A Framework for Financial Markets Modeling and Simulation

Alessio Setzu

March 2007

Contents

Introduction

1	Mo	deling	and Analysis of Financial Markets	1
	1.1	Histor	ical background	3
		1.1.1	Classical Models	4
		1.1.2	Heterogeneous Agent Models	6
		1.1.3	Artificial Financial Markets	11
	1.2	How t	o develop an artificial financial market	16
	1.3	Stylize	ed Facts	19
		1.3.1	Unit Root Property	19
		1.3.2	Fat Tails	20
		1.3.3	Volatility Clustering	21
2	A F	'ramew	vork for Financial Market Simulation	23
	2.1	GASM	I Early History	23
	2.2 The Original Model		24	
		2.2.1	Agents	24
		2.2.2	The Price Clearing Mechanism	27
	2.3	2.3 The Reengineering Process		
	2.4 Simulation Software			34
		2.4.1	Verification and Validation	35

 $\mathbf{i}\mathbf{x}$

3	\mathbf{Ass}	essing	the Impact of Tobin-like Transaction Taxes	37
	3.1	Motiva	ations	38
	3.2	Model	Description	40
		3.2.1	The decision making process	42
		3.2.2	Price clearing mechanism	47
		3.2.3	Financial Resources	47
	3.3	Results	S	49
		3.3.1	One market	50
		3.3.2	Two markets	63
4	Sho	rt Solli	ing and Margin Trading	71
4	4 1			7 I
	4.1	Motiva	itions	(1
	4.2	The E	xtended Model	75
		4.2.1	non-Debt Prone Traders	76
		4.2.2	Debt Prone Traders	77
	4.3	Result	s	78
		4.3.1	Closed Market	79
		4.3.2	Open Market	88
5	The	Interr	play Among Two Stock Markets and the FOREX	92
	5 1	Conta	vion and interdependence	93
	5.2	The E	stended Model	94
	0.2	5.9.1	The sucharge rate cleaning machanism	05
		5.2.1	The exchange rate clearing mechanism	90
		5.2.2	I ne decision making process	96
		5.2.3	Attraction functions	97
	5.3	Results	S	97
		5.3.1	Foreign Exchange Market	98

6	Conclusion	ns 10)8
	5.3.3	The inflationary shock	03
		gether	99
	5.3.2	Two stock markets and the FOREX: putting it all to-	

List of Figures

2.1	Price Clearing Mechanism. The new price p^* is determined by the	
	intersection between the demand and supply curve. The figure is	
	drawn from a simulation	28
3.1	Histogram of the distribution of daily log-returns. The figure shows	
	the data related to a simulation superimposed on the best normal fit.	52
3.2	Survival probability distribution of logarithmic returns. The figure	
	shows the data related to a simulation superimposed on the best	
	normal fit.	52
3.3	Daily time series for prices (top) and returns (bottom) in the case	
	of a single-stock closed market.	53
3.4	$Estimate \ of \ the \ autocorrelation \ function \ of \ logarithmic \ returns \ (top)$	
	and of the autocorrelation of absolute returns (bottom). \ldots .	53
3.5	Dynamics of wealth of the four populations of traders for a simula-	
	tion of 2000 steps	57
3.6	Price variance as a function of tax rate for 10% fundamentalist and	
	10% chartist traders	60
3.7	Daily time series for prices (top) and returns (bottom). \ldots .	62
3.8	Wealth dynamics of the four trader populations for a simulation of	
	2000 steps, for two markets	67

