EXPLICIT MODEL PREDICTIVE CONTROL OF THE BOOST DC-DC CONVERTER

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Abstract: This paper extends previous work done on the optimal control of the dc-dc converter boost circuit topology. The same control problem formulation is maintained but a simpler piecewise affine approximation of the system dynamics is derived and employed to obtain an explicit solution whose complexity renders it viable for implementation on a realistic circuit setup.

Keywords: predictive control, power circuits, piecewise linear controllers.

1. INTRODUCTION

Fixed frequency switch-mode dc-dc converters are a class of electronic power circuits extensively used in regulated dc power supplies and dc motor drive applications due to advantageous features in terms of size, weight and reliable performance. Their role in these applications is to transfer power from a dc input to a load, achieving output voltage regulation in the presence of voltage source and output load variations. The principal control challenge derives from their hybrid nature as their switched circuit topology implies different modes of operation, each with its own related affine continuous-time dynamics. The input variable (duty cycle) also features hard constraints, and safety measures may impose additional constraints such as current limiting.

This paper analyzes the modelling and controller synthesis of the fixed-frequency boost dc-dc converter, in which the semiconductor switch is operated by a pulse sequence with constant switching frequency f_s (resp. period T_s). It is then possible to regulate the dc component of the output voltage through the duty cycle $d = \frac{t_1}{T_s}$, where t_1 denotes the interval within the switching period during which the switch is in the first mode of operation.

By employing this operation principle, the main control objective is to act on the semiconductor switch with a duty cycle such that the dc component of the output voltage reaches the given reference. As the name suggests, for the boost converter this reference value is higher than that of the voltage source, and must be maintained despite variations in the load or the voltage source. Control techniques that are used in practice typically have in common the employment of PI-type controllers tuned on the basis of linearized averaged models (Erickson *et al.*, 1982), (Middlebrook and Cuk, 1976) and that commonly use two control loops for the inductor current and output voltage. Improved controller design strategies involving nonlinear and feed-forward control methods have been formulated (Hiti and Borojevic, 1995), and (Kazimierczuk and Massarini, 1997), but these employ models that do not capture the hybrid dynamics of dc-dc converters. Additionally, none of them allow to directly incorporate constraints in the controller synthesis.

The recent past has witnessed an increased interest in the direction of other alternative control methodologies; passivity-based control design for switched-mode power converters has garnered significant attention (Sira-Ramrez *et al.*, 1997) and has also been employed for the boost circuit in (Jeltsema and Scherpen, 2004), in a way such that the physical structure (energy dissipation and interconnection) is explicitly featured in the model and control scheme, but neglecting variations on the voltage source or on the output load. A hybrid approach is described in (Senesky *et al.*, 2003) and a hybrid automaton synthesized to opportunely switch a boost converter among operating modes, in a way such that however a constant switching frequency, which is crucial from the application point of view, cannot be guaranteed.

Motivated by these issues, this paper extends work recently done on the numerical performance of optimal controller schemes for boost dc-dc converters in (Beccuti *et al.*, 2005); herein the same control problem formulation is maintained but a simpler model is derived in order to allow for the explicit solution of the control problem, so that determining the control law on-line reduces to searching in a look-up table of viable complexity and evaluating an affine function of the state. The approach featured in (Beccuti *et al.*, 2005) is in turn based on the work done in (Geyer *et al.*, 2004*b*) for the dc-dc buck converter topology.

This paper is organized as follows. Section 2 presents the physical setup of the circuit and in Section 3 a model for the boost converter is derived by employing a least square fitting (LSF) approximation of the exact converter dynamics over several regions of the state space to derive a piecewise affine (PWA) system. In Section 4, an optimal control problem incorporating the appropriate control objectives is formulated, and an outline of the employed load estimation setup is given. Section 5 contains simulation results illustrating the performance of the proposed model predictive control (MPC) scheme. Finally, conclusions and further research directions are outlined in Section 6. In the sequel, a normalized time scale will be used, with the time unit being equal to the switching period T_s, and the discrete time instant $t = k$ referring to $t = kT_s$.

