Analysis and Control of Cyber-Physical Systems

Midterm exam — 12 April 2021

Problem 1. [4 pt] Let L be the language on alphabet $E = \{a, b, c\}$ consisting of all strings w containing an odd number of a's and such that in each prefix $u \preceq w$ it holds $-2 \leq |u|_b - |u|_c \leq 2$.

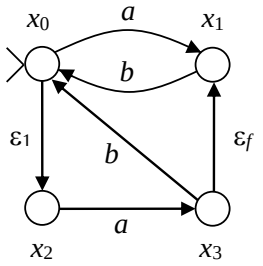
(a) *(3 pts)* Construct a deterministic finite automaton G that accepts language L. Discuss the physical interpretation of each state to show that you understand its structure.

(b) *(1 pt)* Is language L prefix-closed? What is its projection $L' = L \uparrow_{E'}$ on alphabet $E' = \{a\}$?

Note: if you prefer, you may use the concurrent composition operator to construct G.

Problem 2. [10 pt]

Consider the plant G in the following figure subject to faults. The set of observable events is $E_o = \{a, b\}$, the set of non-observable regular events is $E_{reg} = \{\varepsilon_1\}$ and the set of non-observable fault events is $E_f = \{\varepsilon_f\}$.



(a) *(2 pts)* Determine the set of consistent states $\mathcal{X}(w)$ and the set of consistent strings $\mathcal{S}(w)$ for the following observed words:

$$a) \quad w = ab; \qquad b) \quad w = \varepsilon; \qquad c) \quad w = abb.$$

(b) *(4 pts)* Determine the diagnoser $Diag(G)$. You need to show all steps followed to compute the diagnoser from the fault recognizer

(c) *(3 pts)* Discuss if the fault is diagnosable by the analysis of the uncertain and undeterminate cycles of the diagnoser.

(d) *(1 pts)* If the fault is not diagnosable, determine an ambiguous string $s = u\varepsilon_f v \in E^*$ where subfix v can be arbitrarily long.

Problem 3. [6 pt] Consider the DFA G on alphabet $E = \{a, b, c\}$ with initial state x_0, set of final states $X_m = \{x_2, x_5\}$ and transition function

δ	a	b	c
x_0	x_1	x_5	x_3
x_1	$-$	x_2	x_0
x_2	x_4	$-$	$-$
x_3	x_3	$-$	$-$
x_4	x_5	x_4	$-$
x_5	$-$	x_4	$-$

(a) *(1 pt)* Give a graphical representation of G.

(b) *(1 pt)* Determine: $\delta(x_3, \varepsilon)$, $\delta^*(x_5, \varepsilon)$ and $\delta^*(x_3, bbb)$.

(c) *(1 pt)* Discuss if the states of G are: reachable, co-reachable, blocking, dead.

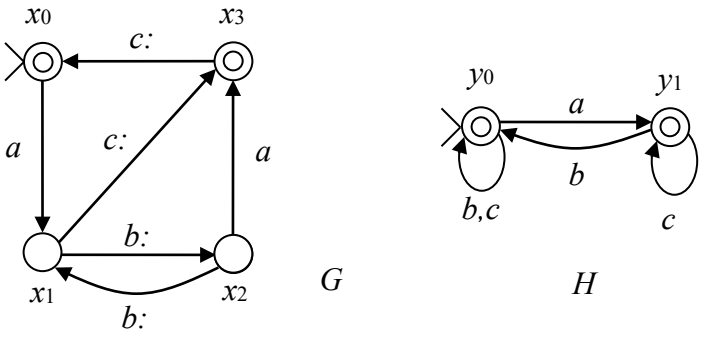
(d) *(1 pt)* Discuss if G is: reachable, co-reachable, blocking, trim, reversible.

(e) *(1 pt)* Determine the strongly connected components of the automaton, specifying which are *transient* and which are *ergodic*. Verify the properties of blocking and reversibility by mean of the components you have determined.

(f) *(1 pt)* Discuss if G is complete. If not determine a complete automaton G' accepting the same language of G.

Problem 4. [10 pts]

The figure below shows a plant G on alphabet $E = \{a, b, c\}$ and a language specifications described by automaton H. The set of controllable events is $E_c = \{b, c\}$ and the set of uncontrollable events is $E_{uc} = \{a\}$.



(a) *(4 pts)* Determine if the specification is controllable.

(b) *(2 pts)* Determine a maximally permissive and non-blocking supervisor capable of enforcing this specification.

(c) *(2 pts)* Is the obtained supervisor a marking or non-marking supervisor? Give the algebraic representation of the closed-loop system.

(d) *(2 pts)* Would the maximally permissive and non-blocking supervisor change if $X_m = \{x_0\}$?