

## DISCRETE-EVENT MODELLING AND FAULT DIAGNOSIS OF DISCRETELY CONTROLLED CONTINUOUS SYSTEMS

Jan Lunze\*

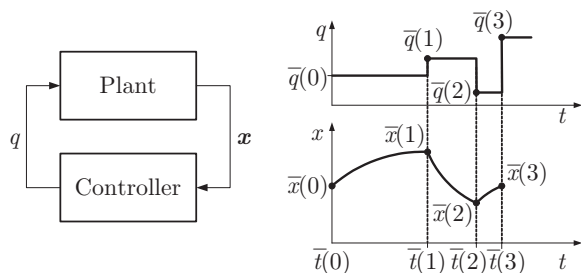
\* Ruhr-Universität Bochum  
Institute of Automation and Computer Control  
44780 Bochum, Germany

Abstract: The papers presents five diagnostic strategies for discretely controlled continuous systems, which differ with respect to the abstraction of the model and measurement information used. From the original hybrid model, four more abstract representations are derived, which have the form of embedded maps, semi-Markov processes, timed automata or nondeterministic automata, respectively. The validity of the diagnostic result is ensured by the claim that the models should be complete and, hence, in the appropriate fault case, consistent with the input-output sequence of the discretely controlled system. In this way a hierarchy of models and of diagnostic results is obtained. *Copyright © 2006 IFAC*

Keywords: Hybrid system, discretely controlled system, fault diagnosis, model hierarchy, embedded map, semi-Markov process, timed automaton.

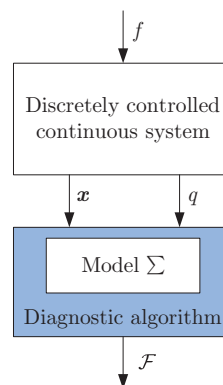
### 1. INTRODUCTION

Discretely controlled continuous systems comprise an important class of hybrid systems, where the continuous system represents a technological process whose operation mode is switched by a feedback controller (Fig. 1). The dynamics is characterised by discrete mode changes  $q(t)$  and a continuous state movement  $\mathbf{x}(t)$ .



**Fig. 1:** Discretely controlled continuous system

A lot of practical examples fall well into this class of dynamical systems like DC-DC converters, combustion engines or simulating moving bed chromatographic processes. These examples have the additional characteristics that their function can only be maintained if the mode switching is carried out for ever.



**Fig. 2:** Diagnosis of discretely controlled systems

**Fault Diagnosis.** Fault diagnosis has to use the measured switching sequence and the measured continuous movement to detect and identify faults (Fig. 2). The main idea is to test the consistency of these measurements with the behaviour of models of the faultless or the faulty system. Inconsistencies indicate that faults have occurred.

In order to get the diagnostic algorithm with the lowest possible complexity, the kind of measurement information and the granularity of the models used have to be chosen in accordance with the faults to be detected. The "best" algorithm for an application is the one with the lowest complexity that detects or identifies the fault.

This paper is concerned with the adaptation of the measurement and modelling information to the diagnostic task. Five models are described which differ in their granularity. The model abstraction process starts with the hybrid model consisting of continuous state-space models for all operation modes and leads to a nondeterministic automaton as the most abstract description. A hierarchy of diagnostic algorithms is obtained. The validity of the diagnostic results is ensured by the claim that all models should be complete in the sense of Definition 10.

**Literature.** There are no specific references to the diagnosis of discretely controlled systems, but several starting points to solve this diagnostic problem. Within a given operation mode, the discretely controlled system can be diagnosed by methods elaborated for continuous systems (Gertler 1998), (Blanke et al. 2006). On the other hand, if a purely discrete-event model is used, diagnostic methods for automata or Petri nets can be applied (Sampath et al. 1995).

The aim of this paper is to develop a connection between these two extremes. Starting from the hybrid model, different more abstract representations of discretely controlled systems are described together with the corresponding diagnostic algorithms. The paper uses the idea of ordered abstractions that has been described in (Raisch and O'Young 1997) for controller design and in (Förstner and Lunze 2001) for the diagnosis of quantised systems and represents it for discretely controlled continuous systems.

