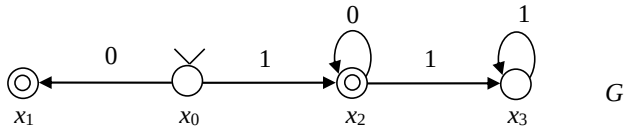
Analysis and Control of Cyber-Physical Systems

Homework 1 — 10 March 2022

Problem 1. Consider the deterministic finite automaton (DFA) shown below.



(a) Determine the algebraic representation of G.

(b) Discuss if the states of G are: reachable, co-reachable, blocking, dead.

(c) Discuss if G is: reachable, co-reachable, blocking, trim, reversible.

(d) Determine the language $L(G)$ generated by G and the language $L_m(G)$ accepted by G.

(e) Determine the languages $L(G) \uparrow \{0\}$ and $L_m(G) \uparrow \{1\}$.

(f) If G is not reversible (resp., blocking), can you add to it a single arc such that the new DFA is reversible (resp., non-blocking)?

Problem 2. For each of the following languages, determine a trim DFA on alphabet $E = \{a, b\}$ that accepts it.

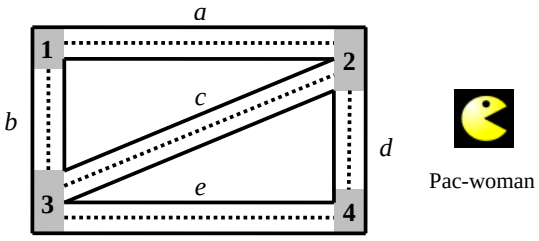
(a) Set of words containing substring aa.

(b) Set of words where the number of a's is even and the number of b's is odd.

(c) Set of words w such that $|w|_a = |w|_b + 1$, i.e., containing one more a's that b's.

Problem 3. In an old videogame, released in 1980, Pac-woman moves along corridors in a maze, eating all the dots she encounters. Ghosts may appear in the maze: if Pac-woman touches a ghost she dies.

Consider the maze in the figure, consisting of four intersections $(1, 2, 3, 4)$ and five corridors (a, b, c, d, e). Initially there are no ghosts in the maze and Pac-woman is in intersection 1. When a corridor is cleared of dots, a ghost appears in it and remains there, so that Pac-woman cannot pass again in the corridor.



(a) Describe this game by means of a DFA. Use alphabet $E = \{a, b, c, d, e\}$ where each event denotes the passage on Pac-woman in the corresponding corridor. You may want to denote each state by means of pair (x, Y) where $x \in \{1, 2, 3, 4\}$ denotes the intersection where Pac-woman is and $Y \subseteq \{a, b, c, d, e\}$ denotes the set of corridors containing a ghost. A state (x, Y) is final if $Y = \{a, b, c, d, e\}$, i.e., Pac-woman has eaten all dots in the maze.

(b) Discuss if Pac-woman can reach a final state.

(c) Compare this game with the solution proposed by Euler to the famous "Seven Bridges of Königsberg" problem (see Wikipedia). Based on Euler theoretical analysis, discuss if Pac-woman can reach a final state starting from a different intersection? Which one?