DYNAMIC OUTPUT FEEDBACK STABILIZATION OF CONTINUOUS-TIME SWITCHED SYSTEMS

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Abstract: This paper considers the closed-loop stabilization problem for continuous time linear switched system. The state variables are assumed to be not accessible so that the feedback strategy hinges on given output variables. The solution of this problem is based on the solution of suitable matrix inequalities for the construction of a full order switched filter and the derivation of the stabilization rule. The main theoretical basis is constituted by the so-called Lyapunov-Metzler inequalities which play a prominent role in the state-feedback stabilization of linear switched systems. Being nonconvex, a more conservative version of the inequalities, expressed in terms of linear matrix inequalities (LMI) plus a line search is given. The theoretical results are illustrated by means of an academic example.

Keywords: Switched systems, Output Feedback, Linear Matrix Inequalities.

1. INTRODUCTION

This paper considers a switched linear system of the following general form

$$\dot{x}(t) = A_{\sigma(t)}x(t) \ , \ x(0) = x_0 \quad (1)$$

$$y(t) = C_{\sigma(t)}x(t) \quad (2)$$

defined for all $t \geq 0$ where $x(t) \in \mathbb{R}^n$ is the state, x_0 is the initial condition, $y(t) \in \mathbb{R}^p$ is the measurement vector and $\sigma(t) \in \{1, 2, \cdots, N\}$ is the switching rule.

Assuming that the two sets of matrices $A_i \in \mathbb{R}^{n\times n}$, $i = 1, \cdots, N$ and $C_i \in \mathbb{R}^{p\times n}$, $i = 1, \cdots, N$ are given, we want to tackle the problem of finding a switching rule $\sigma(t)$ depending on the measurements $y(\cdot)$ in such a way that the closed-loop switched system is globally asymptotically stable. The stability of continuous time linear switched systems have been addressed by many authors, (Branicky, 1998), (Hockerman *et al.*, 1998), (Johansson *et al.*, 1998), (Ye *et al.*, 1998) and (Hespanha, 2004), where the interested reader can find an interesting discussion on a collection of results on uniform stability of switched systems. However, very little attention has been devoted to the design of output feedback control stabilizing laws.

See (Liberzon, 2003) for a rather complete review of stability of continuous time linear switched systems where special attention is given to the case of switching between two linear systems. The same reference also provides a discussion on hybrid feedback based on output measurements which, in our opinion, can not be directly generalized to cope with the problem to be stated afterwards.

In this paper a novel approach is pursued, which is inherited by the recent results obtained for the related state-feedback stabilization problem via the so-called Lyapunov-Metzler inequalities. Precisely, the solution involves a set of symmetric and positive matrices $\{Z_1, \cdots, Z_N\}$, a Metzler matrix Π, a positive matrix X and extra variables $\{L_1, \cdots, L_N\}$. Being the solution of non-convex nature a more conservative but easier to solve asymptotic stability condition is proposed. This condition is expressed in terms of linear matrix inequalities plus a line search, every LMI being solvable in polynomial time, (Boyd *et al.*, 1994).

The notation used throughout is standard. Capital letters denote matrices, small letters denote vectors and small Greek letters denote scalars. For matrices or vectors $(')$ indicates transpose. For symmetric matrices, $X > 0$ (≥ 0) indicates that X is positive definite (nonnegative definite). For square matrices trace(X) denotes the trace function of X being equal to the sum of its eigenvalues. The sets of real and natural numbers are denoted by $\mathbb{R}$ and $\mathbb{N}$ respectively. The $\mathcal{L}_2$ norm of $x(t) \in \mathbb{R}^n$ defined for all $t \geq 0$ equals $\|x(t)\|_2^2 = \int_0^\infty x(t)'x(t)dt$, see (Colaneri *et al.*, 1997) for details.

