Analysis and Control of Cyber-Physical Systems

Homework 1 — 9 March 2023

Problem 1. Consider the DFA G on alphabet $E = \{a, b\}$ with initial state x_0, set of final states $X_m = \{x_1\}$ and transition function

δ	a	b
x_0	x_1	$-$
x_1	x_2	x_0
x_2	$-$	$-$
x_3	$-$	x_0

(a) Give a graphical representation of G.

(b) Discuss if the states of G are: reachable, co-reachable, blocking.

(c) Discuss if G is: reachable, co-reachable, blocking, trim, reversible. If G is blocking, trim it to obtain a new DFA G'.

(d) Determine the language $L(G)$ generated by G and the language $L_m(G)$ accepted by G.

Problem 2. For each of the following languages, determine a trim DFA on alphabet $E = \{a, b, c\}$ that accepts it.

(a) Set of strings whose second symbol is a.

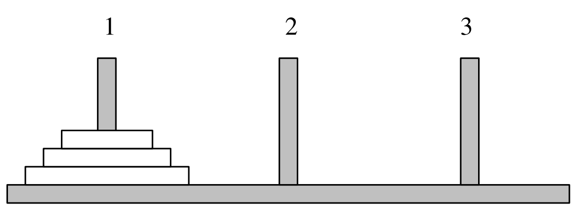
(b) Set of strings that start with cba and end with ba.

(c) Set of strings such that the projection on alphabet $E_1 = \{a, b\}$ is a string where two a's and one b alternatively occur (ex: $aabaab... \cdots$).

(d) Set of strings that do not contain c and that contain an equal number of a and b.

Problem 3. The *tower of Hanoi* is a game with three poles. In the first pole n disks of different diameter are stacked to form a tower. The disks are orderly arranged, the largest on the bottom and the smaller on the top.

There are two rules in this game: (1) only the top disk of a pole can be moved to another pole; (2) a disk cannot be stack over a smaller disk. The objective of the game consists in moving all disks on the third pole.



A Tower of Hanoi with $n = 3$ disks.

(a) Describe this game as a discrete event system, listing states and events. How does the cardinality of the state space depend on the number of disks n? In the following we will assume $n = 3$ as in figure.

(b) Model such a game as a deterministic finite automaton, with the additional constraint that the largest disk can never be moved to the central pole. Clearly describe how you are defining states and events. Does the game still have a solution with such a constraint?

(c) Determine the minimal number of moves, starting from the initial condition in the figure, to reach the final condition where all disks are on pole 3.

(d) Determine if it is possible, starting from the initial condition in the figure, to reach the final condition where all discs are on pole 3 without passing for a configuration where the two smaller disks are both on pole 2.