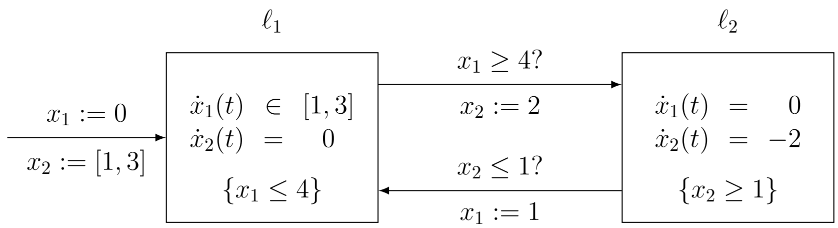
Analysis and Control of Cyber-Physical Systems

Homework 5 — 8 May 2025

Problem 1. Consider the hybrid automaton H shown below.



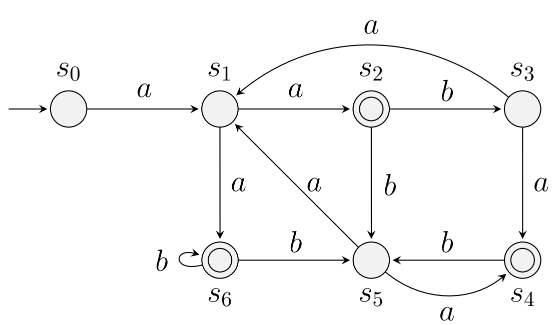
(a) Determine the time-abstract state transition system T that describes H.

(b) Compute the reachability set $Reach(T)$ using the procedure presented in class.

(c) Is state $y = (\ell, x_1, x_2) = (\ell_2, 0.5, 0.5)$ reachable?

(d) Discuss if $Reach(T)$ allows you to verify if state y is reachable at a time $t \geq 10$. If not, can you suggest a way to verify that?

Problem 2. Consider the state transition system T described by the automaton in figure.



(a) Determine the algebraic structure of T.

(b) Compute the set $Pre_b(\{s_5, s_6\})$.

(c) Consider the following partition: $\Pi = \{\pi_1, \pi_2, \pi_3\}$ with $\pi_1 = \{s_0\}$, $\pi_2 = \{s_1, s_3, s_5\}$ and $\pi_3 = \{s_2, s_4, s_6\}$. Determine an equivalence relation on the state space of this automaton whose set of equivalence classes is Π.

(d) Is the equivalence relation R above a bisimulation over the states of T? Justify your answer.

(e) Determine a minimal bisimulation over the states of T and the corresponding quotient state transition system, showing the steps of the procedure you have used.

Problem 3. Let T be an STS with set of states S. Prove that if $R_1, R_2 \subseteq S \times S$ are two bisimulations for T then $R = R_1 \cup R_2$ is also a bisimulation relation for T.