ROBUST MODEL PREDICTIVE CONTROL FOR PIECEWISE AFFINE SYSTEMS SUBJECT TO BOUNDED DISTURBANCES

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Abstract: This paper investigates the robust tracking and regulation control problems for discrete-time, piecewise affine systems subject to bounded disturbances. In particular, the main question addressed is related to the existence of a controller such that the closed-loop system exhibits an attainable desired behavior under all possible disturbances. Checking attainability and calculating the state space regions for which a robust control is assured despite the disturbance is performed using a polyhedral approach. A model predictive control law derived from a quadratic cost function minimization is further examined as a fast sub-optimal robust control. An application of the proposed technique to a two-tank benchmark is finally presented.

Keywords: Piecewise affine systems, hybrid systems, bounded disturbances, reachability, model predictive control.

1. INTRODUCTION

Hybrid systems are now of common use in many control applications in industry, e.g. in control of mechanical systems, process control, automotive industry, power systems, aircraft and traffic control. Hybrid systems are heterogeneous dynamical systems, their behavior is determined by interacting continuous variable and discrete event dynamics. Various approaches have been proposed to model hybrid systems (Branicky *et al*., 1998), such as Automata, Petri nets, Linear Complementary (LC), Piecewise Affine (PWA) (Sontag, 1981), Mixed Logical Dynamical (MLD) models (Bemporad and Morari, 1999a). Different techniques are used to control hybrid systems, for example Model Predictive Control (MPC) (Schutter and van den Boom, 2004; Thomas *et al*., 2003; Olaru *et al*., 2003; Olaru *et al*., 2004) and optimal control (Bemporad, and Morari, 1999a).

An attractive and challenging field of research is currently dealing with hybrid systems subject to uncertainties (Bemporad and Morari, 1999b), either parameters uncertainties or disturbances influences, where problems like safety, reachability, attainability and robust control become interesting questions for researchers.

In this direction, this paper examines a class of discrete-time piecewise affine systems subject to bounded disturbances. For this class of systems, some solutions to the above mentioned problems are already proposed in the literature. For example, in (Lin and Antsaklis, 2003), an attainability checking that employs the predecessor operator, and a controller technique using finite automata and linear programming is presented. In (Necoara *et al*., 2004, Bemporad *et al*., 2003), a control technique based on minimizing the worst-case cost function (min-max problem) is proposed to solve the control problem.

The contribution of this paper is based on a polyhedral approach enabling the elaboration of the state space regions for which a robust control exists which drives the plant to a desired behavior despite the disturbances. The safety, reachability and attainability questions are examined through this framework and a robust Model Predictive Control (MPC) with quadratic cost function is presented as a fast suboptimal robust control.

The paper is organized as follows. A brief description of PWA systems and the related class is given in Section 2. Section 3 develops the polyhedral approach which will elaborate the state space regions where reachability, safety and attainability questions can be assured. A fast and suboptimal robust control is then developed in Section 4 for the considered class. An application of the proposed technique to a two-tank benchmark is presented in Section 5. Finally the conclusions and some remarks are given in Section 6.

2. PIECEWISE AFFINE SYSTEMS SUBJECT TO BOUNDED DISTURBANCES

Piecewise affine systems are powerful tools for describing or approximating both nonlinear and hybrid systems, and represent a straightforward extension from linear to hybrid systems. This paper focuses on the particular class of discrete-time piecewise affine systems subject to bounded disturbances, defined as:

$$S^i : \left\{ \mathbf{x}_{k+1} = \mathbf{A}^i \mathbf{x}_k + \mathbf{B}^i \mathbf{u}_k + \mathbf{C}^i \mathbf{d}_k^i + \mathbf{f}^i \right\} \quad \text{for } \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix} \in \chi_i \tag{1}$$

$\mathbf{x}_k \in \mathbf{X}, \mathbf{u}_k \in \mathbf{U}, \mathbf{d}_k^i \in \mathbf{D}$ denote the state, input and disturbance vector respectively at instant k (for the i th model) with $\mathbf{X}, \mathbf{U}, \mathbf{D}$ assigned polytopes ($\mathbf{D}$ contains the origin).

$\{\chi_i\}_{i=1}^s$ is the polyhedral coverage of the state and input spaces $\mathbf{X} \times \mathbf{U}$, s being the number of subsystems. Each χ_i is given by:

$$\chi_i = \left\{ \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix} \middle| \mathbf{Q}^i \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix} \le \mathbf{q}^i \right\} \tag{2}$$

Exact state measurement $\mathbf{x}$ is supposed to be available. Note that the sets χ_i are assumed here to be not disjoint so that the desired model dynamics can be chosen by the bias of switching (logical) decision variables.