2. STATE-FEEDBACK

In this section we resume some recent results for the state-feedback stabilization of switched systems, that form the basis for the subsequent achievements on dynamic output feedback. For more information and detailed proofs, the reader is requested to see the reference (Geromel *et al.*, 2005). Precisely, consider the continuous-time linear switched system

$$\dot{x}(t) = A_{\sigma(t)}x(t) \ , \ x(0) = x_0 \qquad (3)$$

where all symbols have already been defined earlier. Assume that the state-variable is accessible for feedback. The goal is to determine the function $u(\cdot) : \mathbb{R}^n \to \{1, \cdots, N\}$, such that

$$\sigma(t) = u(x(t)) \qquad (4)$$

makes the equilibrium point $x = 0$ of (1) asymptotically stable. Before we proceed, let us recall the class of Metzler matrices denoted by $\mathcal{M}$ and constituted by all matrices $\Pi \in \mathbb{R}^{N\times N}$ with elements π_{ij}, such that

$$\pi_{ij} \geq 0 \ \forall i \neq j \ , \quad \sum_{i=1}^{N} \pi_{ij} = 0 \ \forall j \qquad (5)$$

It is clear that any $\Pi \in \mathcal{M}$ presents an eigenvalue at the origin of the complex plane since $c'\Pi = 0$ where $c' = [1 \ \cdots \ 1]$. In addition, it is well known that the eigenvector associated to the null eigenvalue of Π is non-negative yielding the conclusion that there exists $\lambda_\infty \geq 0$ with $c'\lambda_\infty = 1$ such that $\Pi\lambda_\infty = 0$. The following result holds.

Lemma 1. Let $Q_i \geq 0$, $i = 1, 2, \cdots, N$, be given. Assume that there exist a set of positive definite matrices $\{P_1, \cdots, P_N\}$ and $\Pi \in \mathcal{M}$ satisfying the Lyapunov-Metzler inequalities

$$A_i'P_i + P_iA_i + \sum_{j=1}^{N} \pi_{ji}P_j + Q_i < 0 \qquad (6)$$

for $i = 1, \cdots, N$. The state switching control (4) with

$$u(x(t)) = \arg \min_{i=1,\cdots,N} x(t)P_ix(t) \qquad (7)$$

makes the equilibrium solution $x = 0$ of (1) globally asymptotically stable and

$$\int_0^\infty x(t)'Q_{\sigma(t)}x(t)dt \leq \min_{i=1,\cdots,N} x_0'P_ix_0 \qquad (8)$$

The result above is important in that it also includes the stability of possible sliding modes. However the numerical determination, if any, of a solution of the Lyapunov-Metzler inequalities with respect to the variables $(\Pi, \{P_1, \cdots, P_N\})$ is not a simple task due to the non-convex nature inherited by the products of variables. Hence, a simpler, although more conservative stability condition can be expressed by means of LMIs being thus solvable by the available machinery plus a line search. The next theorem shows that working with a subclass of Metzler matrices, with the same diagonal elements, this goal is accomplished.

Lemma 2. Let $Q_i \geq 0$, $i = 1, 2, \cdots, N$, be given. Assume that there exist a set of positive definite matrices $\{P_1, \cdots, P_N\}$ and a scalar $\gamma > 0$ satisfying the modified Lyapunov-Metzler inequalities

$$A_i'P_i + P_iA_i + \gamma(P_j - P_i) + Q_i < 0 \qquad (9)$$

for all $j \neq i = 1, \cdots, N$. The state switching control (4) with $u(x(t))$ given by (7) makes the equilibrium solution $x = 0$ of (1) globally asymptotically stable and

$$\int_0^\infty x(t)'Q_{\sigma(t)}x(t)dt \leq \sum_{i=1}^{N} x_0'P_ix_0 \qquad (10)$$

Remark 1. The Lyapunov - Metzler inequalities have been introduced in order to study the *Mean-Square* (MS) stability of Markov Jump Linear Systems (MJLS), where matrix $\Pi \in \mathcal{M}$ is given and Π' represents the infinitesimal transition matrix of a Markov chain $\sigma(t)$ governing the dynamic system (1). In this respect the vector of probabilities $\lambda_i(t)$ to be in the $i-th$ logical state at any time $t \geq 0$ obeys the differential equation

$\dot{\lambda}(t) = \Pi\lambda(t)$ with initial probability vector λ_0. Hence, λ_∞ represents the stationary probability vector. The stochastic system is MS-stable if and only if there exist a set of positive definite matrices $\{P_1, \cdots, P_N\}$ satisfying the Lyapunov-Metzler inequalities (6), see (Fang *et al.*, 2002).

