Analysis and Control of Cyber-Physical Systems

Homework 1 — 10 March 2021

Problem 1. Consider the deterministic finite automaton (DFA) G on alphabet $E = \{a, b\}$ with initial state x_0, set of final states $X_m = \{x_1\}$ and transition function

δ	a	b
x_0	x_1	$-$
x_1	x_2	x_0
x_2	$-$	$-$
x_3	$-$	x_0

(a) Give a graphical representation of G.

(b) Discuss if the states of G are: reachable, co-reachable, blocking, dead.

(c) Discuss if G is: reachable, co-reachable, blocking, trim, reversible. Is this analysis consistent with the partition in strongly connected components?

(d) If G is blocking, trim it to obtain a new DFA G'.

(e) Determine the language $L(G)$ generated by G and the language $L_m(G)$ accepted by G.

(f) If G is not trim and reversible, can one suitably redefine $\delta(x_2, a)$ such that G becomes trim and reversible?

Problem 2. For each of the following languages, determine a trim DFA on alphabet $E = \{a, b, c\}$ that accepts it.

(a) Set of words whose second symbol is a.

(b) Set of words where each substring aba is immediately followed by at least one b.

(c) Set of palindrome words, i.e., word that can be written as

$$w = e_1 e_2 \cdots e_k e_k \cdots e_2 e_1 \qquad \text{(for some } k \in \mathbb{N}\text{)}$$

Problem 3. Three Sardinian students are playing their own version of morra. At each turn, the three players (P1, P2 and P3) simultaneously reveal their right hand extending an even (E) or odd (O) number of fingers. At the first turn the play is OOO, i.e., all players show an odd number of fingers. During the following turns, each player uses their own rule (undisclosed to the other players) to choose the next move based on the past one. Suppose the following rules are adopted.

P1: plays O if at previous turn both P1 and P2 played O, else plays E.

P2: plays O if at previous turn P1 played E, else plays E.

P3: plays O if at previous turn both P2 and P3 played E, else plays E.

(a) Model this game by means of a DFA. What is the set of states? What is the set of events?

(b) Are there plays that can occur more than once in an infinite run of this game?

(c) Does this automaton have ergodic components? If so, what can one infer from their analysis?

Problem 4. Given a language $L \subseteq E^*$ and an integer $q > 0$, we define the language *Kleene star of L minus q* as

$$L^{*-q} = \left(\bigcup_{k=0}^{q-1} L^k \right) \cup \left(\bigcup_{k=q+1}^{+\infty} L^k \right).$$

Discuss if it always holds $L^{*-q} = L^* \setminus L^q$ proving this result or giving a counterexample.