Analysis and Control of Cyber-Physical Systems

Homework 6 — 28 May 2020

Problem 1. Consider the nonlinear systems described by $\dot{x}(t) = f(x(t))$, with state $x(t) \in \mathbb{R}$ and activity

$$f(x) = \begin{cases} -x & \text{if } x \leq 1 \\ -x+1 & \text{if } x > 1 \end{cases}$$

(a) What are its equilibrium point(s)?

(b) Discuss if $V(x) = x^2$ is a Lyapunov function.

(c) Are the equilibrium point(s) stable? Asymptotically stable? Globally asymptotically stable?

(d) Simulate the evolution of the system with initial condition $x_0' = 3$ and $x_0'' = -1$. Discuss if these results are consistent with the previous analysis.

Problem 2. Consider the switched system $\{A_1, A_2\}$ with

$$A_1 = \begin{bmatrix} -2 & -1 \\ 1 & -1 \end{bmatrix} \qquad \text{and} \qquad A_2 = \begin{bmatrix} -1 & 1 \\ -1 & -2 \end{bmatrix}.$$

(a) Show that the systems is stable under arbitrary switching laws by determining a quadratic common Lyapunov function $V(x) = x^T P x$ where matrix $P \succ 0$ is diagonal.

(b) Show that the given Lyapunov function ensures that during a continuous step

$$\frac{d}{dt}V(x(t)) \leq -\varrho\, V(x(t)), \qquad \text{with } \varrho > 0.$$

What is the value of parameter ϱ?

Problem 3. Consider the switched system $\{A_1, A_2\}$ with

$$A_1 = \begin{bmatrix} 0 & 2 \\ 1 & -2 \end{bmatrix} \qquad \text{and} \qquad A_2 = \begin{bmatrix} -2 & -3 \\ -3 & 2 \end{bmatrix}.$$

(a) Show that the system is quadratically stabilizable finding an equivalent stable dynamics A_{eq} and its corresponding state feedback switching law $\ell(x(t))$ that stabilizes the system (a chattering free law is required).

(b) Draw the regions of the state space where each dynamics is active and show the system evolution for three different initial conditions.