4.1	Mean and standard deviation of price variance as a function of	
	Random*. The percentage of DPT was varied from 0% to 100% in	
	steps of 25%, with $m = 0.8$.	80
4.2	Mean and standard deviation of price variance as a function of m.	80
4.3	Mean and standard deviation of price variance as a function of	
	P(in-debt), with the percentage of DPT random traders set at 50%	
	and $m = 0.8.$	81
4.4	Dynamics of wealth of Random and Random* for a typical simula-	
	tion with $m = 0.8$ and $P(in - debt) = 50\%$.	82
4.5	Dynamics of wealth with trend followers for a typical simulation	
	with $m = 0.8$ and $P(in - debt) = 50\%$	84
4.6	Volatility with a population made of 10% of DPT fundamentalists	
	and of 90% random traders.	85
4.7	Volatility with a population made of 5% of DPT fundamentalists,	
	5% of DPT momentum and 5% of DPT contrarian traders	86
4.8	Volatility with a population made of all types of traders, both DPT	
	and non – DPT	88
4.9	Dynamics of wealth with all eight types of traders for a typical sim-	
	ulation with $m = 0.8$ and $P(in - debt) = 50\%$.	88
4.10	Daily time series for prices (a) and returns (b) with random traders	
	and $\sigma = 10^{-4}$. The dotted line represents the population size	90
4.11	Daily time series for prices (a) and returns (b) with 50% non–DPT	
	and 50% DPT random traders, with $\sigma = 10^{-4}$. The dotted line	
	represents the population size.	91
5.1	Daily time series for euro-dollar exchange rate (top) and returns	
	(bottom)	99

5.2	$Survival\ probability\ distribution\ of\ standardized\ logarithmic\ returns.$	
	The bold stars represent an estimate of the cumulative distribution	
	of returns related to a simulation. The solid line represents the	
	survival probability distribution of the best Gaussian fit	100
5.3	Daily time series for stock prices (top) and returns (bottom). \ldots	101
5.4	Daily time series for euro-dollar exchange rate (top) and returns	
	(bottom)	102
5.5	Wealth dynamics of the four trader populations	102
5.6	Daily time series for stock prices (top) and returns (bottom). The	
	inflationary shock is applied at the end of the step number 1000	105
5.7	Daily time series for euro-dollar exchange rate (top) and returns	
	(bottom). The inflationary shock is applied at the end of the step	
	number 1000	105
5.8	Daily time series for euro-dollar exchange rate (top) and volumes	
	(bottom). The inflationary shock is applied at the end of the step	
	number 1000	106
5.9	Daily volatility for the stock prices between step 500 and 1500 of	
	the simulation. The inflationary shock is applied at the end of the	
	<i>step number 1000.</i>	106
5.10	Cross correlations of returns (top) and of absolute returns (bottom)	
	of the two stock price series. Each point in the horizontal axis	
	represents one month of trading, corresponding to 20 simulation	
	<i>steps</i>	107
5.11	Covariance of returns (top) and of absolute returns (bottom) of the	
	stock price series. Each point in the horizontal axis represents one	
	month of trading, corresponding to 20 simulation steps.	107

List of Tables

3.1	Mean and standard error of volatility in a single market with	
	no tax. The results are multiplied by 10^3	55
3.2	Mean and standard error of volatility in a single market with	
	0.1% tax. The results are multiplied by 10^3 .	58
3.3	Mean and standard error of volatility in a single market with	
	0.5% tax. The results are multiplied by 10^3 .	59
3.4	Mean and standard error of volatility computed for different con-	
	trarian traders percentages, p_c . The total percentage of chartists is	
	always 20%. All values are multiplied by 10^3	61
3.5	$Mean and standard \ error \ of \ daily \ volumes \ in \ a \ single \ market$	
	with 0.0% tax. The results are multiplied by 10^3	62
3.6	$Mean and standard \ error \ of \ daily \ volumes \ in \ a \ single \ market$	
	with 0.1% tax. The results are multiplied by 10^3	63
3.7	$Mean and standard \ error \ of \ daily \ volumes \ in \ a \ single \ market$	
	with 0.5% tax. The results are multiplied by 10^3	63
3.8	Mean and standard error of volatility in market one. The	
	results are multiplied by 10^3	66
3.9	Mean and standard error of volatility in market two. The	
	results are multiplied by 10^3	66

