Analysis and Control of Cyber-Physical Systems

Midterm exam — 19 April 2023

Problem 1. [5 pts] Consider a deterministic finite automaton (DFA) G on alphabet $E = \{\square, \lozenge, \triangle\}$ with initial state x_0, set of final states $X_m = \{x_2, x_3\}$ and transition function

δ	$\square$	$\lozenge$	$\triangle$
x_0	x_2	x_3	$-$
x_1	x_4	$-$	x_0
x_2	$-$	x_1	x_0
x_3	$-$	$-$	$-$
x_4	x_4	x_4	$-$

(a) *(1 pts)* Give a graphical representation of G.

(b) *(1 pts)* Discuss if the states of G are: reachable, co-reachable, blocking, dead.

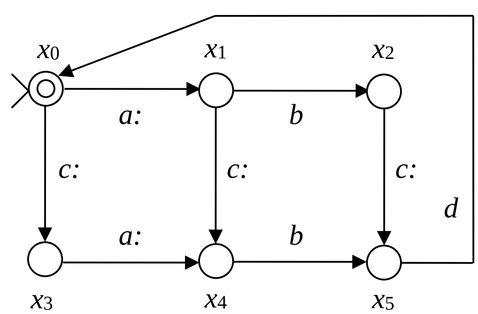
(c) *(1 pts)* Discuss if G is: reachable, co-reachable, blocking, trim, reversible.

(d) *(1 pts)* Does the language identity $\mathit{pref}(L_m(G)) = L(G)$ holds? If not, show a word that belongs to one of the two languages but not to both.

(e) *(1 pts)* If G is blocking, trim it to obtain a new DFA G'. How are the languages generated and accepted by G' related to those generated and accepted by G?

Problem 2. [10 punti] Two machines, M_1 and M_2, work in parallel. Machine M_1 during a working cycle performs two operations in sequence (first event a and then event b). Machine M_2 during a working cycle performs a single operation (event c). When both machines have completed a cycle they are simultaneously reinitialized (event d). The set of controllable events is $E_c = \{a, c\}$.

The system's behavior can be described by the automaton G in figure.



(a) *(1 pts)* Show that you have understood this model, briefly discussing the meaning of each state of G.

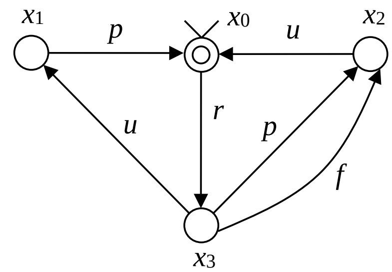
(b) *(3 pts)* In a first operative mode (unsupervised), when machine M_1 has finished a cycle it should not have to wait for the other machine to end its cycle. Describe this constraint as a state specification and determine a maximally permissive supervisor capable of enforcing it. Is this supervisor blocking?

(c) *(2 pts)* In a second operative mode (supervised), a single operator is required to operate both machines. When the operator starts a cycle on a machine it cannot leave the machine (to work at the other one) until the cycle is completed. This means that no event c cannot occur beween an event a and b. Describe this constraint as a language specification.

(d) *(4 pts)* Determine a maximally permissive supervisor capable of enforcing the language specification determined at the previous point. Is this supervisor blocking?

Problem 3. [10 pts] A website adopts the following user/password recovery procedure: when a request arrives (event r) the username (event u) and password (event p) are sent in random order by means of two different email messages. Event u and p are logged in a file while event r is not. When a password is sent before the username, the transmission could fail and in this case no event is logged (event f).

This recovery procedure can be modeled by the DFA G shown in the figure below, where the set of observable events is $E_o = \{u, p\}$, the set of unobservable events is $E_{uo} = \{r, f\}$, and the fault event set is $E_f = \{f\}$.



(a) *(1 pts)* Determine the words in E_o^* that are logged when the following sequences of events are generated:

$$i) \quad s_1 = rup; \qquad\qquad ii) \quad s_2 = rfurp.$$

(b) *(2 pts)* Determine for each logged word $w \in E_o^*$ listed below the set $\mathcal{S}(w)$ of strings consistent with w and the set $\mathcal{X}(w)$ of states consistent with w:

$$i) \quad w_1 = \varepsilon; \qquad ii) \quad w_2 = u; \qquad iii) \quad w_3 = up; \qquad iv) \quad w_4 = pup.$$

(c) *(5 pts)* Determine the diagnoser $Diag(G)$. What is the diagnosis state $\varphi(w)$ for the words listed below?

$$i) \quad w_1 = up; \qquad ii) \quad w_2 = pu; \qquad iii) \quad w_3 = u.$$

(d) *(2 pts)* Discuss if the diagnoser contains uncertain or indeterminate cycles. Is the fault diagnosable?

Problem 4. [5 punti]

Given a language $L \subseteq E^*$ and a word $w \in E^*$, we define the (L, w)-*residual language* as

$$w^{-1}L = \{u \in E^* \mid wu \in L\}.$$

(a) *(1 pts)* Given $L = \{ab, bab, abbab\}$ and $w = ab$ determine $w^{-1}L$.

(b) *(2 pts)* Show that the class of regular languages is closed by residuation. To do this, describe (even informally) an algorithm that, given a DFA G accepting language L and a word w, constructs a new DFA G' accepting $w^{-1}L$.

(c) *(2 pts)* Consider the automaton G discussed in the previous problem. Pick up an arbitrary word w of length 2 generated by G and apply your algorithm to it.