STABILIZATION OF SWITCHED LINEAR SYSTEMS WITH UNKNOWN TIME VARYING DELAYS

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Abstract: We consider continuous time switched systems that are stabilized via a computer. Our goal is to construct a switched digital control for continuous time switched systems that is robust to the varying feedback delay problem. The key idea of this paper is that the control synthesis problem in the case of continuous time systems with uncertain time varying feedback delays can be expressed as a problem of stabilizability for uncertain systems with polytopic uncertainties. For the sake of generality, the problem of switched systems will be considered (the solution for LTI systems can trivially be deduced by eliminating the switching aspect of the problem).

Keywords: Switched linear systems, time varying feedback delays, polytopic uncertainties, robustness, LMI.

1. INTRODUCTION

Nowadays, many control problems are solved via a computer generated feedback. When dealing with such a digital control, uncertain time varying feedback delays are unavoidable. These delays can affect system performances and lead to instability if they are not taken into account (Wittenmark *et al.*, 1995). The problem engaged scientists from both computer science (Buttazzo, 2002; Martí *et al.*, 2001; Stankovic *et al.*, 2001) and control systems (Åström and Wittenmark, 1997; Wittenmark *et al.*, 1995). It has been studied in the general context of real-time control systems (Nilsson *et al.*, 1998) for embedded and networked control systems (Årzen *et al.*, 2000; Hristu-Varsakelis and Levine, 2005; Montestruque and P.J.Antsaklis, 2004; Tzes *et al.*, 2005; Zhang *et al.*, 2001).

Here, we treat the control synthesis problem for continuous time systems affected by uncertain time varying feedback delays in a digital control context. This paper is organized as follows. In Section 2, we mathematically formalize the problem under study: "the robust stabilizability of continuous time switched systems relative to time varying feedback delays" and we introduce its sampled version. In Section 3, we show that, by using a Taylor series approximation, the system affected by time varying feedback delays can be expressed as a polytopic uncertain system. In Section 4, we present the control synthesis for this problem and we apply it to the polytopic version of the system affected by feedback delays. The approach is illustrated by a numerical example in Section 5.

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Notations: For a matrix M we denote by $\|M\|$ the induced matrix norm. By $M > 0$ or $M < 0$ we mean that the matrix M is positive or negative definite respectively. By $\mathbf{I}$ (or $\mathbf{0}$) we denote the identity (or the null) matrix with the appropriate dimension.

2. PROBLEM FORMULATION

We consider a continuous time switched system

$$\frac{dx(t)}{dt} = M_\sigma x(t) + N_\sigma u(t), \qquad (1)$$

The switching function $\sigma(t) : \mathbb{R}^+ \rightarrow P = \{1, 2, .., N\}$ gives a particular index i indicating the active system regime. It is used to represent sudden changing of system dynamic (for example the switch on/off of a pump in a tank system). $\{M_i \in \mathbf{R}^{n\times n} : i \in P\}$ and $\{N_i \in \mathbf{R}^{n\times m} : i \in P\}$ are two families of matrices. Each pair (M_i, N_i), $i \in P$ describes a continuous time model representing different regimes of system behavior. Here σ will be considered a piecewise constant function that may change its value at $t = kT, k \in \mathbb{Z}^+$ with $T > 0$ the sampling period.

In order to design a computer based control, a sampled model of the continuous time system is derived and discrete time control methods are applied. The final digital control is strongly dependent on the sampling period and on the discrete description of the plant.

When sampling and actuation are considered to be periodic and synchronous with the periodicity T, the equivalent discrete representation of the system is given by integrating the solution of the system over one sampling period:

$$x(k+1) = A_\sigma x(k) + B_\sigma u(k), \qquad (2)$$

$$A_\sigma = e^{M_\sigma T}, \qquad B_\sigma = \int_0^T e^{M_\sigma s} ds N_\sigma$$