3. OUTPUT-FEEDBACK

This section contains the new results relative to the output feedback case. Given the measurements

$$y(t) = C_{\sigma(t)}x(t) \tag{11}$$

our main result consists on the determination of a switching rule of the form

$$\sigma(t) = u(y(\cdot)) \tag{12}$$

such that the closed-loop system is asymptotically stable. The function $u(\cdot)$ is indeed a functional of $y(\cdot)$ in the sense that $y(t)$ is viewed as the input of a switched filter that rules out the change of the switching index. To this end, let us introduce the full order switched filter

$$\dot{\hat{x}}(t) = \hat{A}_{\sigma(t)}\hat{x}(t) + \hat{B}_{\sigma(t)}y(t), \quad \hat{x}(0) = \hat{x}_0 \tag{13}$$

where $(\hat{A}_i, \hat{B}_i)$, $i = 1, 2, \cdots, N$ are matrices to be determined. Putting (13) and (1), (2) together we obtain

$$\dot{\tilde{x}}(t) = \tilde{A}_{\sigma(t)}\tilde{x}(t), \quad \tilde{x}(0) = \tilde{x}_0 \tag{14}$$

where $\tilde{x}' = [x' \ \hat{x}']$ and

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ \hat{B}_iC_i & \hat{A}_i \end{bmatrix} \tag{15}$$

Therefore the solution of our problem requires the determination of the switched filter matrices $\hat{A}_i$ and $\hat{B}_i$ and of a switching rule such such that the enlarged switched system is asymptotically stable. However, in doing so, only switching rules that depends exclusively on $\hat{x}(\cdot)$ are permitted. In order to apply the results of the previous section, we limit the structure of the Lyapunov function so as to structurally incorporate switching rules that depends only on $\hat{x}(\cdot)$. Therefore, let

$$\tilde{P}_i = \begin{bmatrix} X & V \\ V' & \hat{X}_i \end{bmatrix}, \quad \det V \neq 0 \tag{16}$$

and notice that

$$\arg \min_i \tilde{x}(t)'\tilde{P}_i\tilde{x}(t) = \arg \min_i \hat{x}(t)\hat{X}_i\hat{x}(t) \tag{17}$$

Hence, to solve the problem under consideration, we need to find a stabilizing rule of the form

$$u(y(\cdot)) = \arg \min_{i=1,\cdots,N} \hat{x}(t)\hat{X}_i\hat{x}(t) \tag{18}$$

Finally consider positive semidefinite matrices Q_i of compatible dimensions and let

$$\tilde{Q}_i = \begin{bmatrix} Q_i & 0 \\ 0 & 0 \end{bmatrix} \tag{19}$$

The following result is a direct consequence of Lemma 1 applied to the composite switched system (14).

Lemma 3. Let $Q_i \geq 0$, $i = 1, 2, \cdots, N$, be given. Assume that there exist a set of positive definite matrices $\{\tilde{P}_1, \cdots, \tilde{P}_N\}$ where each $\tilde{P}_i$ is of the form (16), a set of matrices $\{\hat{A}_1, \cdots, \hat{A}_N\}$, a set of matrices $\{\hat{B}_1, \cdots, \hat{B}_N\}$ and a Metzler matrix $\Pi \in \mathcal{M}$ satisfying the Lyapunov-Metzler inequalities

$$\tilde{A}_i'\tilde{P}_i + \tilde{P}_i\tilde{A}_i + \sum_{j=1}^{N} \pi_{ji}\tilde{P}_j + \tilde{Q}_i < 0 \tag{20}$$

for $i = 1, \cdots, N$. The output switching control (12) with $u(y(\cdot))$ given by (18) makes the equilibrium solution $x = 0$ of (1) globally asymptotically stable and

$$\int_0^\infty x(t)'Q_{\sigma(t)}x(t)dt \leq \min_{i=1,\cdots,N} \tilde{x}_0'\tilde{P}_i\tilde{x}_0 \tag{21}$$