2. PHYSICAL MODEL OF THE BOOST CONVERTER

The circuit topology of the boost converter is shown in Fig.1; only the continuous conduction mode will be considered, that is operating points for which the inductor current remains positive.

Using normalized quantities, r_o denotes the output load resistance, r_c the equivalent series resistance (ESR) of the capacitor x_c and r_ℓ is the internal resistance of the inductor x_ℓ. The boost converter features two operation modes with two different affine dynamics. The controller selects the control input, the duty cycle $d(k)$, for each period k, determining when the switch from the first mode to the second takes place. During the time interval $k \leq t < k + d(k)$ the switch S is in the s_1 position and the inductor is charged. At the end of this interval S is switched to s_2 and power is transferred to the load. The switch is set back to the s_1 position at the end of the period. As a consequence of this principle of operation, the duty cycle lies in the interval $[0, 1]$ by definition.

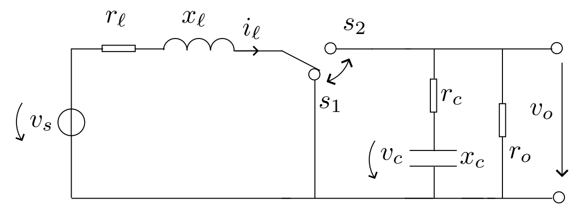


Fig. 1. Topology of the boost converter

By taking $x(t) = [i_\ell(t)\ v_c(t)]^T$ as the state vector, where $i_\ell(t)$ is the inductor current and $v_c(t)$ the capacitor voltage, the system is described by the following pair of affine continuous time state-space equations. The following equations hold

$$\dot{x}(t) = \begin{cases} F_1 x(t) + f_1 v_s, \ k \leqslant t < k + d(k) \\ F_2 x(t) + f_2 v_s, \ k + d(k) \leqslant t < k + 1 \end{cases} \tag{1a}$$

$$v_o(t) = \begin{cases} g_1^T x(t), \ k \leqslant t < k + d(k) \\ g_2^T x(t), \ k + d(k) \leqslant t < k + 1 \end{cases} \tag{1b}$$

Matrices F_1 and F_2 and vectors f_1, f_2, g_1 and g_2 are not given here for the sake of brevity but can be easily obtained by elementary circuit theory.

The state vector of the boost converter model is a continuous function of time, since it comprises the inductor current and the capacitor voltage and there are no degenerate loops or nodes in the circuit topology. However, an important feature is the fact that the output voltage is a discontinuous function of time, due to the existence of the two different output vectors g_1 and g_2. The discontinuity occurs at the time instants of the switch commutations and can be exactly calculated as a function of the inductor current as follows. At the beginning of the switching period, when the model switches from mode 2 to mode 1, the jump of the output voltage is equal to

$$v_o(k^+) - v_o(k^-) = -\frac{r_o r_c}{r_o + r_c} i_\ell(k), \tag{2}$$

while at time $t = k+d(k)$ the model switches from mode 1 to mode 2 and the jump of the output voltage amounts to

$$v_o(k + d(k)^+) - v_o(k + d(k)^-) = \frac{r_o r_c i_\ell(k + d(k))}{r_o + r_c}. \tag{3}$$

3. MODELLING FOR CONTROL DESIGN

3.1 Reformulated Continuous-Time Model

From an implementation point of view, it is preferable that all states used in the prediction model be directly measurable. Thus, the capacitor voltage is replaced by the output voltage in the state vector which leads to setting $x(t) = [i_\ell(t)\ v_o(t)]^T$. Additionally, to account for variations in the voltage source v_s directly, the (to be derived) optimal control law would need to be parameterized over v_s. To obviate this requirement and as will further be explained in sections 3.2 and 4.4, the voltage source v_s is removed from the model equations by redefining the scaled state vector $x'(t) = [i'_\ell(t) v'_o(t)] = [\frac{i_\ell(t)}{v_s}\ \frac{v_o(t)}{v_s}]$. This yields the reformulated state-space equations

$$\dot{x}'(t) = \begin{cases} F'_1 x'(t) + f'_1,\ k \leqslant t < k + d(k) \\ F'_2 x'(t) + f'_2,\ k + d(k) \leqslant t < k + 1 \end{cases} \tag{4a}$$

$$v'_o(t) = g'^T x'(t)\,. \tag{4b}$$

where again matrices and vectors F'_1, F'_2, f'_1, f'_2 and g' can easily be computed from elementary circuit theory.

