FEEDBACK STABILIZATION OF THE OPERATION OF AN HYBRID CHEMICAL PLANT

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Abstract: This paper deals with the feedback stabilization of the operation of a simple hybrid chemical plant at an optimally scheduled operation. The optimally scheduled plant operation is obtained as a solution of a cyclic discrete time scheduling optimization problem and it is open loop unstable. The goal is to illustrate how the stability problem of an hybrid chemical process can be solved by using simple P and PI - like control laws. The sensitivity of the closed loop plant operation to the choice of the controllers parameters is also presented.

Keywords: chemical process, hybrid system, hybrid automaton, open loop scheduling, feedback control, stabilization.

1. INTRODUCTION

This communication deals with the feedback stabilization of an hybrid chemical plant at an optimally scheduled operation which is open-loop unstable. The goal is to assess the efficiency of simple P and PI-like controllers for the stabilization of an hybrid system, through a simple but realistic case-study. The considered plant consists of two tanks (a batch chemical reactor and a buffer tank). The overall system is hybrid in the sense that it is made up of both continuous and discrete event processes. It is modelled by means of the hybrid automaton formalism (Willems, 2003), (Lygeros *et al.*, 2003), (Bemporad and Heemels, 2005).
For this chemical plant, we first address the determination of a cyclic optimal schedule that maximizes the plant productivity. This optimization problem is solved by using a discrete time periodic scheduling method.
However, the state trajectory produced by the optimal schedule is unstable. This fact is easily emphasized by observing that under an arbitrarily small constant disturbance, the actual plant trajectory steadily diverges from the optimal one. As a result, the plant operation becomes not only sub-optimal but even infeasible. Our concern in this communication is to illustrate how this stability problem can be solved by using simple P and PI control laws. The performance of the control is illustrated through various simulation experiments carried out in the Matlab/Simulink/Stateflow environment. With the

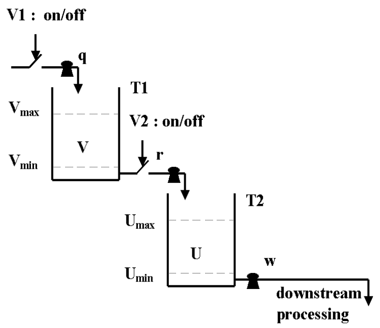

Fig. 1. Hybrid chemical plant

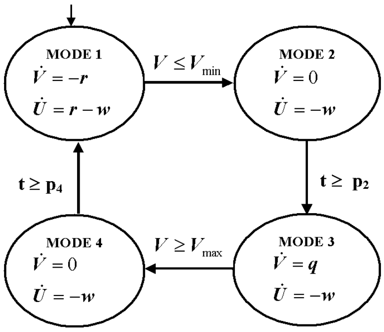

Fig. 2. Hybrid automaton diagram of the plant

considered control laws, it is shown that the operation of the plant is stabilized in the sense that the plant trajectory converges towards a limit cycle which is close to the optimal cyclic schedule. The dependence of the closed loop performance on the choice of the controller parameters is graphically illustrated. The organization of the paper is as follows. The hybrid chemical plant and its hybrid automaton model are presented in Section 2. Section 3 is concerned with the statement and the solution of the optimal cyclic scheduling problem. In Section 4, we present the feedback P and PI controllers. The closed loop performance and the control tuning are then assessed with a benchmark simulator of the plant. The results are given in Section 5. Some final remarks and directions for future work are presented in Section 6.

2. DESCRIPTION OF THE HYBRID CHEMICAL PLANT AND ITS HYBRID AUTOMATON MODEL

The considered hybrid chemical plant is depicted in figure (1). It consists of two tanks: (**T1** and **T2**) and two valves: (**V1** and **V2**). The first tank (**T1**) is a batch chemical reactor which is automatically operated with four successive operation modes: filling with raw material, production by chemical reaction, discharging (i.e. harvesting of final product), cleaning and waiting for the next operation. However, although it is produced in a discontinuous way, the final product has to be continuously delivered to the downstream processing stage. Therefore, there is an intermediate buffer tank **T2** between the batch reactor and the downstream processing plant which is discontinuously fed from the reactor, but continuously withdrawn. The plant is actually an hybrid system that combines time driven and event driven dynamics. The plant is modeled using the hybrid automaton formalism (Willems, 2003), (Lygeros *et al.*, 2003),(Bemporad and Heemels, 2005) which is presented hereafter. It should be remarked that the considered plant is a simplified version of the benchmark chemical plant that we have described in a former publication (Simeonova *et al.*, 2005*b*).