Each subsystem S^i defined by the 6-uple $\left(\mathbf{A}^i, \mathbf{B}^i, \mathbf{C}^i, \mathbf{f}^i, \mathbf{Q}^i, \mathbf{q}^i\right)$, $i \in I = (1,2,\cdots,s)$ is a component of the global hybrid system where I is the collection of all subsystems. $\mathbf{A}^i \in \Re^{n\times n}, \mathbf{B}^i \in \Re^{n\times m}, \mathbf{C}^i \in \Re^{n\times r}, \mathbf{Q}^i \in \Re^{p_i(n+m)}$ and $\mathbf{q}^i \in \Re^{p_i}$ is a suitable constant vector, where n, m, r are respectively the dimension of state, input and disturbance vectors, and p_i is the number of hyperplanes defining the χ_i polyhedral.

In this formalism, a logical control input is taken into account by developing an affine model (1) for each input value (1/0), defining linear inequality constraints linking the model with the relevant input value (2).

3. DIRECT REACHABILITY, SAFETY AND ATTAINABILITY: A POLYHEDRAL APPROACH

Let consider the region $\mathbf{R}_k, k > 1$ as a target region in the global state space $\mathbf{X}$. This section considers the robust one-step control region $\mathbf{R}_{k-1}$ as the region in the state space for which there exist a feasible mode (1) and an admissible control signal able to drive the states from $\mathbf{R}_{k-1}$ into $\mathbf{R}_k$ in one-step despite all allowable disturbances, i.e.:

$$\mathbf{R}_{k-1} = \left\{ \begin{array}{l} \mathbf{x}_{k-1} \in \mathbf{X} \left| \exists i \wedge \mathbf{u}_{k-1} \in \mathbf{U}, \begin{bmatrix} \mathbf{x}_{k-1} \\ \mathbf{u}_{k-1} \end{bmatrix} \in \chi_i \right. \\ s.t. \\ \mathbf{A}^i \mathbf{x}_{k-1} + \mathbf{B}^i \mathbf{u}_{k-1} + \mathbf{C}^i \mathbf{d}_{k-1}^i + \mathbf{f}^i \in \mathbf{R}_k, \\ \qquad \forall \mathbf{d}_{k-1}^i \in \mathbf{D} \end{array} \right\} \tag{3}$$

In the following, the computation of this region $\mathbf{R}_{k-1}$ is achieved through a polyhedral approach.
Consider the global state space defined by the following constraints:

$$\mathbf{X} := \left\{ \mathbf{F}_s \mathbf{x} \le \mathbf{g}_s,\ \mathbf{F}_s \in \Re^{p\times n},\ \mathbf{g}_s \in \Re^p \right\} \tag{4}$$

The control input is supposed to be bounded:

$$\mathbf{U} := \left\{ \mathbf{m}\,\mathbf{u} \le \mathbf{n},\ \mathbf{m} \in \Re^{p_u \times m}, \mathbf{n} \in \Re^{p_u \times 1} \right\} \tag{5}$$

With disturbance given inside an assigned polytope $\mathbf{d}_{k-1}^i \in \mathbf{D}$, with the target region $\mathbf{R}_k$, defined by:

$$\mathbf{R}_k := \left\{ \mathbf{F}\mathbf{x}_k \le \mathbf{g} \right\} \tag{6}$$

and considering each valid model i where $i \in (1,2,\cdots,s)$ and using the system evaluation equation (1), enables to derive the following region:

$$\mathbf{R}_{k-1}^i = \left\{ \mathbf{x}_{k-1} \in \Re^n \middle| \forall \mathbf{d}_{k-1}^i \in \mathbf{D}, \right. \\ \mathbf{F}(\mathbf{A}^i \mathbf{x}_{k-1} + \mathbf{B}^i \mathbf{u}_{k-1} + \mathbf{C}^i \mathbf{d}_{k-1}^i + \mathbf{f}^i) \le \mathbf{g}, \\ \left. \mathbf{Q}^i \left[\mathbf{x}_{k-1}\ \ \mathbf{u}_{k-1} \right]^T \le \mathbf{q}^i, \right\} \cap \left\{ \mathbf{F}_s \mathbf{x}_k \le \mathbf{g}_s \right\} \tag{7}$$

