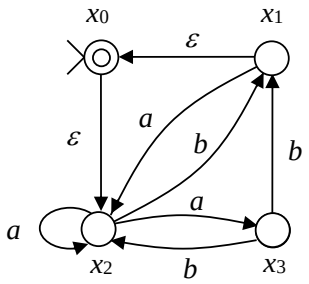
Analysis and Control of Cyber-Physical Systems

Homework 2 — 21 March 2024

Problem 1. Consider the nondeterministic finite automaton G in figure.



(a) Give the algebraic description of this NFA. Which are the nondeterministic structures in this model?

(b) Determine the following set of states:

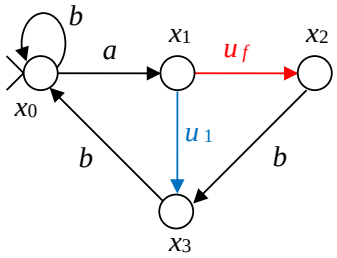
$$(a) \quad \Delta(x_0, ab); \qquad (b) \quad \Delta^*(x_3, \varepsilon); \qquad (c) \quad \Delta(x_3, \varepsilon).$$

where for $e' \in E_\varepsilon$ we define $\Delta(x, e') = \{x' \in X \mid (x, e', x') \in \Delta\}$ and for $w \in E^*$ we define $\Delta^*(x, w) = \{x' \in X \mid (x, w, x') \in \Delta^*\}$.

(c) Determine a DFA G' equivalent to G, i.e., the observer $Obs(G)$.

(d) Are there any observations that allow one to determine the current state of G is certainly x_2?

Problem 2. Consider the DFA G shown in the following figure which represents a system subject to failures. The set of observable events is $E_o = \{a, b\}$, the set of unobservable events is $E_{uo} = \{u, u_f\}$ and the set of fault events is $E_f = \{u_f\}$.



(a) Determine the set of string $\mathcal{S}(w)$ and set of states $\mathcal{X}(w)$ consistent with the following observations:

$$i) \quad w_1 = a; \qquad ii) \quad w_2 = ab; \qquad iii) \quad w_3 = aa.$$

(b) Construct the diagnoser $Diag(G)$ and compute the diagnosis state $\varphi(w)$ for the following observations:

$$i) \quad w_1 = \varepsilon; \qquad ii) \quad w_2 = b; \qquad iii) \quad w_3 = bba.$$

(c) Is this fault diagnosable? If not, determine an ambiguous string $s = wfv \in L(G)$ where v can be arbitrarily long.

(d) If a fault is not diagnosable, does it mean its occurrence can never be detected?

Problem 3. Given a language L on alphabet E, $P(L) = L \uparrow E'$ denotes it projection on alphabet $E' \subseteq E$.

Discuss if it the following result holds.

$$P(L_1 \cap L_2) = P(L_1) \cap P(L_2).$$

If it holds, you should give a formal proof, if it does not hold you should just provide a counterexample.