The voltage source v_s is always considered to be measurable in accordance with common industrial practice, so that the state vector is consistently defined.

An important feature of the reformulated state-space model is the fact that the state vector is now a discontinuous function of time, since it includes the output voltage of the converter. However, the employed hybrid modelling framework can directly incorporate such discontinuities, as shown in the next section.

3.2 Piecewise Affine Discrete-Time Model

The formulation of an adequate model for the boost converter is of fundamental importance for the subsequent derivation and implementation of the optimal control problem. Given the discrete time variation of the input variable, a natural choice is to formulate the model in the discrete time domain by employing a sampling interval equal to the switching period T_s. The employed method considers a direct LSF approximation over several regions of the exact system update equations, yielding a PWA description of the associated non-linear expressions. These can be written as

$$x'(k+1) = \Phi(d(k))x'(k) + \Gamma(d(k)) \tag{5}$$

where $\Phi(d(k))$ and $\Gamma(d(k))$ are matrices that depend nonlinearly on the duty cycle, calculated by integrating (4) from $t = k$ to $t = k+1$, taking into account the discontinuity of the output voltage discussed above.

Expression (5) is approximated by determining the matrices $\bar{A}_i$, $\bar{B}_i$ and $\bar{f}_i$ that describe the system in terms of

$$x'(k+1) = \bar{A}_i x'(k) + \bar{B}_i d(k) + \bar{f}_i \tag{6a}$$

$$\text{if } d(k) \in D_i \quad i = 1, \ldots, \nu \tag{6b}$$

$$0 \leq d(k) \leq 1 \tag{6c}$$

and that minimize the sum of quadratic error terms

$$(\Phi(d(k))x'(k) + \Gamma(d(k)) - (\bar{A}_i x'(k) + \bar{B}_i d(k) + \bar{f}_i))^2 \tag{7}$$

over a gridded series of points $x'(k)$ in the state space $[0, i'_{\ell,max}] \times [0, v'_{o,max}]$, where D_i are the ν intervals $[0, \frac{1}{\nu}]$, ..., $[\frac{\nu-1}{\nu}, 1]$, and $i'_{\ell,max}$, $v'_{o,max}$ are the maximum values of the scaled inductor current and output voltage, respectively.

It should be noted that the choice of normalizing over v_s allows one to obtain matrices $\bar{A}_i$, $\bar{B}_i$, $\bar{f}_i$ that are independent of the voltage source and thus valid for any of its values, since it does not appear in the parameters of the original non-linear update expression (5). However, what needs to be stressed is that the derived PWA model is valid for the given nominal load resistance. In Section 4.5 an estimation scheme connected to the MPC design to account for (unmeasured) changes in r_o is briefly outlined; full details can be found in (Beccuti *et al.*, 2006).

4. THE CONTROL PROBLEM

4.1 Control Issues And Objectives

The main control objective for the boost dc-dc converter is to regulate the dc component of the output voltage v_o to its reference $v_{o,ref}$. This regulation has to be achieved in the presence of the hard constraints on the manipulated variable (the duty cycle) which is bounded between 0 and 1, and needs to be maintained despite the changes in the load r_o and the voltage source v_s whilst rendering steady state operation with constant duty cycle, thus avoiding the occurrence of sub-harmonic oscillations. As described in (Beccuti *et al.*, 2005) however, and as often done in actual industrial practice (Mohan *et al.*, 1989), it is more convenient to formulate the control problem of the boost dc-dc converter as a current (rather than a voltage) regulation problem, aiming at steering the value of the scaled inductor current i'_ℓ to a reference $i'_{\ell,ref}$. The value $i'_{\ell,ref}$ corresponding to the desired $v_{o,ref}$ can be explicitly calculated on the basis of the known parameters of the circuit during nominal system conditions (Kostakis *et al.*, 2000), including variations on v_s. In the case of a load variation, the Kalman filter (cf. 4.5) appropriately updates $i'_{\ell,ref}$ to restore the system to the desired operating point.