2.1 *Hybrid automaton model of the hybrid plant*

The process carried out in the plant follows a sequence of four successive modes (or phases), namely: Mode 1, Mode 2, Mode 3 and Mode 4. This set of four modes represents the discrete state variables of the considered hybrid process. The hybrid automaton diagram of the plant is depicted in figure (2). The vertices of this graph represent the modes of the automaton while the edges represent the time and/or continuous state event driven transitions between the modes. The process behavior during each mode is characterized by a set of continuous differential equations (mass balances) with continuous state variables: V $[m^3]$ - volume of **T1** and U $[m^3]$ - volume of **T2**. A short description of the process dynamics in each mode of the plant operation is as follows.

Initially the plant is in Mode 1, that means: **V1** is closed (off) and **V2** is open (on). The initial volume of **T1**, V_0 is equal to the maximal **T1** volume, $V_{\max}$ $[m^3]$ and the initial volume of **T2**, U_0 is equal to the minimal **T2** volume, $U_{\min}$ $[m^3]$.

- **Mode 1**: During this mode the material in **T1** is discharging with an output flow rate r $[m^3/h]$. Simultaneously **T2** is filling with the same input flow rate r $[m^3/h]$ and in the same time is discharging with an output flow rate w $[m^3/h]$. This is modeled as follows:

$$\dot{V} = -r \quad \dot{U} = r - w \tag{1}$$

  This mode lasts until the volume of the tank **T1** reaches a minimal value $V_{\min}$, then the plant process moves to Mode 2 (figure (2)).
- **Mode 2**: During this mode **V1** and **V2** are closed (off) and therefore **T1** is in stand by, while the material of **T2** is discharging with the output flow rate w $[m^3/h]$, and therefore:

$$\dot{V} = 0 \quad \dot{U} = -w \tag{2}$$

  Mode 2 lasts for a given time duration p_2 $[h]$, assigned by the operator, then the plant goes to Mode 3 (figure (2)).
- **Mode 3**: During this mode **V1** is open (on) and **V2** is closed (off). **T1** is filling with raw material with an inflow rate q $[m^3/h]$ while

T2 is discharging with the output flow rate w $[m^3/h]$. This is modeled as follows:

$$\dot{V} = q \quad \dot{U} = -w \tag{3}$$

The process lasts until the volume of **T1** reaches a maximal value $V_{\max}$, then the plant enters in Mode 4 (figure (2)).

- **Mode 4**: During this mode **V1** and **V2** are closed (off) and the chemical reaction proceeds in **T1** while the material of **T2** is discharging with an output flow rate w $[m^3/h]$. Therefore:

$$\dot{V} = 0 \quad \dot{U} = -w \tag{4}$$

After a certain time duration p_4 $[h]$, the reaction is completed and the plant goes to Mode 1 (figure (2)), and the operation cycle starts again.

We first present the open loop discrete time scheduling optimization of the hybrid chemical plant.

3. DISCRETE TIME SCHEDULING OPTIMIZATION IN OPEN LOOP

We consider the open loop cyclic scheduling problem consisting of maximizing the flow rate (the productivity) w $[m^3/h]$ of material leaving tank **T2**. Open loop means here that there is no feedback controller. Cyclic means that the overall process operation must take the form of a periodic repetition of a basic operation pattern.
First, we define the constant parameters and the variables of the problem. The parameters are: the minimum and maximum volume of the tanks **T1** and **T2** denoted by $V_{\min}$ $[m^3]$, $V_{\max}$ $[m^3]$, $U_{\min}$ $[m^3]$ and $U_{\max}$ $[m^3]$; the minimum $w_{\min}$ $[m^3/h]$ and maximum rate $w_{\max}$ $[m^3/h]$ of material leaving **T2**; the rate of material entering **T1**, q $[m^3/h]$ and the rate of material leaving **T1** and entering **T2**, r $[m^3/h]$; the processing time for each mode p_i for $i \in [1,4]$ where $p_1 = \frac{V_{\max}-V_{\min}}{r}$ $[h]$ and $p_3 = \frac{V_{\max}-V_{\min}}{q}$ $[h]$, while p_2 and p_4 are assigned by the operator.
The variables of the problem are the rate of material leaving tank **T2** denoted by w_i for each mode $i \in [1,4]$ and the value of the stock in **T2** denoted by U_i at the beginning of each mode $i \in [1,4]$.
The constraints of the problem are the following :

$$\begin{aligned}
&U_1 = U_{\min} \\
&U_1 = U_4 - w_4 p_4 \\
&U_2 = U_1 + r p_1 - w_1 p_1 \\
&U_3 = U_2 - w_2 p_2 \\
&U_4 = U_3 - w_3 p_3 \\
&U_{\min} \le U_i \le U_{\max} \ \forall i
\end{aligned} \tag{5}$$