A more realistic discrete representation should consider that the system is affected by several delays (Wittenmark *et al.*, 1995): delays between the sensor and the digital control $\tau^{sc}(k)$, computing delays in the controller $\tau^{c}(k)$ and communication delays between the controller and the actuator $\tau^{ca}(k)$. The total delay in the closed-loop is $\Delta T(k) = \tau^{sc}(k) + \tau^{c}(k) + \tau^{ca}(k)$. These delays have an unknown random varying length. However, the total system delay is bounded, $\Delta T_{min} \leq \Delta T(k) \leq \Delta T_{max}$. Here we will analyze the case where $\Delta T_{max} < T$ and where the switching signal σ and the sampling T are synchronous. When the effect of time delays is taken into account the system input is given by:

$$u(t) = \begin{cases} u(k-1), t \in [kT, kT + \Delta T(k)) \\ u(k), t \in [kT + \Delta T(k), (K+1)T) \end{cases} \qquad (3)$$

Problem: *Assuming that switching signal σ and the sampling T are synchronous, find a switched state feedback that robustly stabilizes the continuous time switched system (1) when the input is affected by time varying delays (3).*

Remark: It is clear that this problem is the same as the classical "timing problem" in the case of LTI systems if we consider N=1.

In order to solve such a problem, we consider the discrete representation of the system (1) over a sampling period:

$$\begin{aligned} x(k+1) = \; & \Phi_\sigma x(k) + \Gamma^0_\sigma(\Delta T(k))u(k) \\ & + \Gamma^1_\sigma(\Delta T(k))u(k-1) \end{aligned} \qquad (4)$$

$$\text{where } \Phi_\sigma = e^{M_\sigma T} = A_\sigma, \qquad (5)$$

$$\Gamma^1_\sigma(\Delta T(k)) = \int_{T-\Delta T(k)}^{T} e^{M_\sigma s} ds N_\sigma$$

$$\Gamma^0_\sigma(\Delta T(k)) = \int_0^{T-\Delta T(k)} e^{M_\sigma s} ds N_\sigma \qquad (6)$$

$$= B_\sigma - \Gamma^1_\sigma(\Delta T(k)). \qquad (7)$$

The proposed system can be written as:

$$z(k+1) = \hat{A}_\sigma z(k) + \hat{B}_\sigma u(k) \qquad (8)$$

$$\text{where } \hat{A}_\sigma(k) = \begin{bmatrix} \Phi_\sigma & \Gamma^1_\sigma(\Delta T(k)) \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\hat{B}_\sigma(k) = \begin{bmatrix} \Gamma^0_\sigma(\Delta T(k)) \\ \mathbf{I} \end{bmatrix}$$

$$\text{and } z(k) = \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix}.$$

This representation has been adapted for switched systems from the real time linear system representation presented by Åström and Wittenmark (Åström and Wittenmark, 1997). In this model $\Gamma^1_\sigma(\Delta T(k))$ and $\Gamma^0_\sigma(\Delta T(k))$ are strongly dependent on the uncertain time varying delay $\Delta T(k)$. Therefore the previous system is an uncertain system with time varying uncertainty.

Now, the problem under study reduces to find a switched state feedback $u(k) = K_\sigma z(k)$ that stabilizes the discrete switched uncertain system (8) when the total system delay $\Delta T_{min} \leq \Delta T(k) \leq \Delta T_{max}$ is time varying and unknown.

In the next section we will show the way the uncertain delay dependent system (8) can be expressed as a switched polytopic uncertain system.

3. EXPRESSING UNCERTAINTIES AS POLYTOPES OF MATRICES

When considering the equivalent discrete representation (8), the two matrices $\Gamma^1_\sigma(\Delta T(k))$ and $\Gamma^0_\sigma(\Delta T(k))$ are delay dependent uncertainties. If they can be expressed as convex polytopes, system (8) can be treated as a switched polytopic uncertain system, for which stability conditions can be expressed in terms of linear matrix inequalities.

