linear observer to determine $\widehat{x}$ an estimation of the continuous state vector x.

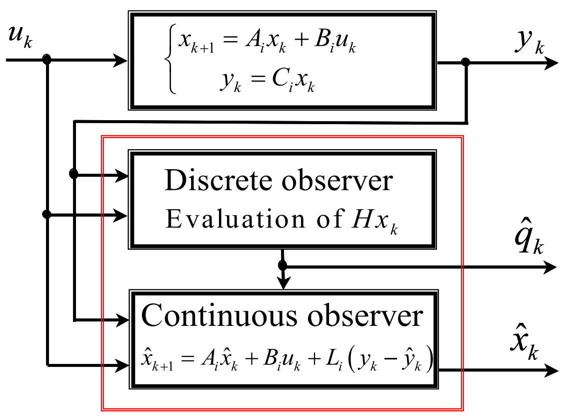

Fig. 1. Hybrid observer structure

The two parts of the observer are described in the following sections.

3. DISCRETE OBSERVER: ACTIVE MODE DETECTION

In this section we present a method for detecting the switching time instant and the corresponding active mode using the input/output data on a time horizon. First, the autonomous case is considered. A generalization is proposed in the next section for the non-autonomous case. We consider the following piecewise-linear system:

$$\begin{cases} x_{k+1} = A_i x_k & \text{if } Hx_k \in [a_i, a_{i+1}] \\ y_k = C_i x_k & \text{for } i = 1, 2, \cdots, s \end{cases} \quad (2)$$

The switching time instants are determined by the value of the quantity Hx_k. As we have only the collected output data y_k on a fixed horizon, the simplest idea is to express Hx_k function of these available outputs. This idea has been investigated in (Benali *et al.*, 2004) in the continuous time case and appears quite easy to exploit. It is possible to express $Hx(t)$ as a linear combination of the output $y(t)$ and its successive derivatives without any assumptions. We show that the transposition in the discrete time case cannot be used without additional assumptions.

As in the continuous cas, we try to express Hx_k as a linear combination of the outputs y_k taken on a time horizon of length N. The relation between the outputs and the switching function will be then written as :

$$Hx_k = [Hx_k]_\alpha = \sum_{j=0}^{N-1} \alpha_j y_{k+j} \quad (3)$$

where $\alpha = \begin{bmatrix} \alpha_0 & \alpha_1 & \cdots & \alpha_{N-1} \end{bmatrix}$ are output weighting coefficients to be computed.
Where in the continuous time case the successive output derivatives $y^{(n)}, n = 1, \cdots, n$ involve always the same active mode, in the discrete time case the samples $y_{k+j}, j = 1, \cdots, N$ do not necessarily correspond to a same active mode. In a consequence it is not possible to find α such that equation (3) is valid when a commutation occurs on $[0, ..., N-1]$.

The idea is, first to find α independent of the discrete state such that the equation (3) is valid whenever the discrete state keeps constant on the interval $[0, ..., N-1]$, second to detect the commutation when it occurs, at last find the new discrete state.

Before starting the theorem let us recall a few definitions associated with the problem of joint observability (Vidal *et al.*, 2004). We define G_k

$$G_k = \begin{pmatrix} C_1 & C_2 & \cdots & C_s \\ C_1 A_1 & C_2 A_2 & \cdots & C_s A_s \\ \cdots & \cdots & \cdots & \cdots \\ C_1 A_1^{k-1} & C_2 A_2^{k-1} & \cdots & C_s A_s^{k-1} \end{pmatrix}$$

Definition 1. (*The joint observability index*) The joint observability index is defined as

$$\mu = \max_k(rank(G))$$

Definition 2. (*The joint observability matrix*) G_μ is called joint observability matrix of the system $S_i = (A_i, C_i), i = 1, ..., s$

Lemma 1. There exist $\alpha^c = \begin{bmatrix} \alpha_0^c & \alpha_1^c & \cdots & \alpha_{\mu-1}^c \end{bmatrix}$ s.t : $Hx_k = [Hx_k]_{\alpha^c}$, where

$$[Hx_k]_{\alpha^c} = \sum_{j=0}^{\mu-1} \alpha_j^c y_{k+j} \quad (4)$$

whenever the discrete state $q \in Q$ keeps constant on $[k, ..., k+\mu-1]$ if and only if

$$rank\left(\begin{bmatrix} G_\mu^T & h^T \end{bmatrix}\right) = \mu \quad (5)$$

with $h = [\underbrace{H \; H \; ... \; H}_{s\ times}]$

If all $S_i = (A_i, C_i)$ is observable $\mu > n$.