These constraints guarantee that the schedule is cyclic. In order to have a relatively smooth material transfer to the downstream processing, we impose also that the w_i for $i \in [1,4]$ do not vary too much. This is guarantied by the following constraints:

$$|w_i - w_j| \le \epsilon \qquad \forall i,j \in [1,4], i \ne j \tag{6}$$

where ϵ is a small value. It is also imposed that:

$$w_{\min} \le w_i \le w_{\max} \ \forall i \tag{7}$$

The objective function is:

$$\max \sum_{i=1}^{4} w_i p_i \tag{8}$$

It is maximized with respect to the values w_i $[m^3/h]$ and U_i $[m^3]$ under the inequality constraints (5− 7).
The scheduling problem is solved under the following conditions for its constant parameters: $V_{\min} = 10$ $[m^3]$, $V_{\max} = 40$ $[m^3]$, $U_{\min} = 70$ $[m^3]$, $U_{\max} = 125$ $[m^3]$, $w_{\min} = 1$ $[m^3/h]$, $w_{\max} = 30$ $[m^3/h]$, $q = 30$ $[m^3/h]$, $r = 30$ $[m^3/h]$, $p_1 = 1$ $[h]$, $p_2 = 4$ $[h]$, $p_3 = 1$ $[h]$, $p_4 = 6$ $[h]$, $\epsilon = 0.2$ $[m^3/h]$.
A cyclic schedule solution of the resulting linear program is represented in figures (3) and (4).

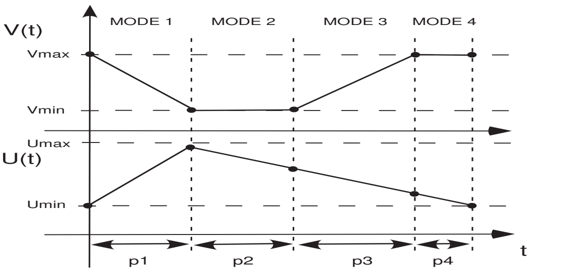

Fig. 3. Evolution of the material in the two tanks

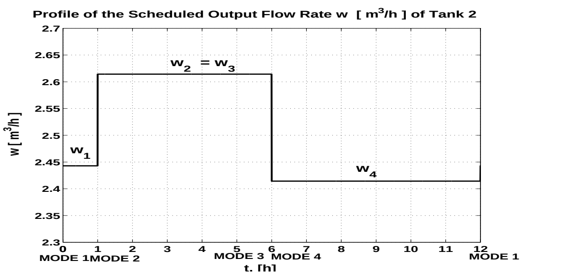

Fig. 4. Scheduled output flow rate of **T2**

This optimal solution of the scheduling problem is however not robust in cases of disturbances. The actual plant operation diverges from the optimal schedule in presence of disturbances. As a result the real plant performance becomes sub - optimal and the schedule may even become infeasible. This means that the plant operation is not stable when driven by the optimal schedule.

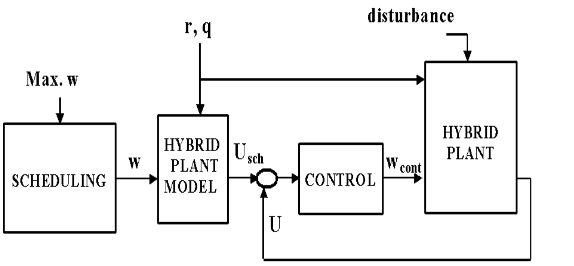

Fig. 5. Feedback control strategy

4. FEEDBACK STABILIZATION OF THE HYBRID CHEMICAL PLANT OPERATION

Let us recall that the objective of this paper is to illustrate how the stability problem of the plant operation can be solved by using simple P and PI control laws. The sensitivity of the closed loop plant operation to the choice of the controller parameters is also presented. The general structure of the feedback control strategy is presented in figure (5).