## 2. DISCRETELY CONTROLLED CONTINUOUS SYSTEMS

### 2.1 Hybrid model

The paper considers hybrid systems shown in Fig. 1. The plant is described by the state-space model

$$\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}(t), q(t)), \quad \mathbf{x}(0) = \mathbf{x}_0, \quad (1)$$

where  $\mathbf{x} \in \mathcal{R}^n$  denotes the state vector and  $q \in \mathcal{Q} = \{1, 2, \dots, n_q\}$  the operation mode. For constant operation mode ( $q(t) = \bar{q}$ ) the model (1) is assumed to have a unique solution  $\mathbf{x}(\bar{q}, \mathbf{x}_0, t)$ .

The controller evaluates the state trajectory and generates an event  $e \in \mathcal{E}$  whenever

$$\Phi_e(\mathbf{x}(t), q(t), t) = 0 \quad (2)$$

is satisfied. The time  $\bar{t}$  denotes the time  $t$  for which Eqn. (2) holds. At this time, the state  $\mathbf{x}(t)$  has the value  $\bar{\mathbf{x}}$ . Then the next operation mode  $\bar{q}'$  is determined according to a discrete state transition function  $C$  of the controller:

$$\bar{q}' = C(\bar{q}, e). \quad (3)$$

The control law (2), (3) can be lumped together

$$\bar{q}' = H_q(\bar{q}, \bar{\mathbf{x}}, \bar{t}) \quad (4)$$

$$\bar{t}' = H_t(\bar{q}, \bar{\mathbf{x}}, \bar{t}), \quad (5)$$

where  $\bar{q}$  and  $\bar{t}$  denote the "old" operation mode and the time instant that this operation mode has been assumed, and  $\bar{q}'$  and  $\bar{t}'$  are the "next" operation mode and event time. The overall model (1), (4) is referred to as the model  $\Sigma_H$ .

### 2.2 Behaviour

The model  $\Sigma_H$  has, for every initial state  $\mathbf{x}_0$  and initial operation mode  $q_0$ , a unique solution. The time instances of the operation mode changes are enumerated by the counter  $k$  and denoted by  $\bar{t}(k)$ . The continuous state at these time points is  $\bar{\mathbf{x}}(k) = \mathbf{x}(\bar{t}(k))$  and the  $k$ -th operation mode is  $\bar{q}(k)$ , where  $q(t) = \bar{q}(k)$  for  $\bar{t}(k) \leq t < \bar{t}(k+1)$  holds. The continuous state movement between two consecutive switching time points is denoted by  $\mathbf{x}_{[\bar{t}(k), \bar{t}(k+1)]}$ . In summary, the behaviour of the discretely controlled system is described by the *hybrid state sequence*:

$$\text{Traj}_H(q_0, \mathbf{x}_0) = \left( \left( \begin{array}{c} \bar{t}(0) \\ \bar{\mathbf{x}}(0) \\ \bar{q}(0) \end{array} \right), \mathbf{x}_{[\bar{t}(0), \bar{t}(1)]}, \left( \begin{array}{c} \bar{t}(1) \\ \bar{\mathbf{x}}(1) \\ \bar{q}(1) \end{array} \right), \mathbf{x}_{[\bar{t}(1), \bar{t}(2)]}, \dots \right) \quad (6)$$

### 2.3 Abstract descriptions of the behaviour

A more abstract representations focuses on the time instants at which the operation mode is changed. It is referred to as the discrete-event behaviour, which is described by the sequences of switching time points, operation modes and the

values of the continuous state at the switching time instances:

$$\text{Traj}_E(q_0, \mathbf{x}_0) = \left( \left( \begin{array}{c} \bar{t}(0) \\ \bar{\mathbf{x}}(0) \\ \bar{q}(0) \end{array} \right), \left( \begin{array}{c} \bar{t}(1) \\ \bar{\mathbf{x}}(1) \\ \bar{q}(1) \end{array} \right), \dots \right) \quad (7)$$

$\text{Traj}_E$  is called the *timed event/state sequence*. A more abstract behaviour results if the continuous state is ignored:

$$\text{Traj}_S(q_0, \mathbf{x}_0) = \left( \left( \begin{array}{c} \bar{t}(0) \\ \bar{q}(0) \end{array} \right), \left( \begin{array}{c} \bar{t}(1) \\ \bar{q}(1) \end{array} \right), \dots \right) \quad (8)$$