Step 1: Taylor series expansion of the uncertainties $\Gamma^1_\sigma(\Delta T(k))$ and $\Gamma^0_\sigma(\Delta T(k))$

Proposition 1. $\Gamma^1_\sigma(\Delta T(k))$ and $\Gamma^0_\sigma(\Delta T(k))$, the elements of system (8) matrices can be expressed as:

$$\Gamma^1_\sigma(\Delta T(k)) = -\sum_{q=1}^{\infty}(-\Delta T(k))^q \frac{M_\sigma^{q-1}}{q!} e^{M_\sigma T} N_\sigma \text{ and} \tag{9}$$

$$\Gamma^0_\sigma(\Delta T(k)) = B_\sigma + \sum_{q=1}^{\infty}(-\Delta T(k))^q \frac{M_\sigma^{q-1}}{q!} e^{M_\sigma T} N_\sigma.$$

Proof. Consider

$$I(x) = \int_{T-x}^{T} e^{M_\sigma s} ds. \tag{10}$$

Using the Taylor series expansion one can write:

$$\begin{aligned} I(x) &= I(0) + \dot{I}(0)x + \ldots + \frac{d^q I}{dt^q}(0)\frac{x^q}{q!} + \ldots \\ &= -\sum_{q=1}^{\infty} \frac{(-x)^q}{q!} M_\sigma^{q-1} e^{M_\sigma T} \end{aligned} . \tag{11}$$

From (10), (11) and (5) Proposition 1 is proved for $x = \Delta T(k)$. □

Step 2: h-order approximation of Taylor series for uncertainties

Focusing on the first h terms of the previous formulation, $\Gamma^1_\sigma(\Delta T(k))$ can be expressed as a finite sum and a remainder

$$\Gamma^1_\sigma(\Delta T(k)) = -\sum_{q=1}^{h}(-\Delta T(k))^q \frac{M_\sigma^{q-1}}{q!} e^{M_\sigma T} N_\sigma + \Theta^h_\sigma \tag{12}$$

where the remainder is

$$\begin{aligned} \Theta^h_\sigma &= -\sum_{q=h+1}^{\infty} (-\Delta T(k))^q \frac{M_\sigma^{q-1}}{q!} e^{M_\sigma T} N_\sigma \\ &= -\left(\int_0^{-\Delta T} e^{M_\sigma \tau} d\tau - \sum_{q=1}^{h} (-\Delta T)^q \frac{M_\sigma^{q-1}}{q!} \right) e^{M_\sigma T} N_\sigma. \end{aligned} \tag{13}$$

$\Gamma^{1,h}_\sigma(\Delta T(k))$ and $\Gamma^{0,h}_\sigma(\Delta T(k))$, given by

$$\begin{aligned} \Gamma^{1,h}_\sigma(\Delta T(k)) &= -\sum_{q=1}^{h}(-\Delta T(k))^q \frac{M_\sigma^{q-1}}{q!} e^{M_\sigma T} N_\sigma, \\ \Gamma^{0,h}_\sigma(\Delta T(k)) &= B_\sigma - \Gamma^{1,h}_\sigma(\Delta T(k)). \end{aligned} \tag{14}$$

will be called the h-order approximation.

Step 3: Polytopic form of the h-order approximation

Consider the notations:

$$\begin{aligned} &G_{\sigma,q} = (-1)^{q+1} \frac{M_\sigma^{q-1}}{q!} e^{M_\sigma T} N_\sigma, \ q = 1..h, \text{ and} \\ &\phi_1 = \left[\underline{\rho}^{\,h}\mathbf{I} \ \underline{\rho}^{\,h-1}\mathbf{I} \ldots \underline{\rho}^{\,2}\mathbf{I} \ \underline{\rho}\mathbf{I}\right]', \\ &\phi_2 = \left[\underline{\rho}^{\,h}\mathbf{I} \ \underline{\rho}^{\,h-1}\mathbf{I} \ldots \underline{\rho}^{\,2}\mathbf{I} \ \overline{\rho}\mathbf{I}\right]', \ldots, \\ &\phi_{h+1} = \left[\overline{\rho}^{\,h}\mathbf{I} \ \overline{\rho}^{\,h-1}\mathbf{I} \ldots \overline{\rho}^{\,2}\mathbf{I} \ \overline{\rho}\mathbf{I}\right]' \end{aligned} \tag{15}$$

with $\underline{\rho} = \Delta T_{min}$ and $\overline{\rho} = \Delta T_{max}$.