4.1 Statement of the closed loop control problem

In order to stabilize the operation of the considered hybrid chemical plant in the presence of disturbances, we consider the problem of controlling the volume of **T2**, U $[m^3]$ at the scheduled volume profile, U_{sch} $[m^3]$ (figure (5)) by acting on the output flow rate of material leaving **T2**, w $[m^3/h]$. It is assumed that the volume of **T2**, U $[m^3]$ is a measured output variable for the plant. In order to solve the control problem we first use a simple continuous feedback P control law which is written as follows:

$$w_{cont} = w - K_p.e_U \tag{9}$$

where $e_U = [U_{sch} - U]$ $[m^3]$ is the error between the scheduled and the real volume profile of **T2**, w $[m^3/h]$ is the scheduled output flow rate of material leaving **T2** (figure (4)), K_p is the coefficient of the P controller. As a second step a simple continuous feedback PI control law is used. It has the form:

$$w_{cont} = w - K_p.\left[e_U + \frac{1}{\tau_I}.\int e_U dt\right] \tag{10}$$

here K_p and τ_I $[h]$ are the coefficients of the PI controller. With these control laws, as shown in the next Section, the plant operation is stabilized in the sense that the plant trajectory converges towards a limit cycle which is close to the optimal cyclic schedule. Similarly to the constraint (6) in the scheduling problem here we impose that w_{cont} $[m^3/h]$ is in some interval $[w_{cont}^{min}\ w_{cont}^{max}]$. It is assumed that w_{cont}^{min} and w_{cont}^{max} are two times smaller and bigger, respectively to the minimal and the maximal values of w $[m^3/h]$. In order to illustrate the efficiency of both feedback control strategies, a Simulator of the hybrid plant has been developed in a Matlab / Simulink / Stateflow environment (Simeonova *et al.*, 2005*a*).

5. SIMULATION RESULTS

The hybrid plant is simulated under the following conditions for the plant constant parameters:
$V_{\min} = 10$ $[m^3]$, $V_{\max} = 40$ $[m^3]$, $V_0 = 40$ $[m^3]$, $U_0 = 70$ $[m^3]$, $p_2 = 4$ $[h]$, $p_4 = 6$ $[h]$, $q = 30$ $[m^3/h]$, $r = 30$ $[m^3/h]$.
The overall operation process of the hybrid chemical plant is presented hereafter.

5.1 Hybrid chemical plant operation

Four case studies of the plant operation are successively considered: scheduled plant operation in the absence of disturbances, plant operation in the presence of process disturbance, plant operation in the presence of process disturbance under P and PI control, respectively.

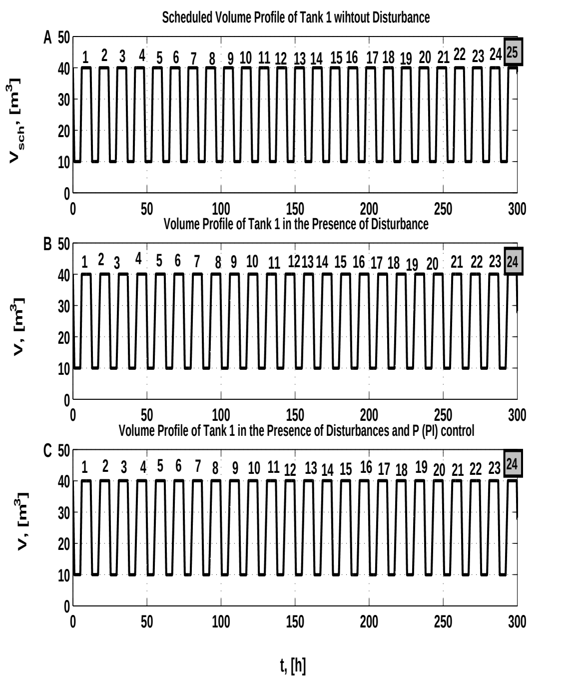

Fig. 6. V $[m^3]$: Volume profile of **T1**

- Scheduled Plant Operation in the Absence of Disturbance

The optimal scheduled plant operation is obtained after the application of the optimal scheduled flow rate w $[m^3/h]$ (figure (4)) to the plant Simulator

