Analysis and Control of Cyber-Physical Systems

Homework 1 — 27 March 2020

Problem 1. Consider the DFA G on alphabet $E = \{a, b\}$ with initial state x_0, set of final states $X_m = \{x_0 x_1\}$ and transition function

δ	a	b
x_0	x_1	$-$
x_1	x_2	x_0
x_2	x_3	x_0
x_3	$-$	x_3

(a) Give a graphical representation of G.

(b) Determine if these words are generated and accepted by showing the corresponding productions:

$$w_1 = abab; \quad w_2 = abb; \quad w_3 = aa.$$

(c) Compute: $\delta(x_3, \varepsilon)$, $\delta^*(x_2, \varepsilon)$ et $\delta^*(x_3, baa)$.

(d) Discuss if the states of G are: reachable, co-reachable, blocking, dead.

(e) Discuss if G is: reachable, co-reachable, blocking, trim, reversible. If G is blocking, trim it to obtain a new DFA G'.

(f) Show that the generated language $L(G)$ is larger than the prefix of the accepted language $L_m(G)$. What is the consequence of this?

Problem 2. For each of the following languages, determine a trim DFA on alphabet $E = \{a, b, c\}$ accepting it.

(a) Set of words which contain at most two a's.

(b) Set of words that start with b and end with ba.

(c) Set of words such that:

- the projection on alphabet $E_1 = \{a, b\}$ is a string where a and b alternatively occur (ex: $abab\cdots$)
- the projection on alphabet $E_2 = \{b, c\}$ is a string where b and cc alternatively occurs (ex: $bccbcc\cdots$).

(d) Set of words that do not contain c and that contain an equal number of a and b.

Problem 3. The *3-puzzle* is composed by three tiles numbered from 1 to 3. The tiles can slide in a 2×2 frame which also contains an empty slot: a tile can move in any of the four directions (up, down, right and left) but only to occupy the empty slot. The initial configuration is shown in figure (a).

(a)

1	**2**
3	

(b)

1	**3**
2	

(a) Give a DFA model of this game. What do the states and the alphabet of this automaton represent?

(b) Determine if the configuration in figure (b) can be reached from the initial state and, if the answer is positive, provide the corresponding run.

(c) If the answer to the previous question is negative, can you prove that the configuration in figure (b) is not reachable without having to explore all the state space?

Problem 4. Given a language $L \in \mathcal{L}_{DFA}$ on alphabet E, let $L' = L \setminus \{\varepsilon\}$ be the language obtained from L removing string ε (should this string belong to the language).

Show that $L' \in \mathcal{L}_{DFA}$ giving a procedure that takes in input the DFA G that accepts L and terminates outputting the DFA G' that accepts L'. Apply this procedure to the DFA in Problem 1.