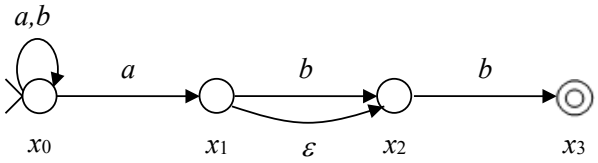
Analysis and Control of Cyber-Physical Systems

Homework 3 — 30 March 2023

Problem 1. Consider the nondeterministic finite automaton G in figure.



(a) Give the algebraic description of this NFA. Which are the nondeterministic structures in this model?

(b) Determine the following set of states:

$$(a) \quad \Delta(x_0, a); \qquad (b) \quad \Delta^*(x_1, \varepsilon); \qquad (c) \quad \Delta(x_2, \varepsilon).$$

where for $e' \in E_\varepsilon$ we define $\Delta(x, e') = \{x' \in X \mid (x, e', x') \in \Delta\}$ and for $w \in E^*$ we define $\Delta^*(x, w) = \{x' \in X \mid (x, w, x') \in \Delta^*\}$.

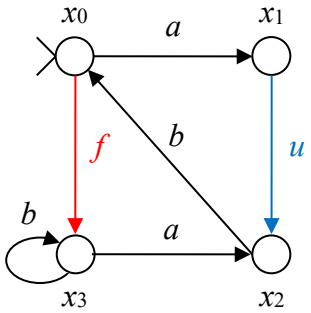
(c) Determine if the following words belong to the language $L(G)$ and to the language $L_m(G)$. You must also write all runs that generate these words if applicable.

$$(a) \ w_1 = ab; \qquad (b) \ w_2 = bb; \qquad (c) \ w_3 = abab;$$

(d) Explain in natural language which is the language accepted by this automaton.

(e) Determine a DFA G' equivalent to G, i.e., the observer $Obs(G)$.

Problem 2. Consider the DFA G shown in the following figure which represents a system subject to failures. The set of observable events is $E_o = \{a, b\}$, the set of unobservable events is $E_{uo} = \{u, f\}$ and the set of fault events is $E_f = \{f\}$.



(a) Determine the set of string $\mathcal{S}(w)$ and set of states $\mathcal{X}(w)$ consistent with all observations $w \in E_o$ of length up to 2.

(b) Construct the diagnoser $Diag(G)$. What is the diagnosis state $\varphi(w)$ for word $w = abab$?

(c) Is this fault diagnosable? If not, determine an ambiguous string $s = wfv \in L(G)$ where v can be arbitrarily long.

(d) If a fault is not diagnosable, does it mean its occurence can never be detected?

Problem 3. Show an NFA with 3 states whose equivalent DFA has 7 states: any alphabet is fine.

For all $n > 3$, does it exists an NFA with n states whose equivalent DFA has $2^n - 1$ states?