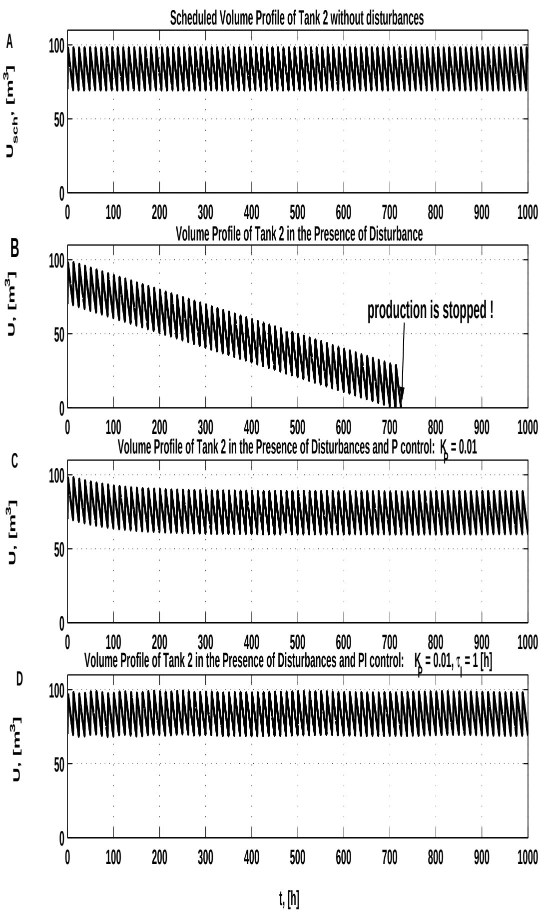

Fig. 7. U $[m^3]$: Volume profile of **T2**

in the absence of disturbances. In figure (6A) and (7A) are given respectively the scheduled volume profiles of **T1**, V_{sch} $[m^3]$ and **T2**, U_{sch} $[m^3]$. As expected the values of V_{sch} are in the interval $[V_{min}\ V_{max}]$ $[m^3]$. In figure (6A) it is also observed that there are 25 batches produced during the time period of approximately 300 $[h]$. In figure (7A) it is seen that the values of U_{sch} are in the approximate interval [70 100] $[m^3]$. Figure (8A) gives a phase plan representation of the optimal cycle. This cycle is actually unstable in open loop. Let us now consider the case when there is a small constant disturbance in the plant process.

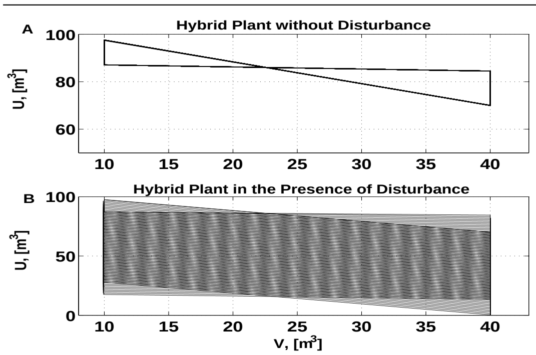

Fig. 8. Phase plane representation of the hybrid process in open loop

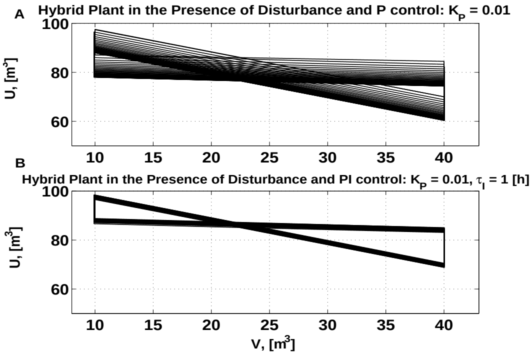

Fig. 9. Phase plane representation of the hybrid process in closed loop

- Plant Operation in the Presence of a Process Disturbance

Let at $t = 17$, $[h]$ the input flow rate $q = 30$ $[m^3/h]$ of material entering **T1** is incidentally decreased. Its new value is: $\widetilde{q} = 20$ $[m^3/h]$. The values of the other input variables are unchanged. As a consequence the real plant operation diverges from the optimal scheduled cycle. In figure (6B) it is observed that the values of the volume of **T1**, V stay in the interval $[V_{min}\ V_{max}]$ $[m^3]$. This is natural because the values V_{min} and V_{max} are used as a switching criteria for the hybrid process carried out in the plant. However due to the disturbance, there is a time delay in the evolution of the volume of **T1** and as a result the number of batches produced at $t \approx 300$ $[h]$ is 24 instead of 25 as scheduled. In figure (7B), we can clearly see the overall plant instability: the actual volume of **T2**, U $[m^3]$ steadily deviates from the scheduled volume (figure 7A). At time $t_{stop} \approx 724$ $[h]$, **T2** is totally empty and the production is stopped. In figure (8B) the plant instability is also observed: the plant trajectory progressively diverges from the optimal cycle (figure 8A). The periodically increasing - oscillatory behavior of the volume error in **T2**, e_U $[m^3]$ is shown in figure (10A). It has a basic period of 12 $[h]$, within an envelope of approximately 300 $[h]$.