Proposition 2. The h-order approximation $\Gamma^{1,h}_\sigma(\Delta T(k))$ and $\Gamma^{0,h}_\sigma(\Delta T(k))$ can be expressed as the convex matrix polytopes:

$$\begin{aligned} \Gamma^{1,h}_\sigma(\Delta T(k)) &= \sum_{i=1}^{h+1} \mu_i(k) U^{1,h}_{\sigma,i}, \\ \Gamma^{0,h}_\sigma(\Delta T(k)) &= \sum_{i=1}^{h+1} \mu_i(k) U^{0,h}_{\sigma,i}, \end{aligned} \tag{16}$$

$$\sum_{i=1}^{h+1} \mu_i(k) = 1, \ \mu_i(k) > 0 \ \forall i = 1,..,h{+}1, \ \forall k \in \mathbb{Z}^+,$$

where the polytope vertices are

$$U^{1,h}_{\sigma,i} = [G_{\sigma,1} \ldots G_{\sigma,h}] \, \phi_i$$

and

$$U^{0,h}_{\sigma,i} = B_\sigma - U^1_{\sigma,i}$$

respectively.

Proof. From equation (14) and the notations (15) $\Gamma^{1,h}_\sigma(\Delta T(k))$ can be expressed as

$$\Gamma^{1,h}_\sigma(\Delta T(k)) = \sum_{q=1}^{h} \rho^q(k) G_{\sigma,q} \tag{17}$$

with $\rho(k) = \Delta T(k)$. The equation (17) can be written as:

$$\Gamma^{1,h}_\sigma(\Delta T(k)) = [G_{\sigma,1} \ldots G_{\sigma,h}] \, \Psi(k) \tag{18}$$

where

$$\Psi(k) = \left[\rho(k)^h\mathbf{I} \ldots \rho^2(k)\mathbf{I} \ \ \rho(k)\mathbf{I}\right]'$$

In the space of parameters $\Psi(k)$ it can be shown that $\rho(k) \in [\underline{\rho}, \overline{\rho}]$, $\forall k \in \mathbb{Z}^+$

$$\Psi(k) = \sum_{i=1}^{h+1} \mu_i(k)\phi_i, \quad \text{with} \tag{19}$$

$$\mu_i(k) > 0 \text{ and } \sum_{i=1}^{h+1} \mu_i(k) = 1 \ \forall k \in \mathbf{Z}^+.$$

The uncertain parameters $\mu_i(k)$ are solutions of the linear system:

$$\begin{bmatrix} 1 & \ldots & \ldots & \ldots & 1 \\ \underline{\rho} & \overline{\rho} & \ldots & \ldots & \overline{\rho} \\ \underline{\rho}^{\,2} & \underline{\rho}^{\,2} & \overline{\rho}^{\,2} & \ldots & \overline{\rho}^{\,2} \\ \vdots & & & \ddots & \vdots \\ \underline{\rho}^{\,h} & \ldots & \ldots & \underline{\rho}^{\,h} & \overline{\rho}^{\,h} \end{bmatrix} \begin{bmatrix} \mu_1(k) \\ \mu_2(k) \\ \vdots \\ \mu_{h+1}(k) \end{bmatrix} = \begin{bmatrix} 1 \\ \rho(k) \\ \rho^2(k) \\ \vdots \\ \rho^h(k) \end{bmatrix}$$

Using the classical Gauss method, the solutions can be computed by the recursive formula:

$$\mu_1 = 1 - \frac{\rho - \underline{\rho}}{\overline{\rho} - \underline{\rho}}, \tag{20}$$

$$\mu_k = \frac{\rho^{k-1} - \underline{\rho}^{k-1}}{\overline{\rho}^{k-1} - \underline{\rho}^{k-1}} - \sum_{i=k+1}^{h+1} \mu_i, \ k = 2..h.$$