4.2 Model Predictive Control

Model Predictive Control (MPC) has been traditionally and successfully employed in the process industry and recently also for hybrid systems. The control action is obtained by minimizing an objective function at each time step over a finite horizon subject to the equations and constraints of the model. The major advantage of MPC is its straight-forward design procedure. Given a model of the system, including constraints, one only needs to set up an objective function that incorporates the control objectives. Further details about MPC can be found in (Maciejowski, 2002).

4.3 Constrained Finite Time Optimal Control Scheme

The control objectives are to regulate the average output voltage to its reference as fast and with as little overshoot as possible, or equivalently, to minimize the absolute scaled inductor current error $i'_{\ell,err}(k) = |i'_{\ell}(k) - i'_{\ell,ref}|$. Let $\Delta d(k) = |d(k) - d(k-1)|$ indicate the absolute value of the difference between two consecutive duty cycles. This term is introduced in order to reduce the presence of unwanted chattering in the input when the system has almost reached stationary conditions. Define the penalty matrix $Q = \text{diag}(q_1, q_2)$ with $q_1, q_2 \in \mathbb{R}^+$ and the vector $\varepsilon(k) = [i'_{\ell,err}(k), \Delta d(k)]^T$. Consider the objective function

$$J(D(k), x'(k), d(k-1)) = \sum_{\ell=0}^{L-1} \|Q\, \varepsilon(k+\ell|k)\|_1 \quad (8)$$

penalizing the predicted evolution of $\varepsilon(k + \ell|k)$ from k over the horizon L using the 1-norm.

The control input at time-instant k is then obtained by minimizing the objective function (8) over the sequence of control moves $D(k) = [d(k), \ldots, d(k + L - 1)]^T$ subject to the model equations and constraints (6a), (6b), (6c); the resulting optimization program is referred to as the constrained finite time optimal control (CFTOC) problem.

4.4 The State Feedback Law

Multi-parametric programming is employed to solve an optimization problem off-line for a range of parameters. In (Baotic *et al.*, 2003) and (Borrelli, 2003) the authors show how to reformulate a discrete-time CFTOC problem for a PWA system as a multi-parametric program by treating the state vector as a parameter and propose an algorithm for its solution.

Note that the CFTOC problem is not only a parametric function of $x(k)$, but also of the last control move $d(k-1)$, as the changes of the duty cycle are penalized in the objective function; furthermore, as it is necessary to solve the CFTOC problem for all possible values of $i'_{\ell,ref}$, the scaled inductor current reference also enters the augmented state vector, which therefore results in being 4-dimensional. Again, it should be noticed that normalizing the system equations over v_s allows to define a model independently of the voltage source, and therefore an explicit state-feedback law that depends on one parameter less (Papafotiou *et al.*, 2004).

Overall the proposed approach, in accordance with common practice, requires the measurement of the inductor current i_ℓ, output voltage v_o and source voltage v_s [1].

As proven in (Borrelli, 2003) the optimal state-feedback control law $d^*(k)$ is a PWA function of the (augmented) state vector defined on a polyhedral partition of the feasible (augmented) state space.

As a result, such a state-feedback controller can be implemented online, since computing the control input amounts to determining the polyhedron in which the measured state lies and then simply evaluating the corresponding affine control law. In many cases, polyhedra with the same control law form a convex union and can thus be optimally merged (Geyer *et al.*, 2004*a*) and replaced by their union, leading to an equivalent PWA control law of reduced complexity.