In order to stabilize the plant operation and to avoid production stopping, the continuous time feedback P and PI control laws are implemented.

- Stabilization of the Plant Operation Under Process Disturbance by P Control

The P control law is tuned with the following controller parameter: $K_p = 0.01$. For both controllers $[w_{cont}^{min}\ w_{cont}^{max}] = [1.2\ \ 5.2]$ $[m^3/h]$.

In figure (7C) it is observed that, due to the use of the P control law, the volume profile of **T2**, U $[m^3/]$ is stabilized in the approximate interval [60 90] $[m^3]$ and the process in no longer interrupted at t_{stop} $[h]$. The settling time of the controller is $t \approx 200$ $[h]$. In figure (9A) it is seen that the plant trajectory converges towards a

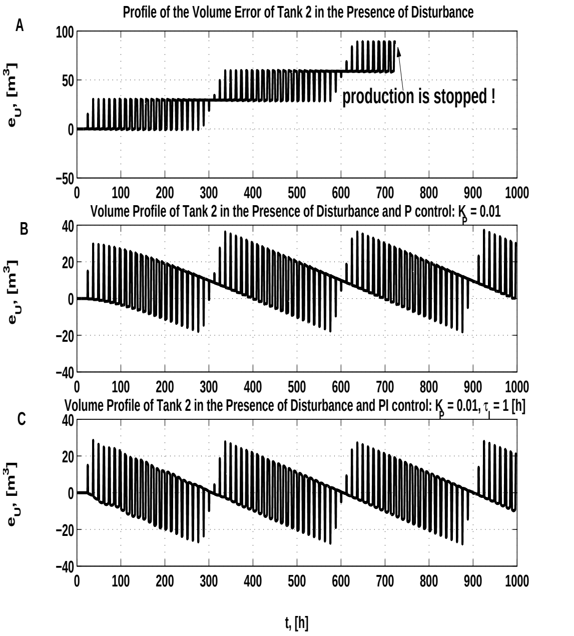

Fig. 10. e_U $[m^3]$: Error profile of $\mathbf{T2}$

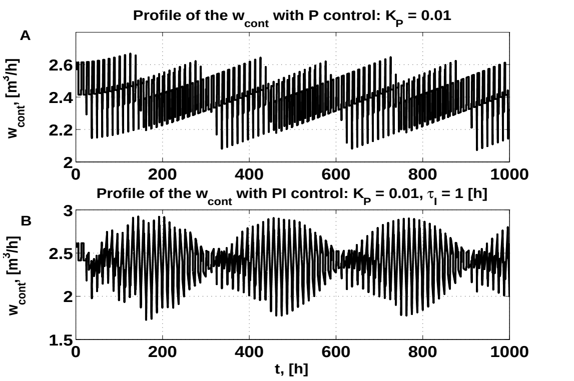

Fig. 11. Controlled output flow rate of $\mathbf{T2}$

limit cycle which is not very far from the optimal cyclic schedule. The availability of a static error compared with the optimal cycle (8A) is observed. Figure 11A shows the corresponding profile of the controlled output flow rate of $\mathbf{T2}$, w_{cont} $[m^3/h]$. The range of its values is not very different from the scheduled one (figure (4)).

Let us now consider the effect of the change of K_p parameter of the P control law, on the closed loop plant stability. In figure (12A,B,C) it is seen that if we decrease the values of the coefficient K_p, $K_p = 0.001$ the plant process becomes unstable. The volume error of $\mathbf{T2}$, e_U $[m^3]$ has a periodically increasing time evolution (figure (12A)). If the value of K_p is increased, $K_p = 0.08$ the plant behavior becomes slightly oscillatory. This is observed from the volume profile of $\mathbf{T2}$(figure (12F)). Moreover the values of the output flow rate w_{cont} $[m^3/h]$ (figure (12E)) increase a lot compared to the scheduled ones (figure (4)). Naturally both effects are undesirable.