The main problem underlying inequality (20) is that the unknowns of the filter and $\tilde{P}_i$ appear in a nonlinear form. Hence, we try to simplify the inequalities by introducing the square matrices

$$\tilde{T}_i = \begin{bmatrix} I & I \\ \Gamma_i & 0 \end{bmatrix} \tag{22}$$

where Γ_i for $i = 1, \cdots, N$ are suitable extra variables to be determined in order to simplify the problem. To this end, notice that

$$\tilde{T}_i'\tilde{P}_i\tilde{A}_i\tilde{T}_i = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \tag{23}$$

where

$$N_{11} = (X + \Gamma_i'V')A_i + (V + \Gamma_i'\hat{X}_i)(\hat{B}_iC_i + \hat{A}_i\Gamma_i)$$
$$N_{12} = (X + \Gamma_i'V')A_i + (V + \Gamma_i'\hat{X}_i)\hat{B}_iC_i$$
$$N_{21} = XA_i + V\hat{B}_iC_i + V\hat{A}_i\Gamma_i$$
$$N_{22} = XA_i + V\hat{B}_iC_i$$

Therefore, a sensible choice of Γ_i is $\Gamma_i = -\hat{X}_i^{-1}V'$ which, by redefining the unknowns as

$$Z_i = X + \Gamma_i'V' \tag{24}$$
$$L_i = V\hat{B}_i \tag{25}$$
$$M_i = -V\hat{A}_i\Gamma_i = V\hat{A}_iV^{-1}(X - Z_i) \tag{26}$$

for all $i = 1, \cdots, N$ leads to the conclusion that the matrix blocks of (23) can alternatively be written as

$$N_{11} = Z_iA_i \tag{27}$$
$$N_{12} = Z_iA_i \tag{28}$$
$$N_{21} = XA_i + L_iC_i - M_i \tag{29}$$
$$N_{22} = XA_i + L_iC_i \tag{30}$$

and, in addition, simple algebraic calculations put in evidence that

$$\tilde{T}_i'\tilde{Q}_i\tilde{T}_i = \begin{bmatrix} Q_i & Q_i \\ Q_i & Q_i \end{bmatrix},\ \tilde{T}_i'\tilde{P}_i\tilde{T}_i = \begin{bmatrix} Z_i & Z_i \\ Z_i & X \end{bmatrix} \quad (31)$$

The importance of the transformation matrices $\tilde{T}_i$ introduced before is apparent since, as indicated in (31), the product $\tilde{T}_i'\tilde{P}_i\tilde{T}_i$ is linearized and the product $\tilde{T}_i'\tilde{P}_j\tilde{T}_i$ for $j \neq i$ can be expressed in terms of LMIs. Indeed,

$$\begin{aligned} \tilde{T}_i'\tilde{P}_j\tilde{T}_i &= \tilde{T}_i'(\tilde{P}_j - \tilde{P}_i)\tilde{T}_i + \tilde{T}_i'\tilde{P}_i\tilde{T}_i \\ &= \tilde{T}_i' \begin{bmatrix} 0 & 0 \\ 0 & \hat{X}_j - \hat{X}_i \end{bmatrix} \tilde{T}_i + \begin{bmatrix} Z_i & Z_i \\ Z_i & X \end{bmatrix} \\ &= \begin{bmatrix} Z_i + \Gamma_i'(\hat{X}_j - \hat{X}_i)\Gamma_i & Z_i \\ Z_i & X \end{bmatrix} \\ &= \begin{bmatrix} Z_j + (Z_j - Z_i)(X - Z_j)^{-1}(Z_j - Z_i) & Z_i \\ Z_i & X \end{bmatrix} \end{aligned}$$

which calculated for all $i, j = 1, \cdots, N$ imply that

$$\tilde{T}_i' \left(\sum_{j=1}^{N} \pi_{ji}\tilde{P}_j \right) \tilde{T}_i = \sum_{j=1}^{N} \pi_{ji} \begin{bmatrix} Z_j + (Z_j - Z_i)(X - Z_j)^{-1}(Z_j - Z_i) & 0 \\ 0 & 0 \end{bmatrix}$$

Based on these calculations, we are now in position to provide the main result of the paper.

Theorem 1. Let $Q_i \geq 0$, $i = 1, 2, \cdots, N$ be given. There exist matrices $\hat{A}_i$ and $\hat{B}_i$, $i = 1, 2, \cdots, N$ for which inequalities (20) are satisfied for some positive definite matrices $\tilde{P}_i$ of the form (16) if and only if there exist a Metzler matrix Π, a positive definite matrix X, a set of positive matrices Z_i, a set of positive matrices R_{ij} and a set of matrices L_i for all $i, j = 1, 2, \cdots, N$, such that the following matrix inequalities