The presence of disturbances can be in a first step ignored, leading to the computation of the set:

$$\widetilde{\mathbf{R}}_{k-1}^i = \left\{ \begin{bmatrix} \mathbf{F}\mathbf{A}^i & \mathbf{F}\mathbf{B}^i \\ \mathbf{Q}^i & \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k-1} \\ \mathbf{u}_{k-1} \end{bmatrix} \le \begin{bmatrix} \mathbf{g} - \mathbf{F}\mathbf{f}^i \\ \mathbf{q}^i \end{bmatrix} \right\} \\ \cap \left\{ \mathbf{F}_s \mathbf{x}_{k-1} \le \mathbf{g}_s \right\} \tag{8}$$

and to the expression of the maximal admissible region for the mode i in the absence of disturbances:

$$\hat{\mathbf{R}}_{k-1}^{i} = \mathrm{Pr}_{\mathbf{X}}\, \widetilde{\mathbf{R}}_{k-1}^{i} \qquad (9)$$

Remark 1: the projection of polyhedral sets can be efficiently handled in a double representation (generators/ constraints) and related tools can be found as for example - POLYLIB (Wilde, 1994).

Due to the fact that the goal is the construction of a control strategy robust with respect to the entire family of possible disturbances realizations, the previous equation finally becomes:

$$\mathbf{R}_{k-1}^{i} = \hat{\mathbf{R}}_{k-1}^{i} - \mathbf{C}^{i}\mathbf{D} \qquad (10)$$

where the subtraction is computed in the Minkowsky sense (exact geometric operation, based on the double representation of polyhedral domains). The set $\mathbf{C}^{i}\mathbf{D}$ is the image of $\mathbf{D}$ by the linear mapping:

$$f : \mathbf{D} \to \Re^{n}, f(\mathbf{d}) = \mathbf{C}^{i}\mathbf{d}$$

With these sets constructed for each linear sub-model, the global one-step robust controllable region of the state space is thus given by:

$$\mathbf{R}_{k-1} = \bigcup_{i=1}^{s} \mathbf{R}_{k-1}^{i} \qquad (11)$$

The procedure presented above can be repeated in a recursive way to find the domain for any limited N steps horizon. Using a dynamic programming approach, after defining the target region $\mathbf{R}_{k+N}$, the state space domain $\mathbf{R}_{k}$ can be recursively calculated, that includes all the states having a feasible control policy that can in N steps derive the states to $\mathbf{R}_{k+N}$ despite the disturbances.

Remark 2: For PWA systems with many sub-models s and for long horizon N, this may imply the exploration of a large number of regions (exponential complexity, Figure 1a). Considering thus “no switch” between sub-models over the N steps horizon (Figure 1b) leads to a lower complexity mechanism. Even if this may imply more conservatism, this suboptimal construction appears to be broad enough for many applications, this will be applied in the following sections.

Safety, a well-known geometric condition for a set to be safe (control invariant) is the following (Lin and Antsaklis, 2002):

the set $\mathbf{R}_{k+1}$ *is safe if and only if* $\mathbf{R}_{k+1} \subseteq \mathbf{R}_{k}$

Attainability, given a finite number of regions $(\mathbf{R}_{k}, \mathbf{R}_{k+1}, \cdots, \mathbf{R}_{k+N}) \in I \times \boldsymbol{\chi}$, the attainability for this sequence of regions is equivalent to the following two different properties:

1. *the direct reachability from region* $\mathbf{R}_{k+j}$ *to* $\mathbf{R}_{k+j+1}$ *for* $0 \le j \le N-1$,
2. *the safety (or control invariance) for region* $\mathbf{R}_{k+N}$.

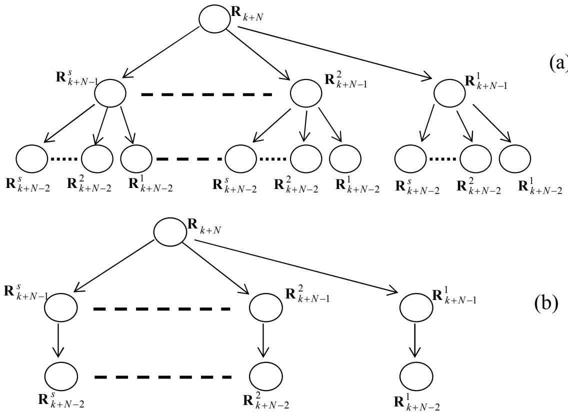

Fig. 1. Regions exploration, (a) complete exploration, (b) exploration with no switch over the N steps.