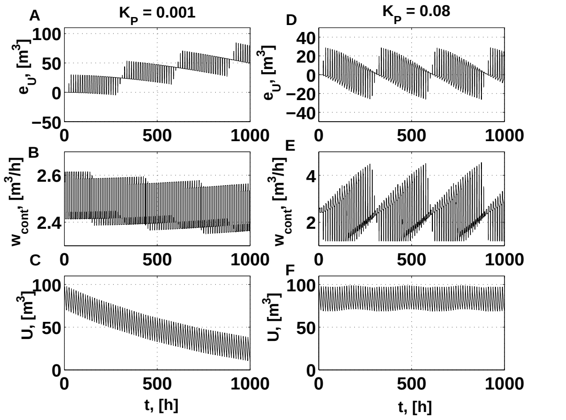

Fig. 12. Influence of K_p on the plant stability with P controller

- Stabilization of the Plant Operation Under Process Disturbance by PI Control

The PI control law is tunned with the following controller parameters: $K_p = 0.01$, $\tau_I = 1$ $[h]$.
The time evolution of the volume of $\mathbf{T1}$, V $[m^3]$ after the application of PI control law is the same as in both previous cases (figure 6C). As observed in figure (7D), similarly to the case when the P control law is used, by means of the PI control law, the volume profile of $\mathbf{T2}$, U $[m^3]$ is stabilized and the process in no longer interrupted at t_{stop} $[h]$. Moreover in this figure it is also seen that there is no static error. In figure (9B) it is seen that the plant operation is stabilized in the sense that the plant trajectory converges towards a limit cycle which is closer to the optimal cycle (figure 8A) compared to the case when the P control law is used (figure (9A)). In figure (10C) is observed that the volume error of $\mathbf{T2}$, e_U $[m^3]$ stays in smaller interval compared to the case when P control is applied. As seen in figure (11B) the range of the controlled output flow rate of $\mathbf{T2}$, w_{cont} $[m^3/h]$ is slightly increased with respect to the P controlled one (figure (11A)).

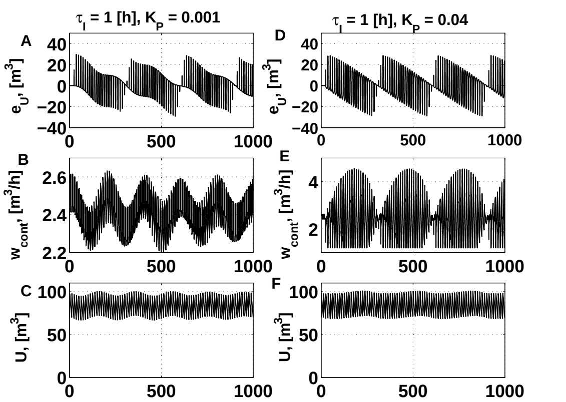

Fig. 13. Influence of K_p on the plant stability with PI controller

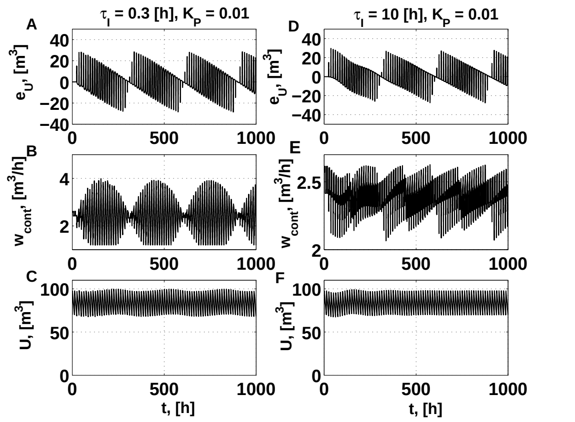

Fig. 14. Influence of τ_I on the plant stability with PI controller

Let us now consider the effect of the change of K_p parameter of the PI control law. In figure (13A, B, C) it is seen that if we decrease the values of the coefficient K_p, $K_p = 0.001$ the plant behavior becomes strongly oscillatory which is definitely undesirable. If the value of the coefficient K_p of the PI controller is increased, $K_p = 0.04$ the plant volume (13F) has a slightly oscillatory behavior compared to the case when $K_p = 0.01$. Moreover the range of the output flow rate is increased a lot compared to the scheduled once (figure (4)). Actually this behavior is similar to the case when we increase the value K_p of the P control law. Let us now observe the effect of the change of τ_I $[h]$. If the value of τ_I $[h]$ is decreased, $\tau_I = 0.3$ $[h]$ the plant behavior is slightly oscillatory and the range of w_{cont} $[m^3/h]$ is increased similarly to the case when we increase the coefficient K_p of the P and PI controllers. If the value of τ_I $[h]$ is increased, $\tau_I = 10$ $[h]$ instead of $\tau_I = 1$ $[h]$ the settling time of the controller is increased but the controlled output flow rate of **T2**, w_{cont} $[m^3/h]$ (figure 14E) is closer to the scheduled once compared to the case when $K_p = 0.01$.
Let us now see what is the quantity of material produced from the hybrid plant.

