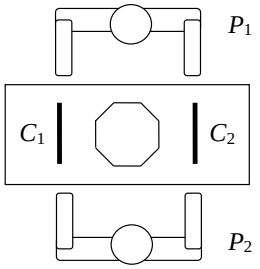
# Analysis and Control of Cyber-Physical Systems

Homework 3 — 21 April 2020

**Problem.** Two Chinese philosophers sit at a table meditating. At the center of the table there is a dish of noodles and between the two philosophers ($P_1$ and $P_2$) are two chopsticks ($C_1$ and $C_2$) as shown in the following figure. When a philosopher feels hungry, she grabs first the chopstick to her left, then the other, and uses them to eat. After eating, she puts the two chopsticks back on the table and goes back to meditate.




1. Model each philosopher and each chopstick by a deterministic finite automaton (DFA). The automaton describing philosopher $P_i$ has three states: the state of meditation, the state in which she has grabbed the chopstick to her left, and the state in which she is eating; the three events will be: $l_i$ (grabs the chopstick on her left), $r_i$ (grabs the chopstick on her right), $f_i$ (finishes eating and releases the two chopsticks). The automaton describing chopstick $C_i$ has three states: the chopstick on the table, the chopstick is in the hand of philosopher $P_1$, the chopstick is in the hand of philosopher $P_2$. The only final state of each DFA is the initial one.

2. Define the set of synchronized events and construct by concurrent composition the DFA $G$ that describes the overall process. Given the cardinality of the state space of each module, what is the maximum number of states that we would expect to find in $G$? How many states does $G$ have? Justify the discrepancy or the equality between these two values.

3. Show that $G$ contains deadlock states and list all of them. Determine a minimal sequence that leads to a deadlock.

4. To prevent reaching a deadlock, one wants to enforce the following constraints: for $i = 1, 2$ if philosopher $P_i$ grabs the stick to her left, then prevent philosopher $P_{3-i}$ to grab the other chopstick. Describe the two automata $H_1$ and $H_2$ corresponding to these constraints and construct, by concurrent composition, the overall specification $H$.

5. Suppose that only the events related to philosopher $P_1$ are controllable. Determine if the specification determined in the previous item is controllable. If not, determine a maximally permissive supervisor that imposes such a constraint.

6. Construct, given the supervisor determined in the previous item, the closed loop system and verify that it is not blocking. Which of the two philosopher you would like to be?

7. Determine, if any exists, a static specification equivalent to the dynamic specification given in item 4.