$\text{Traj}_S$  describes the *timed event sequence*. If the temporal information is ignored, a *logic event sequence*

$$\text{Traj}_N(q_0, \mathbf{x}_0) = (\bar{q}(0), \bar{q}(1), \bar{q}(2), \dots) \quad (9)$$

is obtained. The relation among the behaviours (6) – (9) can be represented by projection operators  $P_E$ ,  $P_S$  and  $P_N$ :

$$\begin{aligned} \text{Traj}_E &= P_E(\text{Traj}_H) \\ \text{Traj}_S &= P_S(\text{Traj}_H) \\ \text{Traj}_N &= P_N(\text{Traj}_H). \end{aligned}$$

### 3. PROBLEM STATEMENT

#### 3.1 Diagnostic problem

A diagnostic algorithm to be developed has to decide whether a fault has occurred (fault detection) and which fault has occurred (fault identification). It uses a model  $\Sigma$  and measurement information, which describe the current system trajectory  $\text{Traj}$ . Four approaches will be presented that use the measured trajectories  $\text{Traj}_H$ ,  $\text{Traj}_E$ ,  $\text{Traj}_S$  and  $\text{Traj}_N$  together with appropriate models to solve the following problem, where the index  $*$  is replaced by  $E$ ,  $S$ ,  $T$  or  $N$ :

Given: Model  $\Sigma_*$   
Trajectory  $\text{Traj}_*$   
Find: Set  $\mathcal{F}_*$  of fault candidates

For the simplicity of presentation, it is assumed that the fault  $f$  does not change if the diagnostic algorithm is applied and that the initial states  $\mathbf{x}_0$ ,  $q_0$  are known.

#### 3.2 Model hierarchy

Four abstract representations of discretely controlled systems will be developed, which comprise a model hierarchy (Fig. 3). It will become clear

that abstract representations, which will be introduced in the next sections, may be nondeterministic. To get a unified notation, the solutions of all models are represented by some sets that are called the model behaviour and denoted by  $\mathcal{B}_H$ ,  $\mathcal{B}_E$ ,  $\mathcal{B}_S$ ,  $\mathcal{B}_T$  or  $\mathcal{B}_N$ , respectively.

	Model	Measurement	Diagnostic result
Abstraction ↑	Nondeterministic automaton $\Sigma_N$	Logic event sequence	$\mathcal{F}_N$
	Timed automaton $\Sigma_T$	Timed event sequence	$\mathcal{F}_T$
	Semi-Markov process $\Sigma_S$	Timed event sequence	$\mathcal{F}_S$
	Embedded map $\Sigma_E$	Timed event/state sequence	$\mathcal{F}_E$
	Hybrid model $\Sigma_H$	Hybrid state sequence	$\mathcal{F}_H$

**Fig. 3:** Model hierarchy

The modelling aim is to set up these models such that the relation  $\mathcal{B}_E = P_E(\mathcal{B}_H)$ ,  $\mathcal{B}_S = P_S(\mathcal{B}_H)$ ,  $\mathcal{B}_T = P_T(\mathcal{B}_H)$  or  $\mathcal{B}_N = P_N(\mathcal{B}_H)$ , respectively, holds. The application of the operator  $P_*$  to the set  $\mathcal{B}_*$  means to apply the operator to all elements of the set. However, as the models to be set up are abstract representations, these equalities cannot be satisfied. In order to ensure the validity of the diagnostic results obtained by means of these models, all models have to be complete:

*Definition 1.* A model  $\Sigma_A$  with the behaviour  $\mathcal{B}_A$  is called **complete** with respect to the discretely controlled system and the abstraction operator  $P_B$  if it satisfies the relation

$$\mathcal{B}_A \supseteq P_B(\mathcal{B}_H). \quad (10)$$

#### 3.3 Consistency-based diagnosis

The idea of consistency-based diagnosis can be briefly explained as follows (Blanke et al. 2003). A fault is known to occur if the model of the faultless system cannot generate the trajectory measured (fault detection). Then it is said that the model and the trajectory are inconsistent. Similarly, fault identification can be accomplished by testing the consistency of the trajectory with the model of the system subject to some faults  $f$ . The result is the set  $\mathcal{F}$  of *fault candidates*, which include all faults  $f$  for which the trajectory is consistent with the model subject to fault  $f$ .