4.5 Load Variations

An estimation scheme to account for variations in the load resistance has been derived and coupled with the previously obtained state-feedback controller (for a time-invariant and nominal load) through an external loop. More specifically, this loop adjusts the scaled inductor current reference $i'_{\ell,ref}$ by a corrective term $\hat{i}'_e$ (equal to zero during nominal system operation) and feeds the obtained value $\tilde{i}'_{\ell,ref} = i'_{\ell,ref} - \hat{i}'_e$, into the controller; the adjustment is done in a way such that the error between the inductor current and its *actual* reference is made small.

This can be achieved through the use of a Kalman filter (Jazwinski, 1970) that yields a zero steady-state inductor current error due to its integrating nature. To address the hybrid nature of the model, a filter with two modes is employed, and switching between the two is done according to the switch transitions in the converter. Such an approach is possible as the mode transitions, which are imposed by the duty cycle, are precisely known. To allow for an easier implementation of the filter, two constant Kalman gains are used (one

[1] Due to the reformulation of the model the knowledge of v_c, which is unmeasurable, is not needed.

for each mode). It should be noted that the employment of a switched filter setup requires specific provisions to be made in order to ensure the stability of the estimation scheme (Alessandri and Coletta, 2001); such provisions, together with a complete description of the filter, can be found in (Beccuti *et al.*, 2006).

5. SIMULATION RESULTS

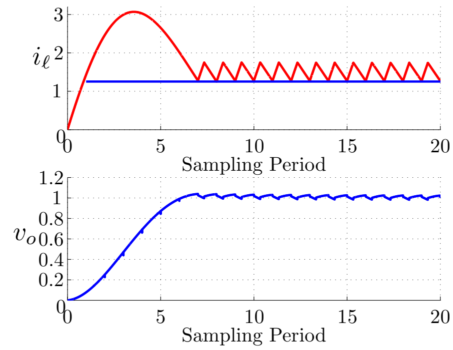
In this section, simulation results demonstrating the performance of the proposed control methodology are presented. The circuit parameters expressed in the per unit system are given by $x_c = 70\,\text{p.u.}$, $x_\ell = 3\,\text{p.u.}$, $r_c = 0.01\,\text{p.u.}$ and $r_\ell = 0.05\,\text{p.u.}$ If not otherwise stated the output resistance is given by $r_o = 1$ p.u. and the voltage source is $v_s = 0.75$ p.u.; the output voltage reference is set to $v_{o,ref} = 1$, to which, for the given circuit parameters, an inductor current reference $i_{\ell,ref} = 1.25$ is associated. The model was derived for a range of values of $[0, 4]$ for the scaled inductor current and $[0, 3]$ for the scaled output voltage; three PWA dynamics were calculated, with the intervals D_i being $[0, \frac{1}{3}]$, $[\frac{1}{3}, \frac{2}{3}]$, and $[\frac{2}{3}, 1]$. For the cost function, the penalty matrix is chosen to be $Q = \text{diag}(10, 1)$ and the prediction horizon is $L = 2$.

As explained in section 4.4 the explicit state-feedback controller is defined in a 4-dimensional space. For the chosen circuit and controller setup its computation yields a polyhedral partition consisting of 239 regions; by utilizing the merging algorithm introduced in (Geyer *et al.*, 2004*a*) the controller can further be simplified to 121 regions.

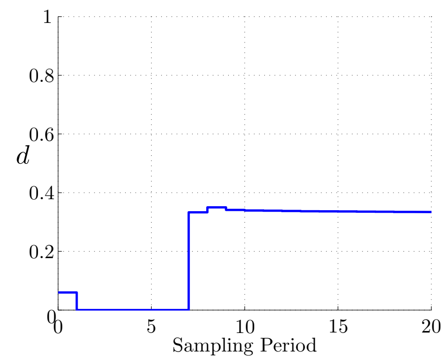
The first case to be analyzed is that of the transient behaviour during startup. Fig.2(a) and Fig.2(b) depict the step responses of the different schemes during start-up, i.e. $x(0) = [0, 0]^T$. The proposed optimal control schemes yields an output voltage that reaches its stationary conditions with an overshoot of about 4% and within 10 switching periods.

