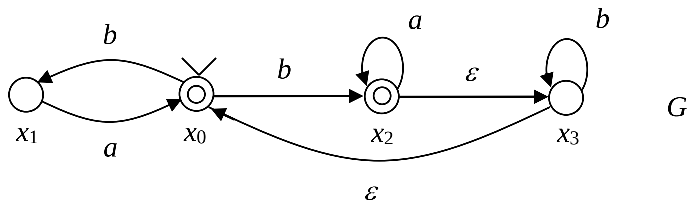
Analysis and Control of Cyber-Physical Systems

Homework 2 — 24 March 2021

Problem 1. Consider the nondeterministic finite automaton shown in figure.



(a) Write the algebraic description of this NFA. Which are the nondeterministic structures in this model?

(b) Determine the following set of states:

$$(a) \quad \Delta(x_2, \varepsilon); \qquad (b) \quad \Delta^*(x_2, \varepsilon); \qquad (c) \quad \Delta^*(x_3, b).$$

where for $e' \in E_\varepsilon$ we define $\Delta(x, e') = \{x' \in X \mid (x, e', x') \in \Delta\}$ and for $w \in E^*$ we define $\Delta^*(x, w) = \{x' \in X \mid (x, w, x') \in \Delta^*\}$.

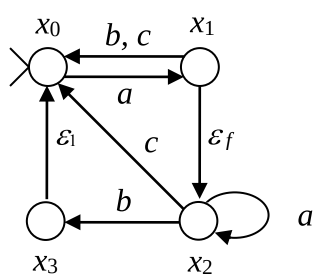
(c) Determine if the following words belong to the language $L(G)$ and to the language $L_m(G)$. You must also write all runs that generate these words if applicable.

$$w_1 = bba; \qquad w_2 = bb; \qquad w_3 = baab.$$

(d) Determine a DFA G' equivalent to G, i.e., the observer $Obs(G)$.

(e) A bookmaker accepts bets 1/3 on what the current state of system is. Can you make money guessing the current state? What would you bet and when?

Problem 2. Consider the DFA G shown in the following figure which represents a system subject to failures. The set of observable events is $E_o = \{a, b, c\}$, the set of unobservable events is $E_{uo} = \{\varepsilon_1, \varepsilon_f\}$ and the set of fault events is $E_f = \{\varepsilon_f\}$.



(a) Determine by direct computations the sets of words $\mathcal{S}(w)$ and of states $\mathcal{X}(w)$ consistent with all words of length up to 3.

(b) Determine the diagnoser $Diag(G)$. What is the diagnosis state $\varphi(w)$ for word $w = acaa$?

(c) Is this fault diagnosable?

Problem 3. Discuss if the following statement is correct:

Any language $L \in \mathcal{L}_{DFA}$ can be accepted by some NFA G' with a unique final state.

If the statement is correct, provide an algorithm that given in input an automaton G produces in output an NFA G' with a unique final state accepting language $L_m(G') = L_m(G)$. Apply this construction to the NFA in Problem 1.

The first two problems can be solved with software UMDES https://wiki.eecs.umich.edu/desuma/index.php/UMDES_Software_Library