3.	10 Mean and standard error of volatility in market one, with $0.1%$	
	transaction tax. The results are multiplied by 10^3	68
3.	11 Mean and standard error of volatility in market two, with 0.1%	
	transaction tax. The results are multiplied by 10^3	68
3.	12 $$ Mean and standard error of volatility in market one, with $0.5%$	
	transaction tax. The results are multiplied by 10^3	69
3.	13 Mean and standard error of volatility in market two, with 0.5%	
	transaction tax. The results are multiplied by 10^3	69
3.	14 Average daily volumes. Tax levyed on market one only. The	
	results are divided by 10^3	70
4.	1 Mean and standard error of volatility with trend followers and	
4.	1 Mean and standard error of volatility with trend followers and random traders. The results are multiplied by 10^3	83
4. 4.:	 Mean and standard error of volatility with trend followers and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists and 	83
4. 4.:	 Mean and standard error of volatility with trend followers and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists and random traders. The results are multiplied by 10³ 	83 85
4. 4.: 4.:	 Mean and standard error of volatility with trend followers and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists 	83 85
4. 4.: 4.:	 Mean and standard error of volatility with trend followers and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists trend followers and random traders. The results are multi- 	83 85
4. 4.2 4.2	 Mean and standard error of volatility with trend followers and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists trend followers and random traders. The results are multi- plied by 10³ 	83 85 86
4. 4. 4.	 Mean and standard error of volatility with trend followers and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists trend followers and random traders. The results are multi- plied by 10³ Mean and standard error of Volatility with random traders. 	83 85 86
4. 4. 4.	 Mean and standard error of volatility with trend followers and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists and random traders. The results are multiplied by 10³ Mean and standard error of volatility with fundamentalists trend followers and random traders. The results are multi- plied by 10³ Mean and standard error of Volatility with random traders. The results are multiplied by 10³ 	83 85 86 90

Introduction

Financial markets belong to the class of things that sound to be simple, but that are indeed very complicated. They are dynamic systems made up of a large number of economic elements engaged in continuous interactions, which give rise to intricate aggregate regularities and to complex phenomena at the macro level. The main result of the trading activity is price time series, that exhibit many well known empirical properties also known as *stylized facts*. In recent years, the large availability of financial data allowed to deepen the knowledge about price processes and, together with the new developments in mathematics, physics and computer science, contributed to transform finance in a quantitative science.

Researchers faced with the analysis and modeling of financial markets for tens of years. But classical theories, based on a single fully rational representative agent, failed to reproduce all the properties of real markets. Also, they have been able to make only limited progress in resolving many important practical and policy relevant open issues, like those related to the instability of financial markets. In contrast, new behavioural approaches, characterized by markets populated with bounded rational, heterogeneous agents emerged. In recent years, this research field has been combined with the realm of agent-based simulation models, and a number of computersimulated, artificial financial markets have been built. This thesis presents an agent-based computer simulation framework for building theoretical models in economics and finance. In an artificial financial market each microscopic element of the overall system, and each kind of interaction among them, has to be modeled and individually represented. The computer simulation approach allows to track the evolution of each component of the system, and to investigate on the aggregate behaviour and to look for emergent phenomena. This approach has been already applied in other sciences to study complex systems, and its main advantage is that it allows to deal with issues where analytic solutions would be impossible.

The proposed model includes many realistic trading features, and has been validated by showing that the simulated time series exhibit the main empirical properties of real financial markets. This artificial market has been developed using object-oriented software techniques, and is aimed to be easily extended and composed, yielding multi-asset and multi-market simulations.

The thesis is organized as follows. Chapter 1 gives the historical background on the models in economics and finance. It also provides a brief guidance in the development of an artificial market, and it presents an overview of the major statistical properties of real economic time series.

The current version of the simulation framework is the result of an incremental and iteractive process: Chapter 2 summarizes its evolution, from the original system, built on the basic ideas of the "Genoa Artificial Stock Market" model, until now. It also provides some details on the simulation software, and on the verification and validation methodology.

The subsequent Chapters run through again the system evolution, and deeply analyse the different versions. Each chapter introduces one major open issue in economics and finance, then provides details on the specific model that has been developed to study that problem, including the description of its extensions and modifications in comparison with the previous version, and finally discusses the results. In particular, Chapter 3 faces the problem of understanding the potential impact of the Tobin tax on a multi-asset financial market. Chapter 4 analyses the impact of margin requirements and of short-sale constraints on prices and volatility, and their connections with stock market crashes. Chapter 5 is about the interaction between two stock markets located into two different countries, and their influence on the Foreign Exchange Market. Chapter 6 concludes the thesis and suggests questions for the future work.