Proof of lemma.1 :

If : if $rank\left(\begin{bmatrix} G_\mu^T & h^T \end{bmatrix}\right) = \mu$ there $h = [\underbrace{H \; H \; ... \; H}_{s\ times}]$ is a linear combination of the rows of the matrix G_μ, then exist $\begin{bmatrix} \alpha_0^c & \alpha_1^c & \cdots & \alpha_{\mu-1}^c \end{bmatrix}$ such that

$$H = \sum_{j=0}^{\mu-1} \alpha_j^c C_i A_i^j \quad \forall i \in Q \quad (6)$$

then $[Hx_k]_{\alpha^c} = \sum\limits_{j=0}^{\mu-1} \alpha_j^c C_i A_i^j x_k = \sum\limits_{j=0}^{\mu-1} \alpha_j^c y_{k+j}$ where y_{k+j} is the output of system S_i.

Only if : if there exist $\begin{bmatrix} \alpha_0^c & \alpha_1^c & \cdots & \alpha_{\mu-1}^c \end{bmatrix}$ s.t

$$Hx_k = \sum_{j=0}^{\mu-1} \alpha_j^c y_{k+j} \ , \forall x_k$$

while y_{k+1} is the output of any system S_i (q is constant on $[k, ..., k+\mu-1]$), then

$$Hx_k = \sum_{j=0}^{\mu-1} \alpha_j^c C_i A_i^j x_k, \forall x_k, \forall i \in Q$$

and $H = \sum\limits_{j=0}^{\mu-1} \alpha_j^c C_i A_i^j$, $\forall i \in Q$ and h is linear combination of the rows G_μ. □

If $S_i = (A_i, C_i)$ is observable then x_k is linear combinaison of the outputs $y_k, y_{k+1}, ..., y_{k+\mu-1}$ and the Hx_k is also a linear combinaison of $y_k, y_{k+1}, ..., y_{k+\mu-1}$, so there exist $\alpha^i = \begin{bmatrix} \alpha_0^i & \alpha_1^i & \cdots & \alpha_{\mu-1}^i \end{bmatrix}$, for all $i \in Q$ s.t $[Hx_k]_{\alpha^i} = \sum\limits_{j=0}^{\mu-1} \alpha_j^i y_{k+j} = \sum\limits_{j=0}^{\mu-1} \alpha_j^i C_i A_i^j x_k$ then

$$H = \sum_{j=0}^{\mu-1} \alpha_j^i C_i A_i^j \quad (7)$$

Theorem 1. Suppose that the following assumptions are verified :

(1) $S_i, i = 1, ..., s$ observable.
(2) $rank\left(\begin{bmatrix} G_\mu^T & h^T \end{bmatrix}\right) = \mu$.
(3) the hybrid system exhibits transitions with time separation greater than or equal to some $\mu > 0$ (dwell time hypothesis).
(4) $C_i A_i \neq C_j A_j, \quad \forall i, j \in Q$.
(5) $\alpha_{\mu-1}^c \neq \alpha_{\mu-1}^i$.

Then : $\begin{cases} \text{If } [Hx_k]_{\alpha^c} = [Hx_k]_{\alpha^i}, \text{ then } q_k = i \\ \text{Else the switching instant is } t_c = k + \mu - 1 \end{cases}$

Proof of theorem.1 :

First it is important to notice that we only prove the result on $[0, N], \ N \geq \mu$. In fact when a commutation will be detected at time $k \in [0, N]$ we will reinitialize the detection procedure. $[Hx_0]_{\alpha_c}$ gives the discrete state $q_0 = i$ on $[0, \mu-1]$ thanks to lemma 1 and dwell-time hypothesis. Let us examine $[Hx_1]_{\alpha_c}$.

Case 1: $[Hx_1]_{\alpha_c} \neq [Hx_1]_{\alpha_{q_0}}$ gives the information of a commutation at μ.