*Definition 2.* Consider a system whose trajectory  $\text{Traj}_*$  has been measured over some time horizon  $t_h$ . The model is represented by its behaviour  $\mathcal{B}_*$  over the same time horizon. The trajectory is called **consistent** with the model, if

$$\text{Traj}_* \in \mathcal{B}_*. \quad (11)$$

In order to determine the set of fault candidates, the consistency of the measured trajectory is tested with respect to the behaviour of models that describe the discretely controlled continuous system subject to the faults  $f \in \bar{\mathcal{F}}$  where  $\bar{\mathcal{F}}$  is the set of all faults under consideration. The behaviour of these models are denoted by  $\mathcal{B}_*(f)$ . Then the set of fault candidates is defined by

$$\mathcal{F}_* = \{f \mid \text{Traj}_* \in \mathcal{B}_*(f)\}. \quad (12)$$

It is important to see that although the models used represent the discretely controlled continuous system with different granularity, the diagnostic results are clearly related to each other:

*Corollary 1.* If the model  $\Sigma_*$  is complete, the following relation holds:

$$f \in \mathcal{F}_H \implies f \in \mathcal{F}_*. \quad (13)$$

That is, if  $f$  is a fault candidate with respect to the model  $\Sigma_H$  it is also a fault candidate obtained by any other model  $\Sigma_*$  discussed below.

#### 4. DIAGNOSIS BY MEANS OF EMBEDDED MAPS

**Model.** If merely the discrete-event trajectory  $\text{Traj}_E$  should be used for diagnosis, the system state has only to be known at the switching instances. Then the diagnosis refers to the time instances and the points in the state space where the discretely controlled system generates events.

It has been investigated, for example in (Krupar et al. 2004), that it is possible to find a mapping  $G_E$  that allows to determine the  $(k+1)$ -st triple  $(\bar{t}(k+1), \bar{\mathbf{x}}(k+1), \bar{q}(k+1))$  from the knowledge of the  $k$ -th triple  $(\bar{t}(k), \bar{\mathbf{x}}(k), \bar{q}(k))$ . This mapping is called an *embedded mapping* of the hybrid system. It has the general form

$$\begin{pmatrix} \bar{\tau}(k+1) \\ \bar{\mathbf{x}}(k+1) \\ \bar{q}(k+1) \end{pmatrix} = \tilde{G} \left( \begin{pmatrix} \bar{\tau}(k) \\ \bar{\mathbf{x}}(k) \\ \bar{q}(k) \end{pmatrix} \right). \quad (14)$$

where  $\bar{\tau}$  is the time span between consecutive switching times  $\bar{\tau}(k) = \bar{t}(k) - \bar{t}(k-1)$  for  $k \geq 1$  and  $\bar{\tau}(0) = 0$ . The embedded map is written here in the implicit form

$$G_E(\bar{\mathbf{x}}', \bar{q}', \bar{t}' - \bar{t}, \bar{\mathbf{x}}, \bar{q}) \in \{0, 1\}, \quad (15)$$

where  $G_E$  has the value 1 if two consecutive switchings of the discretely controlled system occur at the states  $\bar{\mathbf{x}}(k) = \bar{\mathbf{x}}$  and  $\bar{\mathbf{x}}(k+1) = \bar{\mathbf{x}}'$ . The operation mode  $\bar{q}(k+1) = \bar{q}$  is entered at time  $\bar{t}(k) = \bar{t}$  and left towards the mode  $\bar{q}(k+1) = \bar{q}'$  at the time instance  $\bar{t}(k+1) = \bar{t}'$ .