$$A_i'Z_i + Z_iA_i + \sum_{j=1}^{N} \pi_{ji}R_{ij} + Q_i < 0 \quad (32)$$

$$A_i'X + XA_i + C_i'L_i' + L_iC_i + Q_i < 0 \quad (33)$$

$$R_{ii} < Z_i \quad (34)$$

$$\begin{bmatrix} R_{ij} - Z_j & Z_j - Z_i \\ Z_j - Z_i & X - Z_j \end{bmatrix} > 0, \quad i \neq j \quad (35)$$

hold. Finally, assume that (32)-(35) are satisfied. The output switching control (12) defined by

$$u(y(\cdot)) = \arg \min_i \hat{x}(t)V'(X - Z_i)^{-1}V\hat{x}(t) \quad (36)$$

where V is an arbitrary nonsingular matrix, makes the equilibrium solution $x = 0$ of (1) globally asymptotically stable and

$$\int_0^\infty x(t)'Q_{\sigma(t)}x(t)dt \leq \min_{i=1,\cdots,N} \tilde{x}_0'\tilde{P}_i\tilde{x}_0 \quad (37)$$

Proof: Assume first that inequalities (32)-(35) are satisfied and consider the partitioned matrix (23) whose blocks, from the definitions (24) to (26) together with $\Gamma_i = -\hat{X}_i^{-1}V'$, are written as (27)-(30). Consequently, our main purpose is to investigate the structure of the following symmetric matrix expression

$$S_i := \tilde{T}_i' \left(\tilde{A}_i'\tilde{P}_i + \tilde{P}_i\tilde{A}_i + \sum_{j=1}^{N} \pi_{ji}\tilde{P}_j + \tilde{Q}_i \right) \tilde{T}_i$$

and show that $S_i < 0$ for some choice of the filter matrices. Letting S_{11}, S_{21} and S_{22} the three characterizing blocks of S_i, it follows that

$$\begin{aligned} S_{11} &= A_i'Z_i + Z_iA_i + \sum_{j=1}^{N} \pi_{ji}Y_{ij} + Q_i \\ S_{21} &= A_i'Z_i + XA_i + L_iC_i - M_i + Q_i \\ S_{22} &= A_i'X + XA_i + L_iC_i + C_i'L_i' + Q_i \end{aligned}$$

where

$$Y_{ij} = Z_j + (Z_j - Z_i)(X - Z_j)^{-1}(Z_j - Z_i)$$

The filter matrix $\hat{B}_i$ directly follows from equation (25), where V is a selected nonsingular matrix. Notice now that $S_{22} < 0$ in view of (33). Notice also that assumptions (34) and (35) imply $R_{ij} > Y_{ij}$ and $R_{ii} < Y_{ii}$, so that $S_{11} < 0$. Finally, take the filter matrix $\hat{A}_i$ such that $M_i = A_i'Z_i + XA_i + L_iC_i + Q_i$ which implies $S_{21} = 0$. In conclusion, from (25) and (26), the filter matrices are

$$\hat{B}_i = V^{-1}L_i \quad (38)$$

$$\begin{aligned} \hat{A}_i &= V^{-1}M_i(X - Z_i)^{-1}V \quad (39) \\ &= V^{-1}(A_i'Z_i + XA_i + L_iC_i + Q_i)(X - Z_i)^{-1}V \end{aligned}$$

and $\tilde{P}_i$ as in (16) with $\hat{X}_i = V'(X - Z_i)^{-1}V$. Viceversa, assume that inequalities (20) holds for some $\hat{A}_i$, $\hat{B}_i$ and $\tilde{P}_i$ as in (16). Then, apply the state-space transformation (22) with $\Gamma_i = -\hat{X}_i^{-1}V'$ and consider the expressions (24)-(30). Hence $S_i < 0$ and the filter matrices are

$$\hat{B}_i = V^{-1}L_i \quad (40)$$

$$\hat{A}_i = V^{-1}M_i(X - Z_i)^{-1}V \quad (41)$$

In conclusion, condition (33) coincides with $S_{22} < 0$, and (32) comes from $S_{11} < 0$ by letting $R_{ij} = Y_{ij} + \epsilon I$, $i \neq j$ and $R_{ii} = Z_i - \epsilon I$, for $\epsilon > 0$ small enough. From this (34) and (35) are satisfied.