Chapter 1

Modeling and Analysis of Financial Markets

Financial markets are at the heart of each modern economy. They can be described as evolving complex systems, characterized by the interaction of many simple interacting units. Today, financial markets are continuously monitored, and an enormous amount of electronically stored financial data is available. The result is an explosion of interest in this field, that attracts a large number of researchers attempting to model and forecast financial markets. It is well known that economists and mathematicians have a long tradition in studying financial system, but a growing number of physicists is trying to compete with them in explaining economic phenomena. This fact is confirmed by the emerging *Econophysics* research field, which applies theories and methods originally developed by physicists in order to solve problems in economics. Also, the large availability of financial data allowed to deepen the knowledge about price processes, and many so-called stylized facts have been discovered in price series, e.g., the fat tails of return distributions, the absence of autocorrelation of returns, the autocorrelation of volatility, the peculiar distributions of trading volumes and of intervals of trading and so on. The interest in these complex system leaded to an unprecedented cooperation among researchers in economics, physics, mathematics and engineering. In recent past, people with advanced degrees in these sciences have been employed in Wall Street to developed new financial products and new quantitative models. The result was the beginning of a new multidisciplinary field, called *Financial Engineering*, which relies on mathematical finance, numerical methods and computer simulations to exploit financial opportunities, to make trading, hedging and investment decisions, as well as facilitating the risk management of those decisions.

The traditional economic theory is based on simple and analytically tractable models with a representative, fully rational agent, but this classical approach fail to reproduce all the features described above. Financial markets are systems populated with a large number of heterogeneous agents that interact one other using various strategies and that react to external information trying to forecast the best price for a given asset. In recent years, a new approach, based on heterogeneous boundedly-rational agents, has appeared. The new approach goes beyond the limits of the classical models, and fit much better with the characteristics of real markets. The heterogeneous agent models are usually more complex than those with a representative rational agent, and they can be analytically untractable. So, in the recent literature, a number of agent-based simulation models have been developed, and computational and numerical methods have become an important tool of analysis. In particular, over the last fifteen years many computer-simulated, artificial financial markets have been built. The artificial financial markets usually model a subset of the real macro economy or a very specific financial market, and they are made up of different ingredients such as agent preferences, one or more price determination processes, mechanisms of evolution and learning, and methods to present information to the agents.

Artificial financial markets allow the researchers to conduct experiments, in terms of computer simulations, to test hypotheses and to validate ideas and conjectures. They can be employed to model the complex features of real markets that cannot be studied analytically, and they are an important tool for understanding how real markets works: they can help to analyse the price dynamics, the interactions and the performances of numerous trading strategies, the relationship among various the price clearing mechanisms and the market dynamics, but also the reactions of financial markets to the imposition of taxes and trading restrictions, or the economic links among different markets. This work will provide answers to some of these issues.

1.1 Historical background

Economists have faced the problem of studying and modeling economic systems for hundred of years, but in the second part of the 20th century, finance has witnessed an important revolution. The classical representative rational agent paradigm has been replaced by a large number of agents characterized by heterogeneous behaviours. Also, the increasing power of computers has favored a shift from analytically tractable models with a representative agent to complex systems, that require the implementation of simulation models and use numerical methods as an important tool of analysis. Finally, full rationality has been replaced by bounded rationality (see Hommes; 2002).

1.1.1 Classical Models

The study of economic systems has a very long history, and some concepts and ideas behind the models developed in recent times find their fundamentals in classical economics and finance. In the traditional approach, many simple analytically tractable models have been developed, and the mathematics has been the main tool of analysis. These models makes many assumptions regarding the economy and the individuals in order to keep the analytical tractability, but they are often unrealistic. They are based on the rational expectations theory and on the notion of the representative, perfectly rational agent.

The rationality of agents is one key concepts of economics and finance. In a full rational expectations framework, all agents make use of all available information in determining how to best meet their objectives. The *Rational Expectation Hypothesis* (REH) is a theory in economics originally proposed by Muth (1961) and later developed by Lucas (1972). If the agents try to forecast future variables taking into account all available information, they will not make systematic errors, and the value of the observed variables will be equal to the values predicted by the model, plus a random error.