One can prove that

$$\mu_k = \frac{\rho^{k-1} - \underline{\rho}^{k-1}}{\overline{\rho}^{k-1} - \underline{\rho}^{k-1}} - \frac{\rho^{k} - \underline{\rho}^{k}}{\overline{\rho}^{k} - \underline{\rho}^{k}}, \ k = 2..h.$$

which is strictly positive since the function $f : \Re \to \Re$, $f(x) = \frac{\rho^x - \underline{\rho}^x}{\overline{\rho}^x - \underline{\rho}^x}$ is monotone decreasing for $x \in (0, \infty)$. We can remark that while $\rho \in [\underline{\rho}, \overline{\rho}]$, $0 < \mu_k < 1 \ \forall k = 1..h+1$ therefore all the solutions are barycentric coordinates. From (18,19), with the notations (15) and (14) *Proposition 2* is proved. □

Step 4: System polytopic truncated form

Let

$$\hat{A}^h_{\sigma i} = \begin{bmatrix} \Phi_\sigma & U^{1,h}_{\sigma,i} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \hat{B}^h_{\sigma i} = \begin{bmatrix} U^{0,h}_{\sigma,i} \\ \mathbf{I} \end{bmatrix}.$$

Considering the h-order approximation of the uncertainties in equation (14) and Proposition 2, the system (8) can be expressed in the truncated polytopic form:

$$z(k+1) = \hat{A}^h_\sigma z(k) + \hat{B}^h_\sigma u(k) \tag{21}$$

where

$$\hat{A}^h_\sigma(k) = \sum_{i=1}^{h+1} \mu_i(k) \hat{A}^h_{\sigma i}, \quad \hat{B}^h_\sigma(k) = \sum_{i=1}^{h+1} \mu_i(k) \hat{B}^h_{\sigma i}.$$

System (8) truncation is a switched uncertain system with polytopic uncertainty (21).

4. CONTROL SYNTHESIS

In the previous section it was shown that for any switched system with time varying feedback delay there exists a polytopic approximation. Considering the control synthesis in this context, one has to find a switched state feedback that stabilizes the h-order approximation system (21). It is clear that finding such a control law and proving that it is also valid for non truncated form of the system (8) solves the original problem. Robust stability results for switched uncertain systems with polytopic uncertainty are given in this section.

4.1 Switched control design for uncertain switched systems

Until now several control approaches have been presented separately for switched systems (Daafouz *et al.*, 2002) and for polytopic uncertain systems (J.Daafouz and Bernussou, 2001). Here we extend the existing approaches for the following switching uncertain system:

$$z(k+1) = \hat{A}_\sigma(k) z(k) + \hat{B}_\sigma(k) u(k), \tag{22}$$

where

$$\hat{A}_\sigma = \sum_{j=1}^{na_\sigma} \alpha_{\sigma j}(k) A_{\sigma j}, \text{ and } \hat{B}_\sigma = \sum_{l=1}^{nb_\sigma} \beta_{\sigma l}(k) B_{\sigma l},$$

$$\sum_{j=1}^{na_\sigma} \alpha_{\sigma j}(k) = 1, \ \alpha_{\sigma j}(k) \geq 0,$$

$$\sum_{l=1}^{nb_\sigma} \beta_{\sigma l}(k) = 1, \ \beta_{\sigma l}(k) \geq 0, \ \forall\, k \in \mathbb{Z}^+$$

represent the uncertainty on the dynamic and input matrix, respectively. The switching signal is given by σ. Here $\alpha_{\sigma j}$ and $\beta_{\sigma l}$ are the uncertain parameters describing each uncertainty while na_σ and nb_σ represent the number of extreme points in the uncertainty $\hat{A}_\sigma$ and $\hat{B}_\sigma$ respectively. This is a more general form of the switched polyopic system (21); the input and the state matrix are affected by distinct polytopic uncertainty with different uncertain parameters. This system can also be expressed as:

$$z(k+1) = \sum_{i=1}^{N} \xi_i(k) \hat{A}_i(k) z(k) + \sum_{i=1}^{N} \xi_i(k) \hat{B}_i(k) u(k), \tag{23}$$

$$\xi_i : \mathbb{Z}^+ \to \{0, 1\}, \ \sum_{i=1}^{N} \xi_i(k) = 1,$$

The closed-loop dynamics with the switched state feedback

$$u(k) = \sum_{i=1}^{N} \xi_i(k) K_i z(k) \tag{24}$$

is described by the equation:

$$z(k+1) = \sum_{i=1}^{N} \xi_i(k) (\hat{A}_i + \hat{B}_i K_i) z(k).$$

With the uncertainty description (22), the equation becomes

$$z(k+1) = \sum_{i=1}^{N} \sum_{j=1}^{na_i} \sum_{l=1}^{nb_i} \xi_i(k) \alpha_{ij}(k) \beta_{il}(k) H_{ijl} z(k)$$

where

$$H_{ijl} = A_{ij} + B_{il} K_i.$$

We use switched parameter dependent Lyapunov functions given by:

$$V(k) = z^T(k) \mathcal{P} z(k) \tag{25}$$

with

$$\mathcal{P} = \sum_{i=1}^{N} \sum_{j=1}^{na_i} \sum_{l=1}^{nb_i} \xi_i(k) \alpha_{ij}(k) \beta_{il}(k) P_{ijl}.$$

The difference along the system trajectories is:

$$V(k+1) - V(k) = z^T(k)(\mathcal{H}^T \mathcal{P}_+ \mathcal{H} - \mathcal{P})z(k),$$

where

$$\mathcal{H} = \sum_{i=1}^{N} \sum_{j=1}^{na_i} \sum_{l=1}^{nb_i} \xi_i(k)\alpha_{ij}(k)\beta_{il}(k)H_{ijl},$$

$$\mathcal{P}_+ = \sum_{i=1}^{N} \sum_{j=1}^{na_i} \sum_{l=1}^{nb_i} \xi_i(k+1)\alpha_{ij}(k+1)\beta_{il}(k+1), \tag{26}$$

$$P_{ijl} = \sum_{m=1}^{N} \sum_{u=1}^{na_m} \sum_{v=1}^{nb_m} \xi_m(k)\alpha_{mu}(k)\beta_{mv}(k)P_{muv}.$$

Theorem 1. System (23) is stabilizable via the control law (24) if there exists symmetric positive definite matrices S_{ijl} and S_{muv}, and matrices G_i and R_i, solutions of the LMI:

$$\begin{bmatrix} G_i + G_i^T - S_{ijl} & G_i^T A_{ij}^T + R_i^T B_{il}^T \\ A_{ij}G_i + B_{il}R_i & S_{muv} \end{bmatrix} > 0, \tag{27}$$

$i, m = 1..N, j = 1..na_i, l = 1..nb_i, u = 1..na_m, v = 1..nb_m$. The switched state feedback control is given by (24) with $K_i = R_i G_i^{-1}$.

Proof. see (Hetel *et al.*, 2005). □

Remark: Theorem 1 gives robust LMI stabilizability conditions based on switched parameter dependent Lyapunov functions and hence less conservative than all the other results in the literature (Ji *et al.*, 2003; Zhai *et al.*, 2003). Moreover, these results obviously apply to the state reconstruction problem for uncertain switched systems (Daafouz *et al.*, 2003).

4.2 Control synthesis for switched systems with uncertain time varying feedback delay

The truncation (21) of the system with uncertain feedback delay (8) can be considered as a switched uncertain system with polytopic uncertainty (22) for which we developed LMI stabilizability conditions (27). The problem of finding a computer control $u(k) = K_\sigma^h z(k)$ for the discrete h-order truncation (21) of the discrete time switched system (8) is the same as the control synthesis problem for a switched uncertain system with polytopic uncertainty (22).

Proposition 3. If there exists positive definite symmetric matrices S_{ij}, S_{uv} and matrices G_i and R_i solutions of the LMI conditions:

$$\begin{bmatrix} G_i + G_i^T - S_{ij} & G_i^T \hat{A}_{ij}^{h^T} + R_i^T \hat{B}_{ij}^{h^T} \\ \hat{A}_{ij}^h G_i + \hat{B}_{ij}^h R_i & S_{uv} \end{bmatrix} > 0 \tag{28}$$

where $i, u = 1 \ldots N$, $j, v = 1 \ldots h+1$ then the switched state feedback stabilizing gains are given by $K_i^h = R_i G_i^{-1}$.