Case 2: $[Hx_1]_{\alpha_c} = [Hx_1]_{\alpha_{q_0}}$. Assume that a commutation (from $q_0 = i$ to $q_1 = l \neq i$) occurs at μ then

$[Hx_1]_{\alpha^i} = \sum_{j=0}^{\mu-1} \alpha_j^i y_{j+1} = \sum_{j=0}^{\mu-1} \alpha_j^i C_i A_i^j x_1 + (\alpha_{\mu-1}^i C_l A_l A_i^{\mu-2} - \alpha_{\mu-1}^i C_i A_i^{\mu-1}) x_1$.

$[Hx_1]_{\alpha^c} = \sum_{j=0}^{\mu-1} \alpha_j^c y_{j+1} = \sum_{j=0}^{\mu-1} \alpha_j^c C_i A_i^j x_1 + (\alpha_{\mu-1}^c C_l A_l A_i^{\mu-2} - \alpha_{\mu-1}^c C_i A_i^{\mu-1}) x_1$.

we have $\sum_{j=0}^{\mu-1} \alpha_j^i C_i A_i^j x_k = \sum_{j=0}^{\mu-1} \alpha_j^c C_i A_i^j x_k$ then $[Hx_1]_{\alpha^c} - [Hx_1]_{\alpha^i} = (\alpha_{\mu-1}^c - \alpha_{\mu-1}^i)(C_l A_l - C_i A_i) A_i^{\mu-2} x_1 = 0$ only if $\alpha_{\mu-1}^c = \alpha_{\mu-1}^i$ or $(C_l A_l - C_i A_i) = 0$ which contradict the assumptions (4,5).

Remark : The previous theorem gives conditions under which the switching instant can be detected. The only situation where no conclusion can be made is $A_i^{\mu-2} x_1 \in Nul(C_l A_l - C_i A_i)$ which is far from being realistic.

3.1 Generalization to the non-autonomous case

According to the same procedure described for the autonomous piecewise-linear systems, we define Q_k expressed as a linear combination of the output samples taken on a time horizon μ:

$$Q_k = \sum_{j=0}^{\mu-1} \alpha_j y_{k+j} = \alpha Y_k \quad (8)$$

with $Y_k = \begin{bmatrix} y_k & y_{k+1} & \cdots & y_{k+\mu-1} \end{bmatrix}^T$. For each mode, we have :

$$Y_k = \mathcal{O}_i x_k + \Gamma_i U_k \tag{9}$$

where $\mathcal{O}_i$ the observability matrix and Γ_i is the Toeplitz matrix :

$$\mathcal{O}_i = \begin{bmatrix} C_i \\ C_i A_i \\ \vdots \\ C_i A_i^{\mu-1} \end{bmatrix}, \Gamma_i = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ C_i B_i & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ C_i A_i^{\mu-2} B_i & \cdots & C_i B_i & 0 \end{bmatrix}$$

$U_k = \left[u_k \ u_{k+1} \ \cdots \ u_{k+\mu-1} \right]^T$. By substitution of equation (9) in (8), we obtain $Q_k = \alpha \mathcal{O}_i x_k + \alpha \Gamma_i U_k$ with $\alpha \mathcal{O}_i = \sum_{j=0}^{\mu-1} \alpha_j C_i A_i^j$. If $\alpha = \alpha^c$, then $Q_k = H x_k + \alpha^c \Gamma_i U_k$.
Using (8) with $\alpha = \alpha^c$, the switching function Hx_k is then given by

$$Hx_k = \alpha^c Y_k + \beta_i^c U_k \tag{10}$$

where $\beta_i^c = -\alpha^c \Gamma_i$.
The term β_i^c in (10) states that s different evaluations of Hx_k are computed. As in the autonomous case, different coefficients α^i are used. For each subsystem, the quantity Hx_k is evaluated in two different ways, $[Hx_k]_{\alpha^c,\beta_i^c}$ and $[Hx_k]_{\alpha^i,\beta^i}$ given by

$$\begin{cases} [Hx_k]_{\alpha^c,\beta_i^c} = \alpha^c Y_k + \beta_i^c U_k \\ [Hx_k]_{\alpha^i,\beta^i} = \alpha^i Y_k + \beta^i U_k \end{cases}$$

where $\beta^i = -\alpha^i \Gamma_i$

The active subsystem $\widehat{q}_k$ and the switching instant t_c are given by

$$\begin{cases} \text{If } [Hx_k]_{\alpha^c,\beta_i^c} = [Hx_k]_{\alpha^i,\beta^i} \text{ then } \widehat{q}_k = i \\ \text{Else the switching instant is } t_c = k + \mu - 1 \end{cases}$$