The rational expectation theory provides the ground to build models based on the notion of *representative agent*, having rational expectations. A representative agent model is such that the cumulative behaviour of all agents might as well be the actions of one agent maximizing her expected utility function. Its origins can be traced to the 19th century, but it was Lucas (1972) in the 1970s who really popularized the representative agent.

The only trace of heterogeneity in the rational expectations framework resides in the fact that the agents may have different utility functions, but it is not heterogeneity of beliefs, because the agents are given all relevant information. Rational expectations provides an elegant and parsimonious way to exclude "ad hoc" forecasting rules and market psychology from economic modeling (Hommes; 2005).

The REH is closely linked with the financial concept of market efficiency. The Efficient Market Hypothesis (EMH) was formulated in 1960's by Samuelson (1965) and Fama (1970). The EMH states that financial markets asset prices reflect all available and relevant information useful for predicting the future value of the assets themselves. In a efficient market, a fully rational agent can process all available information and take optimal positions on the markets. Under the rational behaviour and the market efficiency, Samuelson showed that the price series prior to period t are not useful to predict the prices for periods t + 1 and beyond, because the price in period t already reflects the fundamental information of all past prices. Most of the empirical studies of the 1960s and 1970s found negligible time correlation between price changes, so corroborating the EMH. Under the EMH, the attempt to beat the market is a game of chance rather then skill, and if markets were not efficient, the rational traders would exploit the arbitrage opportunity, and any foreseeable structure would therefore disappear (see Hommes; 2005). In other words, if investors are perfectly rational and markets are efficient, it follows that strategies using past prices to forecast future prices, such as technical trading, cannot be profitable, except by luck. It was also generally accepted that temporary price overreactions are due to adjustments to market news, that spread quickly through the market and are incorporated into prices without delay. In a such context, notions like "investor sentiment" or "market psychology" do not make sense. Finally, in a world populated only by rational agents that share all relevant market knowledge, the trading volume has to be low or zero, because no one can exploit for profit private

positive (or negative) information in advance of everyone else.

1.1.2 Heterogeneous Agent Models

The EMH, the representative agent and rational expectations have provided the theoretical basis for economics and finance during the seventies and large part of the eighties. But during the eighties new findings shook the classical theories to their foundations (for a good review of these developments refer to Hommes (2005)).

In that years, many empirical studies appeared showing evidence against the EMH. One of the most important findings was that price volatility of many financial time series is clustered. It means that price fluctuations are strongly temporally correlated, and that periods of low volatility are interspersed with high volatility periods (see, e.g. Mandelbrot; 1963; Engle; 1982). Moreover, the largest prices movements often happened even though little or no news about economic fundamentals occurred (Cutler et al.; 1989). Several authors claimed that fluctuations in stock prices are too large compared to those due to the underlying economic fundamentals (Shiller; 1981, 1989; LeRoy and Porter; 1981), and that bubbles can be originated by the difference between real prices and fundamentals values (Summers; 1986; Campbell and Shiller; 1988).

As said in Section 1.1.1, the EMH leads to a *no trade* equilibrium, and many no trade theorems have been obtained. For instance, Milgrom and Stokey (1982) stated that if markets are perfectly efficient, then even though some traders may possess private information, none of them will be in a position to profit from it. The no trade assumption is clearly in sharp contrast with the high trading volume of real markets, and represents a point against the efficient market hypothesis. A large number of laboratory experiments rejected the hypothesis that market participants are fully rational (Kahneman and Tversky; 1973). In a famous paper, Smith et al. (1988) report results from several laboratory financial markets. They showed the occurrence of bubbles in experiments despite the fact that information is made public, so that agents can derive the fundamental values of the assets by backward induction.

These empirical findings pointed out the limits of the classical theories, and a new heterogeneous agents approach was born in order to go beyond these limits and to explain the new observed facts in financial time series.

Maybe one of the first attempt to develop an heterogeneous agent model (HAM) is the one by Zeeman (1974), which includes two kinds of traders: fundamentalists and chartists. The model is very simple and try to explain the temporary bulls and bears in financial markets as a special case of the catastrophe theory. The model is very stylized and is lacking in structural foundations, but some basic ideas can be found also in recent models.