Proof. see (Hetel *et al.*, 2005) □

A procedure based on considering the neglected remainder Θ_σ^h (see Step 2 and Step 5 in Section 3) as a disturbance and using LMI based conditions for stability analysis can be used to check that the control law obtained using Proposition 3 is valid for the non truncated form of the system (8). Consider the notations:

$$H_{\sigma j}^h = A_{\sigma j}^h + B_{\sigma j}^h K_\sigma^h,\ \hat{H}_\sigma^h = \sum_{j=1}^{h+1} \mu_j H_{\sigma j}^h, \tag{29}$$

$$\mu_i(k) > 0,\ \sum_{i=1}^{h+1} \mu_i(k) = 1\ \forall k \in \mathbf{Z}^+, \tag{30}$$

$$K_\sigma^h = \left[K_\sigma^{1,h}\ K_\sigma^{2,h}\right]$$

with

$$K_\sigma^{1,h} \in \mathbf{R}^{m\times n}, \qquad K_\sigma^{2,h} \in \mathbf{R}^{m\times m}$$

$$D = [\mathbf{I}\ \mathbf{0}]^T,\ E_\sigma^h = \left[\mathbf{I}\ \mathbf{I}\right] \begin{bmatrix} -K_\sigma^{1,h} & \mathbf{I} \\ \mathbf{0} & -K_\sigma^{2,h} \end{bmatrix}.$$

Theorem 2. If there exists symmetrical positive definite matrices P_{ij}, and matrices G_{ij} with $i = 1, \ldots, N$ and $j = 1, \ldots, h+1$ solutions of the *LMI*:

$$\begin{bmatrix} -P_{ij} & H_{ij}^{h^T} G_{pq}^T & H_{ij}^{h^T} G_{pq} D & E_i^{h^T} \\ (\bullet)^T & P_{pq} - G_{pq} - G_{pq}^T & \mathbf{0} & \mathbf{0} \\ (\bullet)^T & (\bullet)^T & D^T(G_{pq} + G_{pq}^T)D - \mathbf{I} & \mathbf{0} \\ (\bullet)^T & (\bullet)^T & (\bullet)^T & -\gamma\mathbf{I} \end{bmatrix} < 0 \tag{31}$$

$\forall\ i, p = 1, \ldots, N\ \ j, q = 1, \ldots, h+1$ with $\left\|\Theta_\sigma^h\right\| \leq \gamma^{-\frac{1}{2}}$ for all σ, then the feedback gains K_σ^h ensure the stabilizability of the original discrete time system with time varying delays (8), that is system (4) is stable.

Proof. see (Hetel *et al.*, 2005)□

In practical applications, finding a feedback control for the original discrete time system (8) is a compromise between two constraints: high order uncertainty approximation, for a good discrete system representation, and small number of LMIs. These two constraints are opposite because a high order uncertainty approximation implies a large number of LMIs and the numerical LMI solvers are not able to compute a feedback gain when the number of LMIs is too large.

5. NUMERICAL EXAMPLE

Consider the following system:

$$\frac{dx}{dt} = M_\sigma x(t) + N_\sigma u(t),\ \sigma \in \{1, 2\}\ \text{with}$$

$$M_1 = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}, \qquad M_2 = \begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix},$$

$$N_1 = [1\ 0]' \quad \text{and} \quad N_2 = [1.5\ 0]'$$

The system is sampled with $T = 0.3s$ and the feedback delay is bounded by $\Delta T_{min} = 0s$ and $\Delta T_{max} = 0.1s$. When considering a first order approximation of the system, state feedback K^1_σ gains are found via the LMIs (28) but are not validated by (31). A second order approximation is performed. In this case we obtain the switched feedback gains:

$$K_1^2 = [-2.99\ \ -1.3\ 0.27]$$

and

$$K_2^2 = [-2\ \ -0.38\ 0.21]$$

which can be validated by (31) for the original system.

6. CONCLUSION

This paper was dedicated to the robust control synthesis for continuous time switched systems relative to time varying feedback delays. The sampled version of the system has been considered and the control synthesis in this context has been treated as a problem of stabilizability for uncertain systems with polytopic uncertainties. A method for switched state feedback control synthesis has been presented. A numerical example has been presented to illustrate this approach.

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