4. HYBRID OBSERVER

In this section the continuous observer is analyzed considering the complete hybrid system obtained by composing the hybrid system and the hybrid observer. The continuous observer is a piecewise-linear observer of the form:

$$\begin{cases} \widehat{x}_{k+1} = A_i \widehat{x}_k + B_i u_k + L_i (y_k - \widehat{y}_k) \\ \widehat{y}_k = C_i \widehat{x}_k \qquad \text{if } \hat{q}_k = i \in Q \end{cases} \tag{11}$$

The hybrid observer has $(s \times s)$ situations of type $(q_i, \widehat{q}_i) \in Q \times Q$. To each $(q_k, \widehat{q}_k)$, the error dynamics is given by :

$$e_{k+1} = (A_i - L_i C_i) e_k \quad \text{if} \quad q_k = \hat{q}_k = i \tag{12a}$$

$$e_{k+1} = (A_i - L_i C_i) e_k + v_k \text{ if } q_k = j, \ \hat{q}_k = i \tag{12b}$$

with

$$v_k = ((A_j - A_i) - L_i (C_j - C_i)) x_k + (B_j - B_i) u_k \tag{13}$$

The equation (12a) describes the situation where the active mode is correctly identified, and the second equation (12b) describes a situation where the estimated mode is different from the actual active mode.
In this paper, we will study two possible estimation schemes :

(1) ***Off−line estimation***: The previously presented analyses a set $[y_1, ..., y_{t_f}]$ and gives the discrete state $\hat{q}_k = q_k \ \forall k \in [1, ..., t_f]$. The continuous observer uses the exact knowledge of q_k to construct $\hat{x}_k$.
(2) ***On−line estimation***: The hybrid observer and the hybrid system works simultaneously. In this case the input/output data provided by the hybrid system are analyzed by the hybrid observer in real time.

4.1 Off−line state estimation

To analyze the off−line estimation case, let us consider a time instant $t_f > \mu$. The input/output data information is available for $k \leq t_f$. Using a moving window of size μ, the discrete observer analyzes the data and provides $\widehat{q}_k$. The method proposed in the previous sections guarantees, under section 3, that $\widehat{q}_k = q_k$, this information is used by the continuous observer. The error dynamic is governed by the following equation :

$$e_{k+1} = (A_i - L_i C_i) e_k \tag{14}$$

The observer gains L_i are computed so that the estimated state $\widehat{x}_k$ converges towards the state of the system x_k for all the initial conditions i.e :

$$\forall e_0 \in \mathbb{R}^n \quad \lim_{k \to \infty} \|e_k\| = 0 \tag{15}$$

To obtain these observer gains, we use the LMI (Linear Matrix Inequalities (Boyd *et al.*, 1994) approach developed in (Daafouz *et al.*, 2002). An indicator vector $\xi_k = [\xi_k^1, \ \xi_k^2, \ldots, \xi_k^s]^T$ is defined as

$$\xi_k^i = \begin{cases} 1 \text{ if } \widehat{q}_k = i \\ 0 \quad \text{else} \end{cases} \tag{16}$$

The estimation error is :

$$e_{k+1} = \sum_{i=1}^{s} \xi_k^i (A_i - L_i C_i) e_k \tag{17}$$

Global convergence of (17) is ensured by selecting the gains $L_i; i = 1, \cdots, s$ such that the error stability condition established in (Daafouz *et al.*, 2002) are satisfied. It consists in finding positive definite matrices S_i and matrices F_i and G_i for $i = 1, \cdots, s$ solution of the following inequalities :

$$\begin{bmatrix} G_i + G_i^T - S_i & G_i^T A_i - F_i^T C_i \\ A_i^T G_i - C_i^T F_i & S_j \end{bmatrix} > 0 \qquad (18)$$

with $j = 1, \cdots, s$. In this case, the matrices L_i are given by

$$L_i = G_i^{-T} F_i^T$$

The asymptotic stability is guaranteed by the Lyapunov function $V = \varepsilon^T \sum_{i=1}^{s} \xi_k^i P_i \varepsilon$ with $P_i = S_i^{-1}$.

