Analysis and Control of Cyber-Physical Systems

Homework 4 — 5 May 2020

Problem. In a population of 1.6M individuals, 30 are infected with a virus causing a transmissible disease. The average infective period is $\theta = 10$ days.

1. Assume the average adequate contact rate for this disease and population is $\beta = 0.17$ contacts/day.
   (a) Determine a normalized SIR model for the epidemic and plot its dynamics in the state space (s, i) for different initial conditions.
   (b) What is the basic reproduction number σ for this epidemic?
   (c) What is the herd immunity threshold of this disease (fraction of removed individuals ensuring the disease will not propagate further)?
2. Simulate the evolution in time of the SIR model starting from the given initial condition. How does the actual reproduction number $\mathcal{R}(t)$ change with time? Determine the maximal prevalence and incidence of the disease in the course of its evolution and at what time these maximal values are reached.
3. The public authority, upon realizing in early stages that the epidemic is spreading, has decided to contrast it enforcing a general lockdown which will reduce the average adequate contact rate to $\beta' = 0.07$ contacts/day.
   (a) Will this lockdown be beneficial and why?
   (b) At what time should it be applied, at latest, to ensure the prevalence of the disease will never exceed 5%?
   (c) Describe this switched system as an hybrid automaton.
   (d) What will be the corresponding evolution of the overall system?
   (e) In the steady state, when the disease has been eradicated, has the herd immunity threshold been reached? What will happen if the lockdown is lifted and a few new infectives arrive in this population?
4. Hoping to reach the heard immunity threshold while bounding the disease prevalence, the public authority has decided to adopt the following cyclic policy: no lockdown will be enforced while the prevalence is less than 5% but when this value is reached a lockdown period of one month will be enforced to reduce the average adequate contact rate to $\beta' = 0.07$ contacts/day.
   (a) Describe this switched system as an hybrid automaton.
   (b) Simulate the evolution of this system.
   (c) Is herd immunity eventually reached? At what time?
5. Assume more data concerning the dynamics of this epidemic are available. In particular the total average infective period can be split into an incubation period $\theta_E = 3$ days and a period $\theta_I = 7$ days where symptoms are apparent. In addition, the average adequate contact rate during the incubation period is $\beta_E = 0.17$ contacts/day. Determine a normalized SEIR model for the epidemic. Which value β_I for the adequate contact rate during the period when symptoms are apparent will ensure the SIR and SEIR model have the same basic reproduction number?