**Determination of  $G_E$ .** The embedded map  $G_E$  can be determined from the hybrid model  $\Sigma_H$  by integrating the differential equation (1) from the state  $\mathbf{x}(\bar{t}) = \bar{\mathbf{x}}$  between two switching times  $\bar{t}$  and  $\bar{t}'$  to determine the state  $\mathbf{x}(\bar{t}') = \bar{\mathbf{x}}'$  and by applying the control law (4) to find the next operation mode  $\bar{q}'$ . Hence, for the next switching the following relations hold:

$$\begin{aligned} \bar{\mathbf{x}}' &= \mathbf{x}(\bar{q}, \bar{\mathbf{x}}, \bar{t}' - \bar{t}) \\ \bar{q}' &= H_q(\bar{q}, \bar{\mathbf{x}}, \bar{t}) \\ \bar{t}' &= H_t(\bar{q}, \bar{\mathbf{x}}, \bar{t}) \end{aligned}$$

The embedded map obviously yields the same timed event/state trajectory  $\text{Traj}_E$  as the hybrid model  $\Sigma_H$ . For a given initial state  $(q_0, \mathbf{x}_0)$  this trajectory is unique and, hence, the behaviour

$$\mathcal{B}_E = \{\text{Traj}_E(q_0, \mathbf{x}_0)\} \quad (16)$$

is a singleton. Consequently, the model  $\Sigma_H$  is complete:  $P_E(\mathcal{B}_H) = \mathcal{B}_E$ .

**Consistency test.** To check the consistency of the timed event/state sequence  $\text{Traj}_E$  with the model  $\Sigma_E$ , every single state transition from  $(\bar{q}, \bar{\mathbf{x}})$  towards  $(\bar{q}', \bar{\mathbf{x}}')$  after the time span  $\bar{t}' - \bar{t}$  is consistent with the model if

$$G_E(\bar{\mathbf{x}}', \bar{q}', \bar{t}' - \bar{t}, \bar{\mathbf{x}}, \bar{q}) = 1$$

holds. A sequence (7) is consistent with  $\Sigma_E$  if

$$\prod_{k=0}^{k_e-1} G_E(\bar{\mathbf{x}}(k+1), \bar{q}(k+1), \dots, \bar{t}(k+1) - \bar{t}(k), \bar{\mathbf{x}}(k), \bar{q}(k)) = 1. \quad (17)$$

$k_e$  denotes the number of state changes that are recorded in the trajectory  $\text{Traj}_E$ .

#### 5. DIAGNOSIS BY MEANS OF SEMI-MARKOV PROCESSES

**Model.** If the diagnosis should be carried out by only using the sequence of the discrete state  $\bar{q}(k)$ , the best representation is the timed event sequence  $\text{Traj}_T$ . Then, it suffices to represent the system by a semi-Markov process  $\Sigma_S$ . Then

$$G_S(\bar{q}', \bar{t}' - \bar{t}, \bar{q}) \in [0, 1] \quad (18)$$

describes the probability that the system, which has reached the operation mode  $\bar{q}$  at time  $\bar{t}$ , switches at time  $\bar{t}'$  into the mode  $\bar{q}'$

$$\begin{aligned} G_S(\bar{q}', \bar{t}' - \bar{t}, \bar{q}) &= \\ \text{Prob}(q_p(k+1) = \bar{q}', \bar{\tau}_p(k) = \bar{t}' - \bar{t} \mid q_p(k) = \bar{q}), \end{aligned}$$

where the index "p" identifies stochastic variables. In particular,  $\bar{\tau}_p(k)$  is the stochastic variable that describes the sojourn time in the operation mode  $q_p(k)$ . For details of this model cf. (Lunze 1999).

For a given initial state  $\bar{q}(0) = q_0$  and initial time  $\bar{t}(0) = 0$  the model (18) generates several timed event sequences  $\text{Traj}_S$  all of which are lumped into the set  $\mathcal{B}_S$ .

**Determination of  $G_S$ .** As shown in (Lunze 1999), the state transition relation  $G_S$  of the semi-Markov process can be determined from the given hybrid model  $\Sigma_H$  such that the model is complete. The relation to the embedded map is stated as follows:

$$G_E(\bar{x}', \bar{q}', \bar{t}' - \bar{t}, \bar{x}, \bar{q}) = 1 \Rightarrow G_S(\bar{q}', \bar{t}' - \bar{t}, \bar{q}) > 0.$$

As the behaviour  $\mathcal{B}_S$  of the semi-Markov process includes all timed event sequences that occur with a positive probability, this relation implies

$$P_S(\text{Traj}_H) \in \mathcal{B}_S, \quad (19)$$

i. e. the model is complete.