4. ROBUST MODEL PREDICTIVE CONTROL

The min-max control technique is proposed in the literature as a robust control for such problems, which minimizes the maximum cost, to try to counteract the worst disturbance. This paper focuses on the model predictive control for PWA systems with quadratic cost function as a fast suboptimal robust solution.

The model predictive control proposed here requires solving at each sampling time the following problem:

$$\begin{aligned} &\min_{\mathbf{u}_{k}^{k+N-1}} J(\mathbf{u}_{k}^{k+N-1}, \mathbf{x}_{k}) = \\ &\quad \sum_{j=1}^{N} \left\| \mathbf{x}_{k+j} - \mathbf{x}_{e} \right\|_{\Lambda}^{2} + \sum_{j=0}^{N-1} \left\| \mathbf{u}_{k+j} \right\|_{\Gamma}^{2} \\ &\quad \text{s.t.:} \quad \mathbf{Q}^{i} \begin{bmatrix} \mathbf{x}_{k+j} \\ \mathbf{u}_{k+j} \end{bmatrix} \le \mathbf{q}^{i}, \\ &\quad \mathbf{x}_{k+j} \in \mathbf{R}_{k+j}, \quad \text{for } j = 0,1,\cdots,N \end{aligned} \qquad (12)$$

where $\mathbf{x_e}$ is the states reference, Λ, Γ are the weighting diagonal matrices in the sense $\|x\|_{\Lambda}^{2} = x^{T} \Lambda x$.

(12) is solved according to the following steps:

1. solve this quadratic problem for each dynamic among the s sub-models, staying on the dedicated branch of Figure 1.b (i.e. as assumed with Remark 2, the open-loop control sequence of the predictive law is elaborated without switching),
2. compare all the resulting costs,
3. retain the model with the lowest cost and the associated control sequence,
4. apply only the first value of this sequence and restart the procedure at the next sampling time.

Remark 3: At each sampling time, the decision process can drive the system to any particular feasible mode due to receding horizon implementation of the optimal open-loop sequence. To sum up, the conservatism is only related to the feasible set coverage and not directly to the chosen performance index.

Remark 4: If the initial state $\mathbf{x}_k$ is included in the union of regions $\mathbf{R}_{k-N}^{i}$ of different modes (i), the MPC technique can select a suboptimal solution among all feasible modes. The feasibility at instant k implies feasibility at any instant $k+1$ to $k+N$. The longest the prediction, the largest the feasible domain will be.

5. APPLICATION

Let consider as application of the previous theory the following benchmark consisting of two tanks (Figure 2), filled by pump acting on tank 1, continuously manipulated from 0 up to a maximum flow Q_1.

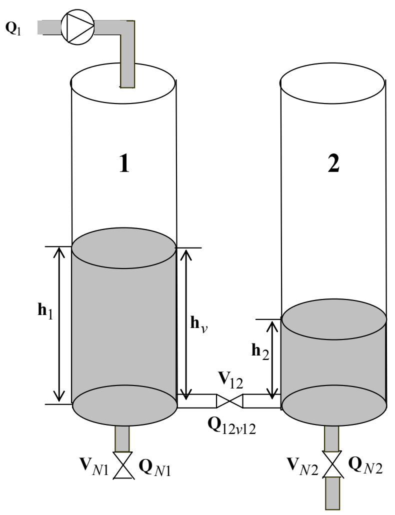

Fig. 2. Two-tank benchmark.

One switching valve V_{12} controls the flow between the tanks, this valve is assumed to be either completely opened or closed ($V_{12}=1$ or 0 respectively). The V_{N2} manual valve controls the nominal outflow of the second tank. It is assumed in further simulations that the manual valves, V_{N1} is always closed and V_{N2} is open. The liquid levels to be controlled are denoted h_1 and h_2 for each tank respectively.

The conservation of mass in the tanks provides the following differential equations:

$$\begin{aligned} \dot{h}_1 &= \frac{1}{A}(Q_1 - Q_{12V12}) \\ \dot{h}_2 &= \frac{1}{A}(Q_{12V12} - Q_{N2}) \end{aligned} \qquad (13)$$

where the Qs denote the flows and A is the cross-sectional area of each of the tanks. The Toricelli law defines the flows in the valves by following expressions:

$$\begin{aligned} Q_{12V12} &= V_{12} a S_{12}\, sign(h_1 - h_2)\sqrt{\left|2g(h_1 - h_2)\right|} \\ Q_{N2} &= V_{N2} a S_{N2} \sqrt{2gh_2} \end{aligned} \qquad (14)$$

where S_i represents the area of valves V_i and a is a constant depending on the liquid. From this, a simplified linear model can be obtained under the form:

$$\begin{aligned} Q_{12V12} &\approx k_{12} V_{12} (h_1 - h_2) \\ Q_{N2} &\approx k_{N2} V_{N2} h_2 \end{aligned} \qquad (15)$$

where: $k_{12} = a S_{12}\sqrt{\dfrac{2g}{h_{\max}}},\ k_{N2} = a S_{N2}\sqrt{\dfrac{2g}{h_{\max}}}$

