Analysis and Control of Cyber-Physical Systems

Homework 5 — 5 May 2022

Problem 1. A cosmetic company produces a face cream on-demand using a plant which operates in short cycles.

The company receives orders from customers at a rate $u(t) \geq 0$ and the total number of orders $x(t)$ grows according to

$$\dot{x}(t) = u(t).$$

When the total number of orders reaches level $x_{\max}$, the company starts a production cycle. During this cycle, while orders continue to arrive, the cream is produced at a constant rate $\varrho > 0$ and thus the inventory of the cream $y(t)$ grows according to

$$\dot{y}(t) = \varrho.$$

The production cycle ends when the inventory level reaches a value $y_{\max} = 1.2 \cdot x_{\max}$ and the company satisfies in a single step all orders it can based on the current inventory, thus reducing the current number of orders and the inventory level. More precisely, all current orders are satisfied if they do not exceed the inventory, else only a quantity equal to the inventory can be satisfied.

(a) Describe this system by means of a deterministic hybrid automaton, showing its graphical structure.

(b) Assume that $u(t) = 1$ (const), $\varrho = 5$ and $x_{\max} = 25$ and consider an initial state in which the plant is not operating and $x(0) = y(0) = 0$.

Determine the time-abstract state transition system T associated to this automaton.

(c) Determine the reachability set $Reach(T)$.

(d) Since the company does not have a buffer to stock inventory, it is required that the plant operation satisfies the following the specification: "The inventory should never exceed the number of orders."

Discuss what type of specification this is. Is the specification satisfied?

Problem 2. A scalar signal $x(t) \in \mathcal{R}$ satisfies the scalar differential inclusion

$$\dot{x}(t) \in F(x(t)) \qquad \text{where} \qquad F(x) = [-x, x].$$

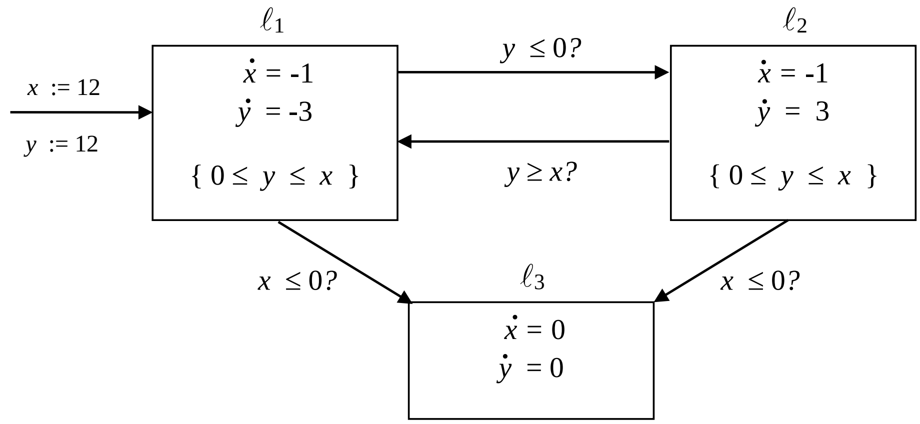
(a) Discuss if the differential inclusion admits solutions for an initial conditions $x(0) = 2$.

(b) Determine the set of possible values $X(t)$ that a solution may assume at an arbitraty value of time $t \geq 0$.

(c) Is $x(t) = 2$ a solution?

(d) Model this time-driven system as an STS (time should not be abstracted).

Problem 3. Consider the hybrid automaton in figure.



(a) Plot the evolution of this automaton for $t \in [0, 11]$.

(b) Show that the automaton is zeno.

(c) Compute a non-zeno model by time regularization.

(d) The physical interpretation of this model is the following. Variable $x(t)$ denotes the distance from a solid wall of a partition that is moving towards the wall at a speed of $1\ m/s$. Variable $y(t)$ denotes the distance from the solid wall of a ball that is bouncing between the wall and the partition at a speed of $3\ m/s$ (bounces are purely elastic).

Describe the meaning of the different locations.

(e) Given the physical interpretation above, can you propose a better way than time-regularization to modify the hybrid automaton so that it describes in a more accurate way this physical phenomenon?