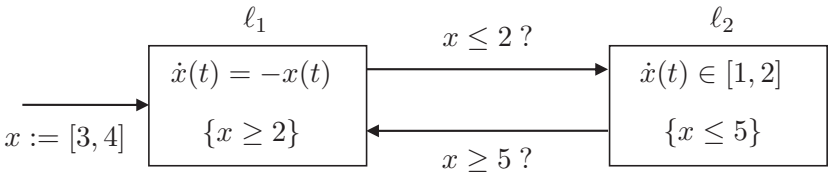
Analysis and Control of Cyber-Physical Systems

Homework 5 — 20 May 2021

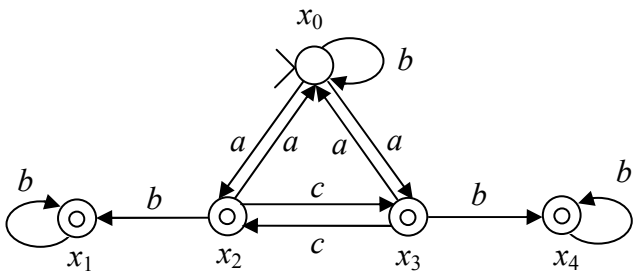
Problem 1. Consider the hybrid automaton in figure.



(a) Determine its time abstract state transition system T.

(b) Compute the reachability set of T.

Problem 2. Consider the state transition system T described by the automaton in figure.



(a) Determine a state transition system T' that is simulated by T but that does not simulate it. Justify your answer.

(b) Compute the set $Pre_b(\{x_0, x_1\})$.

(c) Consider the following partition: $\Pi = \{\pi_1, \pi_2, \pi_3\}$ with $\pi_1 = \{x_0\}$, $\pi_2 = \{x_1, x_2\}$ and $\pi_3 = \{x_3, x_4\}$. Is the corresponding equivalence relation a bisimulation over the states of T? Justify your answer.

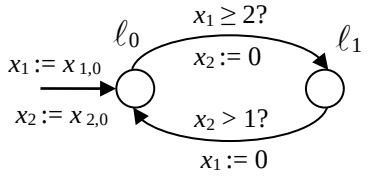
(d) Determine a minimal bisimulation over the states of T and the corresponding quotient state transition system, showing the steps of the procedure you have used.

Problem 3. Determine a timed automaton with external inputs to describe the following system. A lamp is initially "off". When a button is pushed the lamp is switched on with low intensity: in this "low" state if the button is pushed again fast enough (within 3 seconds) the lamp will become brighter else pushing the button will turn the light off. When the light is "bright" if no button is pushed after 20 seconds it goes back to "low" while pushing the button will turn it off.

Problem 4. Consider the timed automaton in figure where to each location two timers x_1 and x_2 are associated.

Let m denotes the number of the month in which you are born. The initial value of the timers is

$$x_0 = \begin{cases} x_{0,1} := 0, & x_{0,2} := (0,1), & \text{if } m \text{ is even;} \\ x_{0,1} := (0,1), & x_{0,2} := 0, & \text{if } m \text{ is odd.} \end{cases}$$

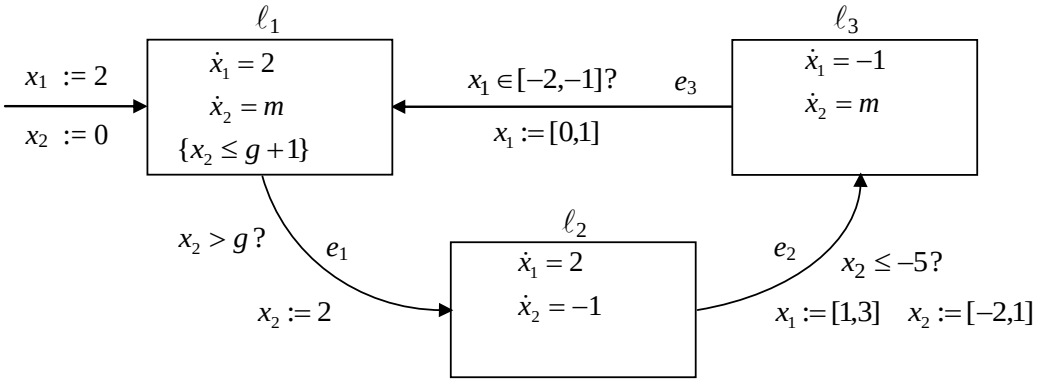


(a) Determine the regions in the continuous state space X of this automaton.

(b) Determine the number of equivalence classes of the corresponding bisimulation for its time-abstract state transition system and discuss if this number is smaller than or identical to the bound N_S.

(c) Determine the region graph of this automaton starting from the given initial condition.

Problem 5. Consider the hybrid automaton H in figure, where (g, m) denote the day and the month in which you were born.



(a) Discuss to which subclasses of HA automaton H belongs.

(b) Discuss if there exists a timed automaton equivalent to H. If the answer is positive, determine such a timed automaton.

Note that not all invariants of the multirate automaton take the form $X = \mathbb{R}$: this exercise aims to show that the conversion procedure can also be applied in such a case.