**Consistency test.** Any trajectory that may occur with positive probability is called consistent with the model  $\Sigma_S$ . Hence the timed event sequence  $\text{Traj}_S$  is consistent with this model if and only if

$$\prod_{k=0}^{k_e-1} G_S(\bar{q}(k+1), \bar{t}(k+1) - \bar{t}(k), \bar{q}(k)) > 0 \quad (20)$$

## 6. DIAGNOSIS BY MEANS OF TIMED AUTOMATA

**Model.** A more abstract representation  $\Sigma_T$  of the discrete-event behaviour of the system uses a timed automaton, which needs only to have a single clock, which is reset for every state transition. The state transition function is

$$G_T(\bar{q}', \bar{t}' - \bar{t}, \bar{q}) \in \{0, 1\}, \quad (21)$$

where  $G_T$  has the value 1 if the discretely controlled system can reside in the operation mode  $\bar{q}$  for the time duration  $\bar{t}' - \bar{t}$  before the operation mode is changed to become  $\bar{q}'$ .

**Determination of  $G_T$ .** The function  $G_T$  can be determined from  $G_S$  as follows:

$$G_S(\bar{q}', \bar{t}' - \bar{t}, \bar{q}) > 0 \Rightarrow G_T(\bar{q}', \bar{t}' - \bar{t}, \bar{q}) = 1.$$

Hence, all trajectories  $\text{Traj}_S(q_0)$  that the semi-Markov model generates with a positive probability are also generated by the timed automaton and  $\mathcal{B}_S = \mathcal{B}_T$  holds. The completeness of the model  $\Sigma_T$  follows directly from the completeness of the model  $\Sigma_S$ .

**Consistency test.** The consistency test is the same as for the semi-Markov model  $\Sigma_S$  but it does not yield the additional information about the probability that the timed event sequence is consistent with the model  $\Sigma_T$ . The sequence  $\text{Traj}_S$  is consistent with the timed automaton if

$$\prod_{k=0}^{k_e-1} G_T(\bar{q}(k+1), \bar{t}(k+1) - \bar{t}(k), \bar{q}(k)) > 0. \quad (22)$$

## 7. DIAGNOSIS BY MEANS OF NONDETERMINISTIC AUTOMATA

**Model.** If the temporal information is ignored and the diagnostic task should be solved by the logical event sequence  $\text{Traj}_N$ , a nondeterministic automaton  $\Sigma_N$  is the suitable model of the discretely controlled system. Its state transition function

$$G_N(\bar{q}', \bar{q}) \in \{0, 1\} \quad (23)$$

has the value 1 if the discretely controlled system can change its operation mode from  $\bar{q}$  towards  $\bar{q}'$ .

**Determination of  $G_N$ .** The function  $G_N$  results from the state transition relation of the timed automaton by ignoring the temporal information:

$$\exists \bar{t}, \bar{t}' : G_T(\bar{q}', \bar{t}' - \bar{t}, \bar{q}) = 1 \Rightarrow G_N(\bar{q}', \bar{q}) = 1. \quad (24)$$

Due to the completeness of the timed model  $\Sigma_T$  and the relation (24), the model  $\Sigma_N$  is complete.

**Consistency test.** For the nondeterministic automaton the temporal information is ignored. Hence, from the consistency test (22) the following conditions are obtained. The logic event sequence  $\text{Traj}_N$  is consistent with the model  $\Sigma_N$  if

$$\prod_{k=0}^{k_e-1} G_N(\bar{q}(k+1), \bar{q}(k)) > 0. \quad (25)$$

## 8. HIERARCHIES OF MODELS AND DIAGNOSTIC RESULTS

This section summarises the relation among the different models and the diagnostic results obtained by these models.

**Model hierarchy.** The more abstract the representation  $\Sigma_*$  is the more trajectories are included in the model behaviour  $\mathcal{B}_*$ . As these trajectories describe the system on different abstraction levels, a comparison can only be done by projecting them to the same abstraction level.

