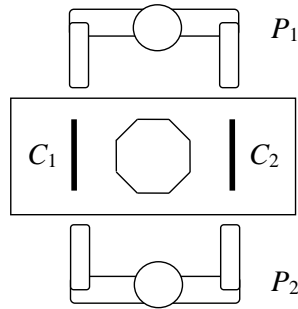


Analysis and Control of Cyber-Physical Systems

Homework 3 — 3 April 2025

Problem 1. Two Chinese philosophers sit at a table meditating. At the center of the table there is a dish of noodles and between the two philosophers (P_1 and P_2) are two chopsticks (C_1 and C_2) as shown in the following figure. When a philosopher feels hungry, they grab first the chopstick to their left, then the other, and use them to eat. After eating, they put the two chopsticks back on the table and go back to meditate.



1. Model each philosopher and each chopstick by a deterministic finite automaton (DFA). The automaton describing philosopher P_i has three states: the state of meditation, the state in which they have grabbed the chopstick to their left, and the state in which they are eating; the three events will be: l_i (grabs the chopstick on the left), r_i (grabs the chopstick on the right), f_i (finishes eating and releases the two chopsticks). The automaton describing chopstick C_i has three states: the chopstick on the table, the chopstick is in the hand of philosopher P_1 , the chopstick is in the hand of philosopher P_2 . The only final state of each DFA is the initial one.
2. Define the set of synchronized events and construct by concurrent composition the DFA G that describes the overall process. Given the cardinality of the state space of each module, what is the maximum number of states that we would expect to find in G ? How many states does G have? Justify the discrepancy or the equality between these two values.
3. Show that G contains dead states and list all of them. Determine a minimal sequence that leads to a deadlock.
4. To prevent reaching a deadlock, one wants to enforce the following constraints: for $i = 1, 2$ if philosopher P_i grabs the stick to their left, then prevent philosopher P_{3-i} to grab the other chopstick. Describe the two automata H_1 and H_2 corresponding to these constraints and construct, by concurrent composition, the overall specification H .
5. Suppose that only the events related to philosopher P_2 are controllable. Determine if the specification determined in the previous item is controllable. If not, determine a maximally permissive supervisor that imposes such a constraint.
6. Construct, given the supervisor determined in the previous item, the closed loop system and verify that it is not blocking. Which of the two philosophers you would like to be?
7. Determine, if any exists, a static specification equivalent to the dynamic specification given in item 4.

You may want to check your solution using the software DESops developed at the University of Michigan.

Problem 2. Let G be a DFA on alphabet E with set of uncontrollable events E_{uc}

1. Under what conditions is it possible to determine a supervisor S such that $L(S/G) = \{\varepsilon\}$?
2. Let $K_1, K_2 \subseteq E^*$ be two language specifications controllable with respect to $L(G)$ and E_{uc} . Discuss if specification $K = K_1 \cup K_2$ is also controllable.