Analysis and Control of Cyber-Physical Systems

Final exam — 1 June 2021

Problem 1. [8 pt] A cosmetic company produces a face cream on-demand using a plant which operates in short cycles.

The company receives orders from customers at a rate $u(t) \geq 0$ and the total number of orders $x(t)$ grows according to

$$\dot{x}(t) = u(t).$$

When the total number of orders reaches level $x_{\max}$, the company starts a production cycle. During this cycle, while orders continue to arrive, the cream is produced at a constant rate $\varrho > 0$ and thus the inventory of the cream $y(t)$ grows according to

$$\dot{y}(t) = \varrho.$$

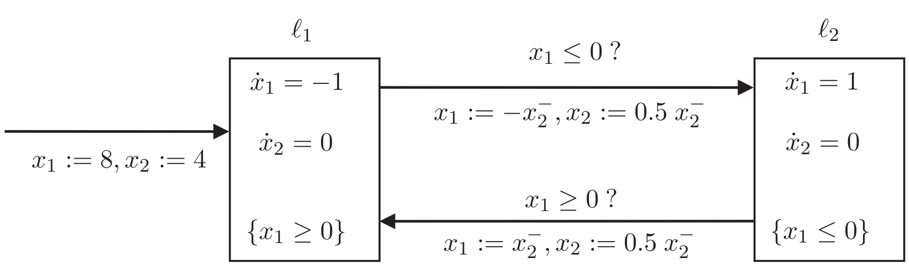
The production cycle ends when the inventory level reaches a value $y_{\max} = 1.2 \cdot x_{\max}$ and the company satisfies in a single step all orders it can based on the current inventory, thus reducing the current number of orders and the inventory level. More precisely, all current orders are satisfied if they do not exceed the inventory, else only a quantity equal to the inventory can be satisfied.

(a) *(5 pts)* Describe this system by means of a deterministic hybrid automaton, presenting its algebraic and graphical structure.

(b) *(3 pt)* Assume that $u(t) = 1$ (const), $\varrho = 15$ and $x_{\max} = 25$ and consider an initial state in which the plant is not operating and $x(0) = y(0) = 0$.

Describe the evolution of the system in the time interval $[0, 60]$ in terms of hybrid temporal trajectory and hybrid signals.

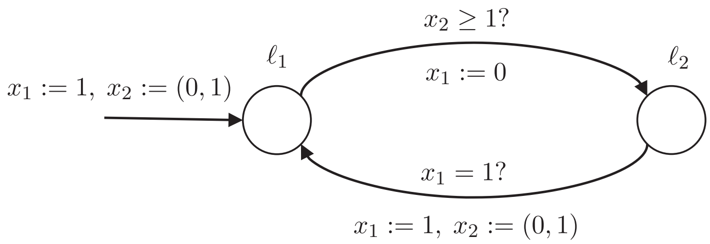
Problem 2. [7 pt] Consider the hybrid automaton in figure.



(a) *(3 pts)* Plot the evolution of the discrete and continuous state per $t \in [0, 15]$.

(b) *(4 pt)* Discuss if this system is Zeno. If such is the case, compute the instant T_{zeno} beyond which there is no progression of time.

Problem 3. [15 pt] Consider the timed automaton in figure.



(a) *(3 pt)* Determine the time-abstract state transition system T associated to this automaton.

(b) *(5 pt)* Compute the reachability set of the automaton based on T.

(c) *(4 pt)* Determine the region graph of the timed automaton.

(d) *(3 pt)* Do the two approaches for reachability analysis considered in item (b) and (c) provide the same results? What does each approach allow you to conclude concerning the reachability of the following states:

$$y_1 = (\ell_1, (0.5, 0.5)), \qquad y_2 = (\ell_2, (0, 1)), \qquad y_3 = (\ell_1, (1.1, 1.5)).$$