4.2 On−line state estimation

We recall that to detect the active mode, we use the input/output samples available on a time horizon of size μ. Let us consider a switch occurring at time t_c. The discrete observer detects at $t_c + 1$ this commutation. However the new mode is only identified at the instant $t_c+\mu$. In the on−line state estimation case the continuous observer work simultaneously with the discrete observer, therefore on the interval $[t_c, t_c + \mu - 1]$, we must provide $\hat{q}_k$ for the continuous observer. Different schemes can be investigated during the interval time $[t_c, t_c + \mu - 1]$ as the discrete state is unknown and the continuous state cannot be correctly calculated. In this paper we propose to keep the discrete state with its last known value $t_c - 1$ and estimate the continuous state with the observer designed for value $q_{t_c - 1}$, with this option it can prove that the estimation error remains bounded.

The corresponding error dynamics is given by the equation (12).
A new indicator vector $\hat{\xi}_k = (\hat{\xi}_k^1, \hat{\xi}_k^2, \cdots, \hat{\xi}_k^s)^T$ is associated in order to establish a correspondence between discrete $\hat{q}_i$ and the triplet (A_i, B_i, C_i) :

$$\hat{\xi}_k^i = \begin{cases} 1 \text{ if } \hat{q}_k = i \\ 0 \quad \text{if } \ \hat{q}_k \neq i \\ \hat{\xi}_k^i = \hat{\xi}_{k-1}^i \text{ if } \hat{q}_k \text{ is not available.} \end{cases} \qquad (19)$$

the estimation error is given by:

$$e_{k+1} = \sum_{i=1}^{s} \hat{\xi}_k^i (A_i - L_i C_i) e_k + \quad v_k \qquad (20)$$

This error cannot be asymptotically stable because of the term v_k. However, we can calculate the gain L_i, such that the error estimation is Input-to-State stable (Jiang *et al.*, 1999). For that, we use the result proposed in (Daafouz *et al.*, 2005). The gain L_i is calculated by solving the optimization problem :

$$\underset{\substack{P_i = P_i' \\ G_i = G_i' \\ F_i, \alpha}}{\mathrm{M}in} \quad \eta$$

under

$$\begin{bmatrix} \mathbf{1} - P_i & A_i' G_i - C_i' F_i' & A_i' G_i - C_i' F_i' \\ G_i A_i - F_i C_i & P_j - 2G_i & \mathbf{0} \\ G_i A_i - F_i C_i & \mathbf{0} & 2G_i - \eta \mathbf{1} \end{bmatrix} < \mathbf{0} \qquad (21)$$

if the minimization problem has a solution $P_i^* \in \mathbb{R}^{n \times n}$, $G_i^* \in \mathbb{R}^{n \times n}$, $F_i^* \in \mathbb{R}^{n \times m}$ et $\eta^* \in]1, \infty[$, the gains L_i are given by :

$$L_i = G_i^{*-1} F_i^*$$

The Input-to-State stability is guaranteed by the Lyapunov function $V(e_k, \hat{\xi}_k^i) = e_k^T P_k e_k$ with $P_k = \sum_{i=1}^{s} \hat{\xi}_k^i P_i^*$. The estimation error verifies:

$$\|e_k\| \leq \sqrt{\eta^*} (1 - \frac{1}{\eta^*})^{k/2} \|e_0\| + \eta^* \|v\|_\infty \qquad (22)$$

As the LMI resolution implies that η^* is necessarily greater than 1, when $k \longrightarrow \infty$ the estimation error is bounded $\|e_k\| \leq \eta^* \|v\|_\infty$ if v_k is bounded. exist $X > 0$ and $U > 0$, such that $\|x_k\|_\infty \leq X$, $\|u_k\|_\infty \leq U$ and

$$\|v_k\|_\infty \leq V = \max_{i \neq j} \|((A_j - A_i) - L_i (C_j - C_i))\| X + \|(B_j - B_i)\| U$$

5. ILLUSTRATIVE EXAMPLE

To illustrate the observation scheme proposed here, we consider a piecewise-linear system given by (1) with:

$$A_1 = \begin{bmatrix} 0.80 & 0.22 \\ -0.22 & 0.80 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0.79 & 0.29 \\ -0.29 & 0.50 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.20 \\ 0.20 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0.50 \\ -0.50 \end{bmatrix}$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad H = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

