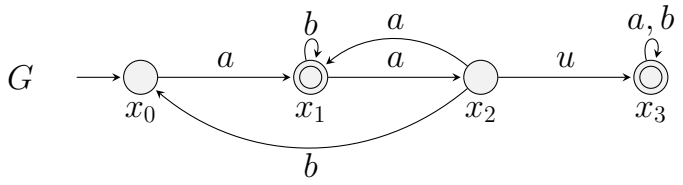
Analysis and Control of Cyber-Physical Systems

Midterm exam — 16 April 2025

Problem 1. (10 points) Consider the automaton G in figure.



(a) *(2 points)* Discuss whether the automaton is reachable, blocking, reversible and complete.

(b) *(1 points)* Modify this automaton by adding or removing a single transition in order to make it blocking (if it is non-blocking) or non-blocking (if it is blocking). If it is not possible to determine such a new automaton, explain why.

(c) *(2 points)* Suppose event u is unobservable and consider observation $w = aaa$. Determine: (a) the set $\mathcal{S}(w)$ of words in $L(G)$ consistent with w; (b) the set of states $\mathcal{X}(w)$ consistent with w.

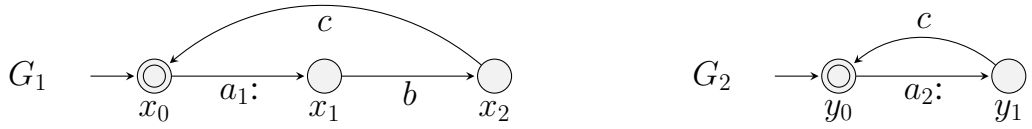
(d) *(2 points)* Let G_n be the NFA equivalent to this partially observed system. Show the graphical and algebraic representation of G_n.

(e) *(3 points)* Compute the DFA G_d equivalent to NFA G_n. Show how to use G_d to verify the value of $\mathcal{X}(aaa)$ you have previously computed.

Problem 2. (3 points) Given a language L over an alphabet E, discuss if it may be possible that some of the following conditions hold. If possible, provide an example of a language satisfying the condition.

$$(a) \quad L \subsetneq L^2; \qquad (b) \quad L = L^2; \qquad (c) \quad L \supsetneq L^2.$$

Problem 3. (12 points) Two processes accessing a database are modeled by the following automata.



The first process, G_1, reads from a memory cell (event a_1), then writes to the same cell (event b) and is finally reinitialized (event c). The second process, G_2, reads from the same memory cell (event a_2) and is then reinitialized (event c). The two processes can access the memory cell in any order, but are synchronized through the reinitialization event, which can occur only after both have completed their respective operations.

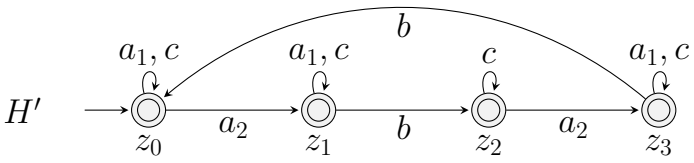
The set of controllable events is $E_c = \{a_1, a_2\}$.

(a) *(3 points)* Determine automaton $G = G_1 \parallel G_2$ that represents the overall system.

(b) *(2 points)* The evolution of the overall system G should alternate between two modes. Mode (i): the second process reads the same data as the first process (i.e., event a_2 occurs **before** b). Mode (ii): the second process reads the data written by the first process (i.e., event a_2 occurs **after** b).

Define an automaton H that describes this specification.

If you are unable to find automaton H, you may still complete the exercise using the alternative specification described by automaton H', shown in the figure, over the alphabet $\{a_1, a_2, b\}$.



(c) *(3 points)* Discuss whether this specification is controllable. If the specification is not controllable, determine whether there exists a legal word w such that, when followed by an uncontrollable event, it leads to a forbidden word.

(d) *(2 points)* Determine a maximally permissive supervisor S for system G that enforces the given specification. Is S a marking supervisor?

Also, describe the structure of the closed-loop system S/G. Are S and S/G different automata?

(e) *(2 points)* Determine, if possible, a supervisor S' with fewer states than S but such that the behavior of the closed-loop system S'/G coincides with the behavior of S/G.