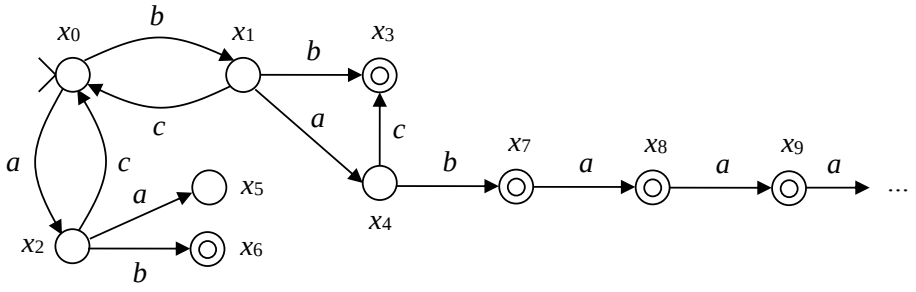
Analysis and Control of Cyber-Physical Systems

Final exam — 8 June 2022

Problem 1. [14 pts] Consider the state transition system T shown below, whose state space is countably infinite.



(a) *(4 pts)* Apply the procedure presented in class for computing the reachability set of T. Show sets R_k and $Reach_K$ for $k = 0, \ldots, 6$.

(b) *(2 pts)* Discuss if the following relation on the state set S of T is an equivalence relation or a bisimulation: $\mathcal{R} = \{(x_i, x_j) \subseteq S \times S \mid i + j < 3\}$.

(c) *(6 pts)* Determine a minimal bisimulation over the states of T and the corresponding quotient state transition system, showing the steps of the procedure you use.

(d) *(2 pts)* Suppose one is interested in determining if a given state (say, x_8) is reachable in a given number of steps (say, $k = 15$). Can this property be verified by means of the quotient?

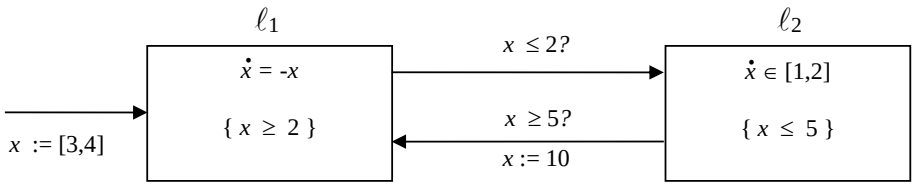
Problem 2. [5 pts] Consider the scalar differential equation $\dot{x}(t) = 3\, x^{2/3}(t)$, with initial condition $x(0) = 0$.

(a) *(2 pts)* Verify that for $T \geq 0$, all functions belonging to the following family are a solution:

$$x_T(t) = \begin{cases} 0 & t \leq T \\ (t - T)^3 & t \geq T \end{cases}$$

(b) *(3 pts)* Discuss which property, ensuring the uniqueness of a solution, does not hold in this case. Are there different initial conditions admitting unique solutions?

Problem 3. [6 pts] Consider the hybrid automaton H whose graphical representation is shown below.



(a) *(2 pts)* Describe its algebraic structure.

(b) *(4 pts)* Determine its time-abstract state transition system.