The switching function Hx_k is defined by :

$$\begin{cases} \text{if } \ (0.2 \leq Hx_k \leq 10) \ \text{ then } \ i = 1 \\ \qquad\qquad \text{else } \ i = 2 \end{cases}$$

For this system we find $\mu = 4$ and

$$\alpha^c = \begin{bmatrix} -11.87 & 39.65 & -45.18 & 18.65 \end{bmatrix}$$

In off−line state estimation, the gains of the observer are obtained solving the LMIs (18):

$$L_1 = \begin{bmatrix} -0.7660 \\ 2.5341 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.5441 \\ 0.1030 \end{bmatrix}$$

Figure (2) shows the actual active mode (full line), the reconstructed active mode (stars $*$) and estimation error norm.

Notice that the modes are perfectly identified, and the error estimation norm decreases towards zero. A zoom shows that after the 15^{th} sample the error norm is almost null.

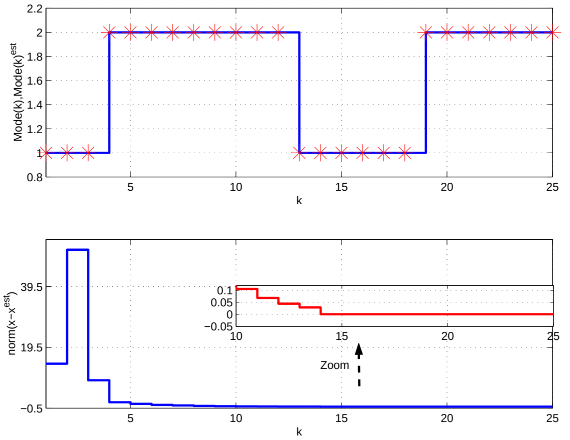

Fig. 2. off−line estimation case : mode evaluation (in the top) and the error estimation norm (in the bottom).

In on−line state estimation, the gains of the piecewise-linear observer are obtained by solving the optimization problem (21) :

$$\eta^* = 11.3122,, \quad L_1 = \begin{bmatrix} -0.0621 \\ 1.3035 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.0524 \\ 0.8879 \end{bmatrix}$$

The actual mode (full line) and the evaluated mode (dotted line) and estimation error norm are given on the figure (3).

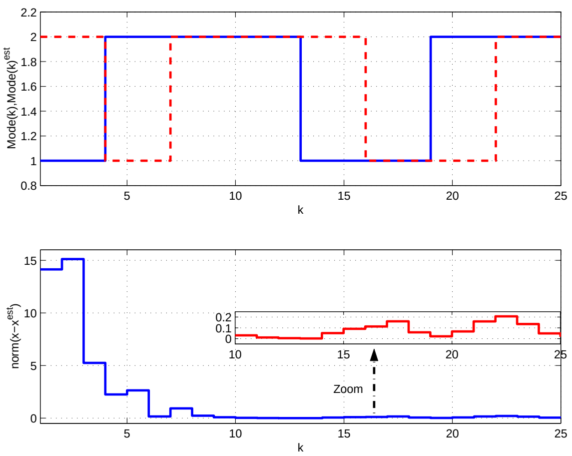

Fig. 3. on−line estimation case : mode evaluation (in the top) and the error estimation norm (in the bottom).

At the beginning of the estimate $k = 1$, we initialized the discrete state estimated at $\widehat{q}_k = 2, k = \{1, 2, 3\}$, whereas the true discrete state is $q_k = 1$. Since the acquisition of the 4^{th} sample, the discrete observer provides $\widehat{q}_k = 1$, at the instant time $k = 5$ a switch occurs: $q_k = 2$. The discrete observer keeps its old value $\widehat{q}_k = 1$ for $k = \{5, 6, 7\}$. For the continuous component, we notes that starting from the sample $k = 10$ the norm of the estimation error is stabilized around zero. A zoom on the interval $[10, 25]$, shows that the error norm is not null but it is bounded.

6. CONCLUSION

In this paper, we propose a hybrid observer for a class of piecewise-linear systems. The association of a discrete state detection method and a piecewise-linear switched observer leads to a hybrid observer witch may operates off-line or on-line. In the off-line case, the observation error is guaranteed to converge toward zero whereas in the on-line case, it is guaranteed to be bounded.

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