Finally, assume that (32)-(35) are satisfied, and recall Lemma 3. Therefore, from $\hat{X}_i = V'(X - Z_i)^{-1}V$, it follows that the output switching control (12) with $u(y(\cdot))$ given by (36) makes the equilibrium solution $x = 0$ of (1) globally asymptotically stable and (37) is satisfied. □

It is important to stress the fact that formulas (40) and (41) provide a parametrization of all filters (13) for which (20) is satisfied with $\tilde{P}_i$ as in (16). In order to provide less stringent conditions, the equality $M_i = A_i'Z_i + XA_i + L_iC_i + Q_i$ has been set in the proof of the sufficient part of the theorem so as to put to zero the off-diagonal block entries of S_i.

The structure of the full-order filter is not in the observer form, i.e. $\hat{A}_i \neq A_i - \hat{B}_iC_i$. To recover this condition, an additional constraint, unfortunately non linear, has to be added, namely (the simple check is left to the reader)

$$\begin{aligned} M_i &= (VA_i - L_iC_i)V^{-1}(X - Z_i) \\ &= A_i'Z_i + XA_i + L_iC_i + Q_i \end{aligned} \tag{42}$$

A notable exception can be devised by letting $Q_i = 0$, so overlooking the guaranteed cost property (37). Indeed, in this case, we have the following result.

Theorem 2. Assume that there exist a Metzler matrix Π, a positive definite matrix X, a set of positive matrices L_i and a set of positive matrices Z_i for all $i = 1, 2, \cdots, N$, such that the following matrix inequalities are satisfied:

$$A_i'Z_i + Z_iA_i + \sum_{j=1}^{N} \pi_{ji}Z_j < 0 \tag{43}$$

$$A_i'X + XA_i + C_i'L_i' + L_iC_i < 0 \tag{44}$$

Then, the switching rule

$$u(y(\cdot)) = \arg \min_i \hat{x}(t)Z_i\hat{x}(t) \tag{45}$$

makes the equilibrium solution $x = 0$ of (1) globally asymptotically stable where $\hat{x}$ satisfies the differential equation of the filter (13) in observer form with

$$\hat{B}_i = -X^{-1}L_i \tag{46}$$

$$\hat{A}_i = A_i - \hat{B}_iC_i \tag{47}$$

Proof: The proof relies to Theorem 1, by letting $Z_i \to \epsilon Z_i$ with $\epsilon > 0$ arbitrarily small and $V = -X$. Indeed, notice that the condition (42) for the filter to be in observer form is satisfied for ϵ going to zero and that

$$\begin{aligned} &\arg \min_i \hat{x}(t)V'(X - \epsilon Z_i)^{-1}V\hat{x}(t) = \\ &\arg \min_i \hat{x}(t)(X + (Z_i^{-1}/\epsilon - X^{-1})^{-1})\hat{x}(t) \sim \\ &\arg \min_i \hat{x}(t)\epsilon Z_i\hat{x}(t) \sim \arg \min_i \hat{x}(t)Z_i\hat{x}(t) \end{aligned}$$

holds. □

The conclusion is that if there exist N gains that make the filter quadratically stable, see equation (44), then the usual solution to the Metzler-Lyapunov inequality (state feedback, equation (43)) provides a stabilizing switching rule. It is important to keep in mind that if we want to determine a good switching strategy by minimizing a guaranteed quadratic cost then this solution although stabilizing is not the best that can be done. Moreover, it should be noticed that the output feedback strategies invoked by the theorems presented so far require the existence of a state-observer injection matrices $\hat{L}_i = X^{-1}L_i$, $i = 1, \cdots, N$ that render the set of matrices $A_i + \hat{L}_iC_i$ quadratically stable (see e.g. equation (33)).

Remark 2. There is no difficulty to get the versions of Theorem 1 and Theorem 2 associated to the modified Lyapunov-Metzler inequalities appearing in Lemma 2. The BMIs are replaced by LMIs with an additional parameter that can be determined by line search. The results follow the same pattern of each mentioned theorem, being thus omitted.