The Euler discretisation technique is used to further derive the discrete form :

$$\begin{aligned} & h_1(k+1) = h_1(k) + \\ & \quad \frac{T_s}{A}(Q_1(k) - k_{12}V_{12}(h_1(k) - h_2(k))) \\ & h_2(k+1) = h_2(k) + \\ & \quad \frac{T_s}{A}(k_{12}V_{12}(h_1(k) - h_2(k)) - k_{N2}V_{N2}h_2(k)) \end{aligned} \qquad (16)$$

where T_s is the sampling time, equal to 10 s.

This benchmark can be considered as a piecewise system of form (1), with two subsystems (two modes) described as follows:

For mode one, the valve V_{12} is open:

$$\mathbf{A}^1 = \begin{bmatrix} 0.9269 & 0.07305 \\ 0.07305 & 0.8539 \end{bmatrix},\quad \mathbf{B}^1 = \begin{bmatrix} 649.351 \\ 0 \end{bmatrix}$$

$$\mathbf{Q}^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix},\quad \mathbf{q}^1 = \begin{bmatrix} 0.62 \\ 0.62 \\ 0 \\ 0 \\ 0.0001 \\ 0 \end{bmatrix}$$

For mode two, the valve V_{12} is closed:

$$\mathbf{A}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.9269 \end{bmatrix},\quad \mathbf{B}^2 = \begin{bmatrix} 649.351 \\ 0 \end{bmatrix}$$

$$\mathbf{Q}^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix},\quad \mathbf{q}^2 = \begin{bmatrix} 0.62 \\ 0.62 \\ 0 \\ 0 \\ 0.0001 \\ 0 \end{bmatrix}$$

The previous constraints have integrated limitations on the global state space:

$$\mathbf{X} := \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}}_{\mathbf{F}_s} \mathbf{x} \leq \underbrace{\begin{bmatrix} 0.62 \\ 0.62 \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{g}_s} \tag{17}$$

and limitations on the control signal:

$$\mathbf{U} := \underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{\mathbf{m}} Q_1 \leq \underbrace{\begin{bmatrix} 0.0001 \\ 0 \end{bmatrix}}_{\mathbf{n}} \tag{18}$$

The target region, to which system states will be derived to, is defined by the following constraints:

$$\mathbf{R}_{k+N} := \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}}_{\mathbf{F}} \mathbf{x} \leq \underbrace{\begin{bmatrix} 0.55 \\ 0.25 \\ -0.45 \\ -0.15 \end{bmatrix}}_{\mathbf{g}} \tag{19}$$

A polytope for bounded disturbance is finally considered with:

$$\begin{bmatrix} -0.01 \\ -0.01 \end{bmatrix} \leq \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \leq \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix} \tag{20}$$

The approach presented above is first applied to elaborate the region $\mathbf{R}_k$ in the state space which includes the states that can be derived in finite N steps to $\mathbf{R}_{k+N}$ despite the disturbance. However, a suboptimal approach is used here (see Remark 2) as a compromise with the computational load.

With this assumption, Figure 3 presents the regions for mode one for $N = 5$, and Figure 4 for mode two with $N = 5$ as well. For both modes, the regions are presented in Figure 5 with $N = 3$.

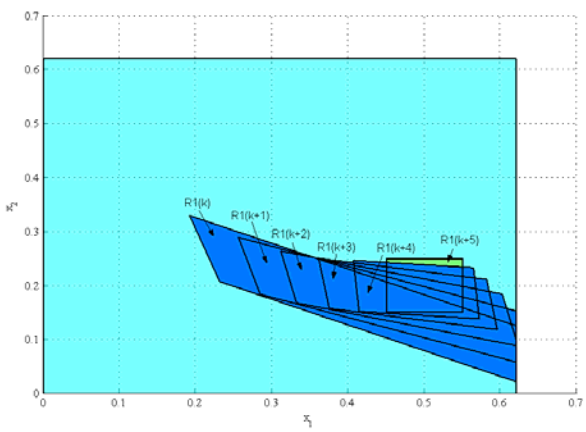

Fig. 3. Regions for mode one with $N = 5$.

