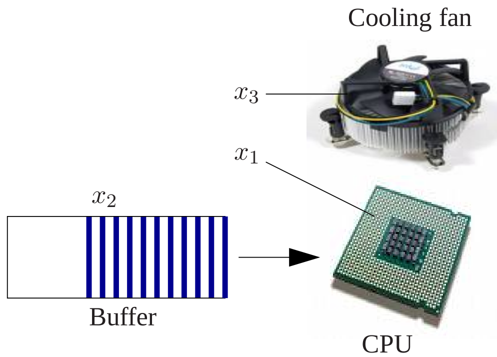
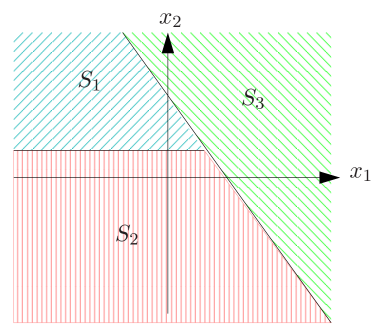
Analysis and Control of Cyber-Physical Systems

Homework 4 — 28 April 2021

Problem 1. Consider the system composed by a CPU, a buffer and a cooling fan as shown in figure (left). Here the state variables denote variations from the nominal value and are defined as follows: x_1 is the CPU temperature, x_2 is the amount of CPU tasks in buffer and x_3 is the angular velocity of the cooling fan.





The dynamical behavior of this model around the nominal values is given as follows:

$$\begin{cases} \dot{x}_1 = -a_1 x_1 - a_2 x_3 + b_1 u_1, \\ \dot{x}_2 = -b_2 u_1, \\ \dot{x}_3 = -a_3 x_3 + b_3 u_2, \end{cases}$$

where the control inputs are

- u_1: the deviation of the CPU clock frequency;
- u_2: the deviation of the fan voltage control.

We consider the following choice of parameters: $a_1 = 1$, $a_2 = 2$, $a_3 = 1$, $b_1 = 1$, $b_2 = 1$, and $b_3 = 1$.

Consider the regions in the subspace (x_1, x_2) shown in figure (right) and defined as follows:

- $S_1 = \{(x_1, x_2) \mid x_1 \leq 4 - x_2,\ x_2 \geq 2\}$,
- $S_2 = \{(x_1, x_2) \mid x_1 \leq 4 - x_2,\ x_2 \leq 2\}$,
- $S_3 = \{(x_1, x_2) \mid x_1 \geq 4 - x_2\}$.

We assume that:

- if $(x_1, x_2) \in S_1$ then $u_2 = 0$, i.e., only u_1 is used as control input;
- if $(x_1, x_2) \in S_2$ then $u_1 = 0$, i.e., only u_2 is used as control input;
- if $(x_1, x_2) \in S_3$ then both u_1 and u_2 are used as control inputs.

(a) Model this system as an hybrid automaton with inputs. You should give both the algebraic and graphical representation of the automaton.

(b) Derive a control law for the continuous inputs such that the trajectory reaches and remain in a safe set

$$S = \{(x_1, x_2, x_3) \mid x_1 \leq 2,\ x_2 \leq 2,\ x_3 \leq 0\}.$$

(c) Model the controlled system as an autonomous hybrid automaton. It is sufficient to give just a graphical representation of the automaton.

(d) Simulate the evolution of this system from the following different initial conditions: $x_{0,a} = (-3, 4, 1)$, $x_{0,b} = (4, 4, 1)$, $x_{0,c} = (8, -2, 1)$, and print the following plots: state space trajectory in the (x_1, x_2) subspace, the evolution of all components of the continuous state and of the continuous input in time, the evolution of the discrete state in time.

Problem 2. Consider a time-driven system modelled by the differential equation

$$\dot{x}(t) = f(x(t))$$

where $x = (x_1, x_2) \in \mathbb{R}^2$ and the piece-wise constant activity is

$$f(x_1, x_2) = \begin{cases} (1, -1) & \text{if } x_2 \geq 0 \\ (2, 1) & \text{if } x_2 < 0 \end{cases}$$

(a) Determine the discontinuity points of this activity function.

(b) Determine if in the discontinuity points there exists a Cauchy solution; if not determine a Filippov solution.

(c) Determine a hybrid automaton with two locations and constant activity functions that describes this system.

(d) Determine analytically the evolution of the hybrid automaton for $t \geq 0$ starting from the initial condition $x(0) = (0, 1)$. You should represent this solution as a hybrid signal.

(e) Discuss if the solution determined at the previous point is affected by chattering and if so determine[1], by time or space regularization, a chattering-free model.

(f) Simulate the evolution of the regularized automaton for $t \geq 0$ from the initial condition $x(0) = (0, 1)$.

[1] Solutions by space regularization will be awarded 1 bonus point.