For the second case results stemming from a 25% decrease in the voltage source v_s during steady state operation are shown in Fig.3(a)-3(b); the new value of the voltage source is measured, the current reference updated accordingly and the system restored to its desired output voltage value; as the control problem is formulated in terms of a current tracking scheme the system is steered in such a manner as to quickly reach its required inductor current value.

The third and final case concerns a 100% increase in the load resistance r_o during steady state operation; results are displayed in Fig.4(a)-4(b). The Kalman filter adjusts the current reference and

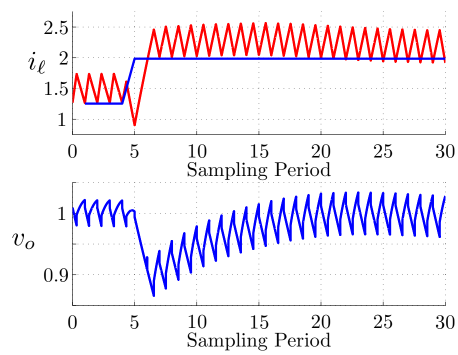


(a) Inductor current (above, with reference) and output voltage (below), in p.u.

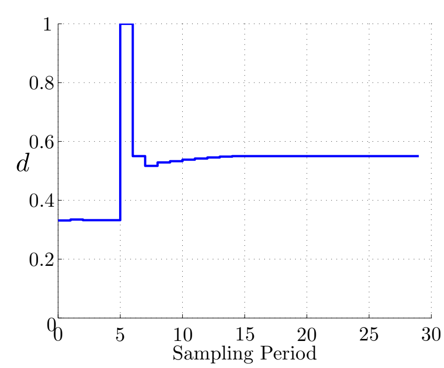


(b) Duty cycle

Fig. 2. Simulation results for the startup scenario

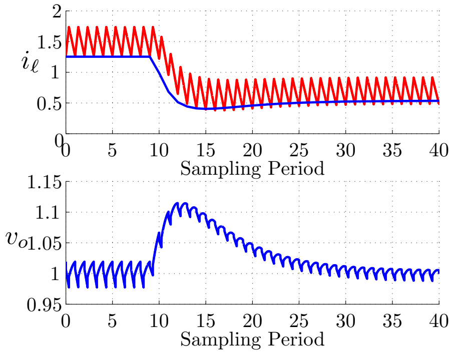


(a) Inductor current (above, with reference) and output voltage(below), in p.u.

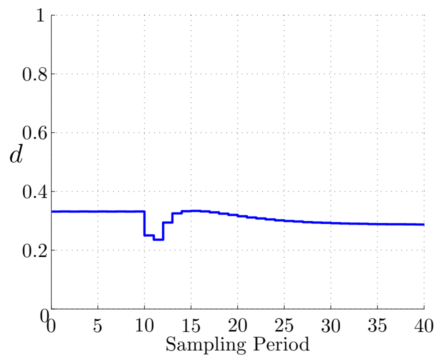


(b) Duty cycle

Fig. 3. Simulation results for the scenario featuring a 25% decrease of v_s



(a) Inductor current (above, with reference) and output voltage(below), in p.u.



(b) Duty cycle

Fig. 4. Simulation results for the scenario featuring a 100% increase of r_o

the output voltage reaches its desired value after approximately 20 switching periods.

6. CONCLUSIONS AND OUTLOOK

An extension of a previously presented MPC scheme for boost switch mode dc-dc converters has been formulated, whereby the solution is explicitly obtained off-line. This reduces the task of solving the optimal control problem to a search in a look-up table, thus rendering the proposed approach viable for the successive phase of experimental validation on a physical converter. The described setup is additionally integrated with an estimator that allows to consider variations on the load, as typically required by any realistic industrial application. Future research work will be directed at formulating the MPC scheme directly in terms of a voltage reference problem and at considering the case of the discontinuous conduction regime, wherein the inductor current drops to zero and thereafter remains constant, thus introducing a third mode of operation for the circuit.

7. ACKNOWLEDGEMENTS

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