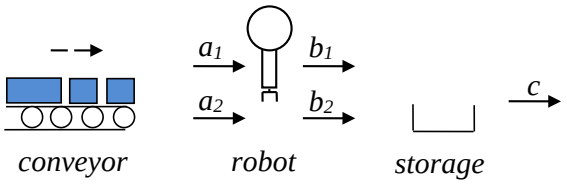
Analysis and Control of Cyber-Physical Systems

Homework 2 — 16 March 2023

Problem 1. Consider a small logistic cell where a robot transports packages from a conveyor to a storage space. There are two types of packages: small-size and large-size. The robot picks up a package (either small or large) from the conveyor which is always full, and deposits the package into the storage space which contains two slots: a small-size package takes up one slot and a large-size package takes up two slots. Thus the storage space is essentially a buffer.

The events of picking up and depositing a small-size package are a_1 and b_1, respectively. Likewise, the events of picking up and depositing a large-size package are a_2 and b_2, respectively. The storage space can be emptied by a human operator: we denote this event by c. The layout of the cell is displayed in the following figure.



(a) Model the robot and the storage space each by a (deterministic) finite-state automaton. The automaton **R** (say) describing the robot should have three states: the state of idle (no picking), the state of picking up a small-size package, and the state of picking up a large-size package. The event set of this robot automaton should contain a_1, b_1, a_2, b_2. Next, the automaton **B** (say) describing the storage space should also have three states: the state of containing no package, the state of containing a small-size package, and the state of containing two small-size packages or a large-size package. The event set of this storage space automaton should contain b_1, b_2, c. Note that event c may occur at all states.

(b) Construct the synchronous product of the two automata **R** and **B** in step (a) above by showing all steps. Is the product automaton reachable, coreachable, nonblocking, and trim (explain your answers)?

Problem 2. We now consider a supervisory control problem for the above logistic cell. Let the controllable event set be $\Sigma_c = \{a_1, a_2\}$ and the uncontrollable event set be $\Sigma_{uc} = \{b_1, b_2, c\}$. The plant **G** to be controlled is the robot **R** with event c selflooped. Thus the event set of **G** is $\{a_1, a_2, b_1, b_2, c\}$.

The control specification is to protect the storage space against overflow, when there are not sufficient empty slots to accommodate a package the robot wants to deposit. This specification **H** can be described by the storage space automaton **B** with events a_1, a_2 selflooped. Thus the event set of **H** is also $\{a_1, a_2, b_1, b_2, c\}$.

(a) Construct the extended specification $\widehat{\mathbf{H}}$ and use it to determine if the specification language $L_m(\mathbf{H})$ is controllable with respect to plant **G** (explain your answer).

(b) Determine a maximally permissive supervisor **S** for this supervisory control problem.

(c) Explain the control logic of the optimal supervisor **S**: at each state of **S**, which events are enabled and which are not?

(d) Can you find a different Σ_c and $\Sigma_{uc} (\neq \emptyset)$ such that $L_m(\mathbf{H})$ is controllable with respect to plant **G** in step (a) above (explain your answer)?