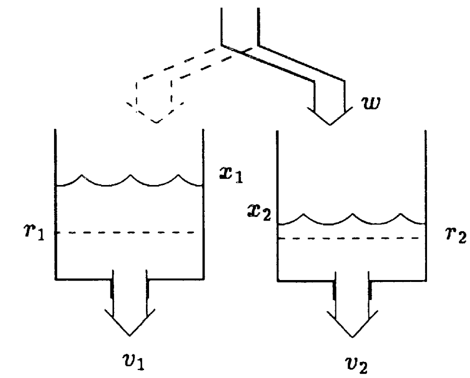
Analysis and Control of Cyber-Physical Systems

Homework 5 — 15 May 2020

Problem 1. The system shown in the picture consist of two tanks S_1 e S_2 and the volume of water they contain at time t is denoted, respectively, $x_1(t)$ and $x_2(t)$.



A single hose, producing a constant flow w, feeds the two tanks. The hose can be in two positions: when on the left it feeds the first tank, when on the right it feeds the second one. The time required to move the hose between the two positions is negligible and can be assumed null.

The output flow of tank S_i (for $i = 1, 2$) is denoted v_i and is constant. The volume of water in tank S_i should not drop below a reference level r_i and the adopted control policy consisting in switching the position of the hose to feed it when its volume reaches the reference level.

(a) Determine the graphical and algebraic structure of a hybrid automaton describing the controlled system assuming the hose is initially feeding the first tank.

You should write the model without specifying the numerical value of the parameters. However, to solve the following points (b), (c), (d) and (e) you may assume $w = 3\, m^3/s$, $v_1 = v_2 = 2\, m^3/s$, $r_1 = r_2 = 1\, m^3$, $x_1(0) = 2\, m^3$, $x_2(0) = 3\, m^3$.

(b) Describe the state evolution of the automaton up to the first three switches of the hose position in terms of hybrid temporal trajectory and hybrid signals.

(c) Simulate the evolution of this system for $t \in [0, 5]$ (in s). You should print the following plots: state space trajectory, the evolution of all components of the continuous state in time, the evolution of the discrete state in time.

(d) Show that the automaton is zeno and explain why in plain words. Determine the time instant T_{zeno}.

(e) Apply time regularization with a minimum dwell time $\delta = 0.1\ s$ to determine a non-zeno model and simulate its evolution.

(f) Determine a condition on w, v_1 and v_2 ensuring that the original automaton is non-zeno.

Problem 2. Modify the model of the bouncing ball assuming that when the ball reaches the ground the change of velocity $v = -\alpha v^-$ occurs linearly in a fixed time Δ.

(a) Model this system as an autonomous hybrid automaton. You should give both the algebraic and graphical representation of the automaton.

(b) Discuss if this system is zeno.

(c) Simulate the evolution of this system from the initial condition $(h_0, 0)$ with $h_0 = 1\text{m}$ assuming $\alpha = 0.8$ and $\Delta = 0.1\text{s}$.

Problem 3. Consider a time-driven system modelled by the differential equation

$$\dot{x}(t) = f(x(t))$$

where $x(t) \in \mathbb{R}$ and the activity function is

$$f(x) = \begin{cases} -2x & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ -2x & \text{if } 1 \leq x \end{cases}$$

(a) Draw the plot of the activity function.

(b) Discuss for which values of the initial condition one can ensure the existence of a local solution. Are these solutions unique? Are they global?

(c) Determine the evolution of the system for $t \geq 0$ starting from the initial condition $x(0) = 0.5$, both analytically and via simulation. What happens when the state reaches a point of discontinuity of the activity function?

(d) Determine, starting from the previously given initial condition, the evolution of the system for $t \geq 0$ using a Filippov solution and draw a plot of this signal.

(e) Determine, if possible, a hybrid automaton with continuous activity functions, that can also describe this system.