Analysis and Control of Cyber-Physical Systems

Homework 4 — 28 April 2022

Problem 1. Consider the hybrid automaton H whose graphical representation is shown below.

	ℓ_1	_	ℓ_2		ℓ_3
$x := (x_{1,0}, x_{2,0})$	$\begin{aligned} \dot{x}_1 &= 1\\ \dot{x}_2 &= 1 \end{aligned}$	$x_1 \ge 2$?	$\dot{x}_1 = 2$ $\dot{x}_2 = 1$	$x_2 \ge 3$?	$\begin{aligned} \dot{x}_1 &= 1\\ \dot{x}_2 &= 0 \end{aligned}$
ŗ	$\{x_1 \leq 2\}$	$x_1 := 1$	$\{x_2 \leq 3\}$	$x_1 := 0$	

- (a) Describe the algebraic structure of H.
- (b) Write an OpenModelica program to model H and simulate its evolution from the two different initial conditions

 $x_0' = (1,0)$ and $x_0'' = (0,1).$

For each simulation shown $x_1(t)$, $x_2(t)$ and $\ell(t)$.

(c) Describe as hybrid signals the two evolutions previously computed.

Problem 2. Consider the circuit with a diode studied in the Example 10.7 of the class notes, which is shown in the figure below.

The hybrid automaton model of this device is shown in the next figure.

$$x := 0$$

$$i = -\frac{x}{RC} + \frac{u}{R_1C}$$

$$i = -\frac{x}{R_2C} + \frac{u}{R_1C}$$

$$i = -\frac{x}{R_2C}$$

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(a) Write an OpenModelica program to model this automaton.

The following values are suggested: $R_1 = 500 \Omega$, $R_2 = 700 \Omega$, C = 0.01 F.

(b) Simulate its evolution for $t \in [0, 40]$ assuming that $u(t) = \sin(t)$ and that the switch is opened for $t \in [20, 30]$ and closed otherwise.