Analysis and Control of Cyber-Physical Systems

Homework 2 — 25 March 2022

Problem 1. Consider the nondeterministic finite automaton $G = (X, E, \Delta, x_0, X_m)$ with

$$X = \{x_0, x_1, x_2, x_3, x_4\}; \qquad E = \{a, b\}; \qquad X_m = \{x_3\};$$

$$\Delta = \left\{ \begin{array}{lllll} (x_0, a, x_1), & (x_0, b, x_2), & (x_1, b, x_2), & (x_1, b, x_3), & (x_2, a, x_1), \\ (x_2, a, x_2), & (x_2, a, x_3), & (x_3, \varepsilon, x_4), & (x_4, \varepsilon, x_0) & \end{array} \right\}.$$

(a) Show the graphical representation of this NFA. Which are the nondeterministic structures in this model?

(b) Determine the following set of states:

$$(a) \quad \Delta(x_2, a); \qquad (b) \quad \Delta^*(x_3, \varepsilon); \qquad (c) \quad \Delta^*(x_2, a).$$

where for $e' \in E_\varepsilon$ we define $\Delta(x, e') = \{x' \in X \mid (x, e', x') \in \Delta\}$ and for $w \in E^*$ we define $\Delta^*(x, w) = \{x' \in X \mid (x, w, x') \in \Delta^*\}$.

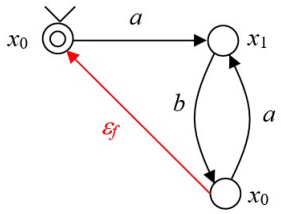
(c) Determine if the following words belong to the language $L(G)$ and to the language $L_m(G)$. You must also write all runs that generate these words if applicable.

$$w_1 = baa; \qquad w_2 = aa; \qquad w_3 = abb.$$

(d) Determine a DFA G' equivalent to G, i.e., the observer $Obs(G)$.

(e) If an NFA G is reversible, is $Obs(G)$ reversible? If an NFA G is non-reversible, is $Obs(G)$ non-reversible?

Problem 2. Consider the DFA G shown in the following figure which represents a system subject to failures. The set of observable events is $E_o = \{a, b\}$ and the set of unobservable and fault events are $E_{uo} = E_f = \{\varepsilon_f\}$.



(a) Determine by direct computations the sets of words $\mathcal{S}(w)$ and of states $\mathcal{X}(w)$ consistent with all observations of length up to 3.

(b) Determine the diagnoser $Diag(G)$. What is the diagnosis state $\varphi(w)$ for word $w = abab$?

(c) Is this fault diagnosable? If not, determine an ambiguous string $s = u\varepsilon_f u \in L(G)$ where v can be arbitrarily long.

Problem 3. Given a language $L \subseteq E^*$ and alphabet $\hat{E} \subseteq E$, consider the language $\hat{L} = \{\hat{w} \in \hat{E}^* \mid w \in L, \hat{w} = w \uparrow \hat{E}\}$. Show that

$$L \in \mathcal{L}_{DFA} \quad \Longrightarrow \quad \hat{L} \in \mathcal{L}_{DFA}$$

describing a procedure that, given in input a DFA G accepting L, determines a DFA $\hat{G}$ accepting $\hat{L}$.

The first two problems can be solved with software DESUMA https://wiki.eecs.umich.edu/desuma/index.php/DESUMA