*Theorem 1.* The models of the discretely controlled continuous system form the hierarchies

$$\begin{aligned} \mathcal{B}_N &\supseteq P_{NT}(\mathcal{B}_T) \supseteq P_{NS}(\mathcal{B}_S) \supseteq P_{NE}(\mathcal{B}_E) \\ &\supseteq P_{NH}(\mathcal{B}_H). \\ \mathcal{B}_T &\supseteq \mathcal{B}_S \supseteq P_{SE}(\mathcal{B}_E) \supseteq P_{SH}(\mathcal{B}_H) \\ \mathcal{B}_E &\supseteq P_{EH}(\mathcal{B}_H), \end{aligned}$$

where  $P_{N*}$ ,  $P_{S*}$  and  $P_{EH}$  denote the projection operators that maps the behaviour of the model  $\Sigma_*$  towards the set of logic event sequences, timed event sequences or timed state/event sequences, respectively, if the state transition functions  $G_*$  are chosen according to eqns. (16), (19), and (24).

**Diagnostic results.** Due to the completeness of all models used and the model hierarchy, the set of fault candidates obtained by the diagnostic algorithm with the different models form a hierarchy.

*Theorem 2.* If the models are complete, the diagnostic results obtained by means of these models satisfy the following relations:

$$\mathcal{F}_N \supseteq \mathcal{F}_T \supseteq \mathcal{F}_S \supseteq \mathcal{F}_E \supseteq \mathcal{F}_H. \quad (26)$$

In the theorem,  $\mathcal{F}_N, \dots, \mathcal{F}_H$  are the sets of fault candidates obtained by means of the models  $\Sigma_N, \dots, \Sigma_H$ . The proof of this theorem uses the relations among the models given in the last sections. For example, the relation  $\mathcal{F}_N \supseteq \mathcal{F}_T$  follows directly from the consistency conditions (25) and (22) for both models and the relation (24) between both models.

The theorem shows that the more information about the system is used for diagnosis, the fewer elements are in the set of fault candidates. Although an improvement of the diagnostic result by using more information about the system is an intuitively clear result, the strict inclusion described by eqn. (26) can only be obtained only due to the additional requirement that all models should be complete.

## 9. CONCLUSION

The paper has shown that discretely controlled continuous systems can be diagnosed by using different levels of information. If the model used is

complete, the diagnostic result is valid in the sense that it includes all possible fault candidates.

The hierarchy of models and diagnostic results shown make it possible to adapt the measurement and modelling information used to the practical circumstances of the system under consideration.

Due to space limitations, the diagnostic methods have been explained under two assumptions, which can be released. First, if the initial state is unknown, the diagnostic algorithms have to be applied by considering all possible initial states  $q_0 \in \mathcal{Q}$  and  $\mathbf{x}_0 \in \mathcal{R}^n$ . Then the diagnostic algorithm includes a state observer. Second, if the fault changes during the run of the diagnostic algorithm, fault models have to be used to restrict the temporal behaviour of the fault.

## REFERENCES

- M. Blanke, M. Kinnaert, J. Lunze, M. Staroswiecki, *Diagnosis and Fault-Tolerant Control* (2nd edition), Springer-Verlag, Heidelberg 2006.
- D. Förstern, J. Lunze, Discrete-event models of quantized systems for diagnosis, *Intern. J. Control* **74** (2001), pp. 690-700.
- J. Gertler, *Fault Detection and Diagnosis in Engineering Systems*, Marcel Dekker, New York 1998.
- J. Krupar, A. Mögel, W. Schwarz, Continuous-discrete systems - modelling and statistical analysis, *Intern. Symposium on Nonlinear Theory and its Applications (NOLTA)*, Fukuoka 2004, pp 55-58.
- J. Lunze, A timed discrete-event abstraction of continuous-variable systems, *Intern. Journal of Control* **72** (1999), 1147-1164.
- J. Lunze, Diagnosis of quantized systems based on a timed discrete-event model, *IEEE Trans. on Systems, Man, and Cybernetics - Part A SMC-30* (2000), 322-335.
- J. Raisch, S. O'Young, A totally ordered set of discrete abstractions for a given hybrid or continuous system, In *Hybrid Systems IV*, Springer-Verlag, Berlin 1997, pp. 342-360.
- M. Sampath, R. Sengupta, S. Lafortune, K. Srinamohedeen, D. Teneketzis, Diagnosability of discrete event systems, *IEEE Trans.* **AC-40** (1995), 1555-1575.
- P. Supavatanakul, *Modelling and Diagnosis of Timed Discrete-Event Systems*, Shaker-Verlag, Aachen 2004.
- C. K. Tse, *Complex Behaviour of Switching Power Converters*, CRC Press, Boca Raton, 2003.