The next example illustrates some aspects of the theoretical results obtained so far. Consider the system (1) with $N = 2$ and matrices $\{A_1, A_2\}$ given by

$$A_1 = \begin{bmatrix} 0 & 1 \\ 2 & -9 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0 & 1 \\ -2 & 2 \end{bmatrix} \tag{48}$$

which, as it can be easily verified by inspection, are both unstable. Setting $Q = I$ our goal is to design an output feedback stabilizing control which minimizes the quadratic cost on the left hand side of (37). Considering the filter initial condition $\hat{x}(0) = 0$, it is readily seen that

$$\int_0^\infty x(t)'Q_{\sigma(t)}x(t)dt \leq x_0'Xx_0 \tag{49}$$

whenever the switching rule satisfies the conditions provided by Theorem 1. Setting

$$\Pi = \begin{bmatrix} -20 & 20 \\ 20 & -20 \end{bmatrix} \tag{50}$$

matrices X, $Z_1, \cdots, Z_N$ and $L_1, \cdots, L_N$ have been determined from the solution of the convex programming problem

$$\min\{\text{trace}(X)\ ,\ (32) - (35)\} \tag{51}$$

which means that the minimum guaranteed cost (49) is calculated with x_0 being a random variable with zero mean and unitary covariance matrix. The optimal solution provided the minimum guaranteed cost trace$(X) \approx 338.04$. The associated matrix variables and the choice $V = -X$ allow the determination of the stabilizing output feedback switching rule $\sigma(t) = u(y(\cdot))$ given by (36) as well as the switching filter (13).

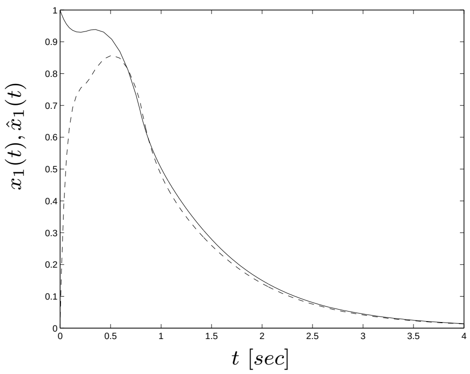

Fig. 1. Time simulation - first state variable

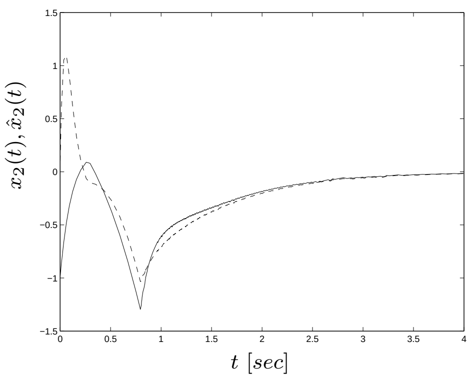

Fig. 2. Time simulation - second state variable

Time simulation has been performed from the initial conditions $x_0 = [1 \ -1]'$ and $\hat{x}_0 = [0 \ 0]'$. Figures 1 and 2 show the trajectories of the state variable $x(t) \in \mathbb{R}^2$ of the system (solid line) and the state variable $\hat{x}(t) \in \mathbb{R}^2$ of the switching filter (dashed line) versus time for the system and filter controlled by the output switching rule $\sigma(t) = u(y(\cdot))$ given by (36). Even though, in this case, the filter does not exhibit an observer structure, it estimates with good precision - under switching - the state variable of the system. Indeed, as it is clearly indicated in the mentioned figures, after a small period of time ($\approx$ 1.0 sec.) $\hat{x}(t)$ provides $x(t)$ from the available measurements $y(\tau), \tau \leq t$. From the same figures, it can be seen, that the proposed control strategy is very effective to stabilize the system under consideration.

4. CONCLUSION

In this paper we have introduced a new stability condition for continuous time switched systems based on output measurements. Furthermore, the determination of a guaranteed cost associated to the proposed control strategy has been addressed. The results reported are necessary and sufficient for the existence of a switching filter and a switching rule based exclusively on the output vector of the original system in such a way the so called Lyapunov-Metzler inequalities have a feasible solution. In a precise case, identifying in the paper, the switching filters are of the form of classical observers. Special attention has been devoted towards the numerical solvability of the design problems by means of methods based on linear matrix inequalities. There are several points that deserve further investigation. Among them, it is important to mention the possible introduction of robustness issues against parameter uncertainty as considered, for instance, in (Geromel *et al.*, 1991) appearing in the system and filters parameters. The example showed that the switching rule is particulary sensitive to parameter uncertainty which may in particular cases lead to instability of the closed loop system.

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