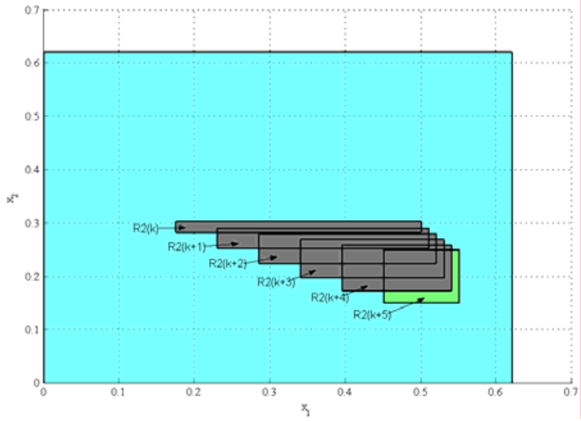

Fig. 4. Regions for mode two with $N = 5$.

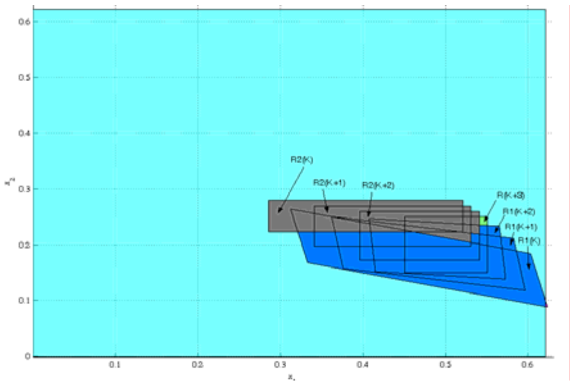

Fig. 5. Regions for both modes with $N = 3$.

The robust model predictive control presented above, is applied considering many different initial states inside the region $\mathbf{R}_k$, and at each time step, a random disturbance is added to the system states. The weighting diagonal terms in the cost function are chosen such that $\Lambda = 1000 * I_2$ and $\Gamma = 1$, and the states reference is $(0.5, 0.2)$.

Figure 6 shows some results of robust MPC with $N = 2$ for extreme initial states inside $\mathbf{R}_k$ with random disturbance, and as Figure 6 shows, all the states in $\mathbf{R}_k$ are derived in two steps ($N = 2$) to the desired region $\mathbf{R}_{k+2}$ despite the disturbance.

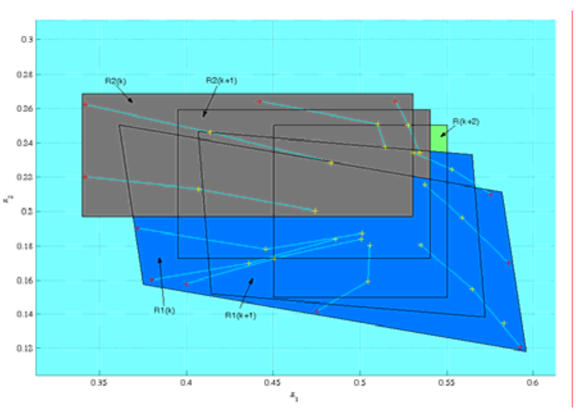

Fig. 6. Robust MPC for different initial states, with $N = 2$.

When talking about complexity, one has to mention that the convex regions computed here, $\mathbf{R}_{k-1}^{i}$ as in (11), are obtained in a dual representation (extreme points/constraints), which does not represent a computational challenge (the MPT toolbox was used here – (Kvasnica *et al*., 2004)) as long as the number of vertices does not increase (there are polytopic regions with either 4 or 5 vertices). This fact is strongly related to the particular shape of the target region. In this case, neither the projections nor the difference of polyhedral regions should require an important computational effort.

Finally, in Figure 6 one can remark several state trajectories generated based on random disturbance realizations validating any physical extreme combination of states.

6. CONCLUSION

This paper has examined a class of discrete-time piecewise affine systems with bounded disturbance, for which a polyhedral technique has been proposed to find the regions in the state space where a feasible mode and a robust control is assured to derive the system states to the desired region despite the disturbance. Model predictive control technique has been proposed as a fast and suboptimal robust control for the considered problem.

Future work will consider applying the same techniques on discrete-time piecewise affine systems with parameter uncertainties and exogenous disturbances.

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