- Quantity of Produced Material

In figure (15A,B) the dotted line represents the optimal scheduled plant production W_{opt30} $[m^3]$ in the absence of disturbances during $t = 1000$ $[h]$ of operation. The solid line represents the closed loop quantity of material produced by P and PI control, respectively. As seen due to the availability of unknown disturbance the difference between the scheduled and actual quantity of produced material is progressively increasing. In figure (15C,D) the dotted line is the optimal plant production W_{opt20} $[m^3]$ supposing that at $t = 17$ $[h]$ (the time when the disturbance appears) a rescheduling is done knowing that the new value of q is $\tilde{q} = 20$ $[m^3/h]$. As observed the actual plant production converges to the optimally rescheduled one.

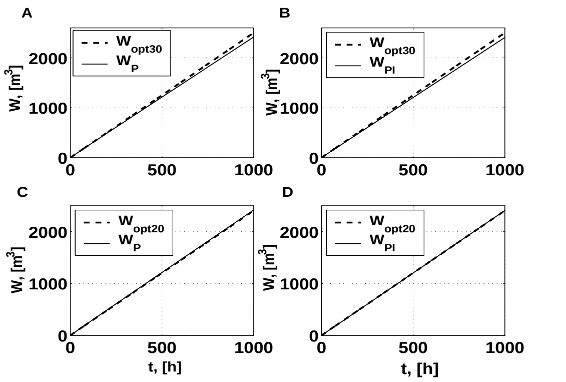

Fig. 15. $W[m^3]$: Quantity of produced material

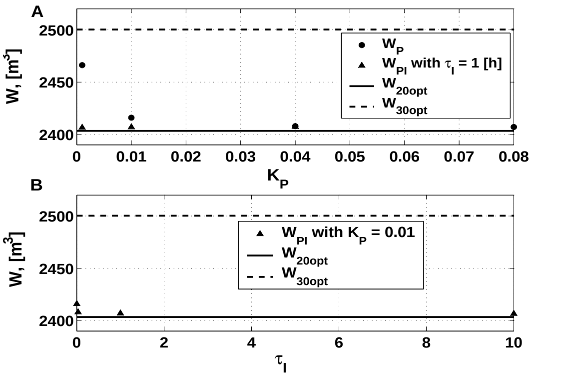

Fig. 16. Influence of the controller parameters on the closed loop plant production

Let us now see what is the influence of the controller parameters on the quantity of produced material. In figure (16A,B) the dotted line is the initial optimal scheduled quantity of produced material, W_{opt30} $[m^3]$ in the absence of disturbances after $t = 1000$ $[h]$ of operation. In the same figure the solid line is the optimal quantity of produced material, W_{opt20} $[m^3]$ assuming that at $t = 17$ $[h]$ (the time when the disturbance appears) the rescheduling is done knowing that the new value of q is $\tilde{q} = 20$ $[m^3/h]$. In figure (16A) the dotes illustrate the dependence of the production W_p $[m^3]$, after the application of P control, on the coefficient $K_p = \{0.001\ 0.01\ 0.4\ 0.8\}$. As seen the bigger is the coefficient K_p the closer is the value of the quantity W_p to the rescheduled quantity value, W_{opt20} $[m^3]$. In contrast the smaller is the value of K_p the closer is W_p $[m^3]$ to the initial optimal value W_{opt30} $[m^3]$. In the same figure the triangles represent the dependence of the production, obtained after the application of PI control, on the coefficient $K_p = \{0.001\ 0.01\ 0.4\}$. As seen this produced quantity is closer the the optimal rescheduled value W_{opt20} $[m^3]$ compared to the P control only. It is also seen that it does not depend a lot on the choice of K_p. In figure (16B) it is observed that the bigger is the parameter τ_I $[h]$

the closer is the productivity W_p $[m^3]$ to W_{opt20} $[m^3]$. The quantity W_{PI} in both cases are far from the initial optimal quantity W_{op30} $[m^3]$.

6. CONCLUSIONS AND FUTURE WORK

In this communication, we have dealt with the feedback stabilization of an hybrid chemical plant at an optimally scheduled operation which is open-loop unstable. The goal has been to illustrate how this stabilization problem can be solved by using simple P and PI control laws. The performance of the control has been illustrated through various simulation experiments carried out in the Matlab/Simulink/Stateflow environment. With the considered control laws, it is shown that the operation of the plant is stabilized in the sense that the plant trajectory converges towards a limit cycle which is close to the optimal cyclic schedule. An interesting challenge that is under study is to analyze the stability of these limit cycles through the fixed point stability of the corresponding Poincarre maps (Hiskens, 2001) (Girard, June 2003).

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