One of the most important models is the one proposed by Grossman and Stiglitz, also known as the *noisy rational expectations* model (see Grossman; 1976; Grossman and Stiglitz; 1980). The model try to extend the EMH by addressing the problem of costly information, by allowing the agents to know pieces of information that are not immediately absorbed into the market. Who obtain or analyse information faster can earn positive returns, and the profits obtained can be used to cover the costs related to the acquisition and the analysis of information itself.

The subsequent models departed from the EMH more and more, and new models characterized by groups of heterogeneous agents with bounded rationality and different beliefs appeared.

Beja and Goldman (1980) proposed a model with fundamentalist and

trend follower (chartist) traders. They were among the first to develop a market maker who adjusts prices with respect of the aggregate excess demand. The excess demand both of fundamentalists and chartists is computed using linear rules. They found that if the percentage of chartist is too high the market can become unstable, and concluded that the interaction of different agents with different behaviour could explain some features of the dynamics of prices. Chiarella (1992) considered a non-linear extension of the model, and showed that the non-linear system is characterized by a stable equilibrium, but if the number of chartists is too high the price trend tends to destabilize the system and prices exhibit periodic limit cycles.

The models introduced above represent just some examples of a huge number of studies that analyse the interaction between chartists and fundamentalists, and that can be considered as a branch of the HAM also known as *the fundamentalist and chartist approach*. The reported models are not fully rational, because each group of agents does not know anything about the other. But what happens if there are fully rational agents too? Friedman (1953) has been the first to argue that non rational investors cannot survive the market competition because they will be driven out of the market by rational investors eventually in the process of natural selection. There are many HAM that try to test the so called *Friedman's hypothesis*, for instance those that include two further kinds of traders: *rational traders and noise traders*.

The notion of noise trader was introduced by Black (1986): noise traders are individual who trade on what they think is information, but is in fact merely noise. This idea allowed Black to justify the large volumes of trading activity that occurs in real markets. The activity of noise traders makes it difficult to understand what is noise and what is good information, so rational traders are obliged to look for more information without a break. This behaviour favors large volumes and gives traders the opportunity to earn profits by exploiting their information.

More recently, De Long et al. (1990a,b) provided evidence that noise traders may survive in the long run, and that they may gain more money than rational ones. They found that rational traders perceive the risk introduced by the presence of the other traders and, under certain conditions, they are not able to get control over the dynamics generated by the non-rational traders. It follows that the presence of many categories of agents cannot be considered only a temporary condition, so contradicting the Friedman's hypothesis.

The Wall Street stock market crash in October 1987 fed the interest in financial market models and reinforced the idea that the classical models based on a representative rational agent cannot explain the behaviour of real markets. Also, new empirical studies showed that there is not direct relation between fundamental news and stock price movements (Cutler et al.; 1989), and that the strange behaviour of the US dollar during the mid eighties was absolutely unrelated to economic fundamentals (Frankel and Froot; 1986).

During the seventies and the eighties there were many developments in mathematics and physics, such as chaos theory and complex systems. These concepts stimulated many HAM works of the eighties and of the nineties, because they can be used to model the unpredictable price paths by using simple laws. For instance, the models by Beja and Goldman (1980) and Chiarella (1992) exhibit chaotic dynamics. One influential paper is that by Day and Huang (1990), who proposed a discrete time model with a fundamentalist and a noise representative agent. The model shows complicated deviations from the equilibrium price, that are similar to real stock market fluctuations with chaotic switchings between bull markets and bear market regimes.

During the nineties an impressive number of HAMs have been developed. They explored a wide set of assumptions and proposed new kinds of heterogeneity, in terms of new kinds of trading strategies, learning capabilities, adaptive techniques, and interactions among individuals. However, the most part of these models concentrated on behavioural assumptions while neglecting the market structural assumptions. Structural assumptions are those related to the "structure" of the market, for instance the trading procedures which define the rules of the market and the price clearing mechanisms. Behavioural assumptions are the trading strategies and the roles by which the traders take their decisions LiCalzi and Pellizzari (2002). Raberto (2003) and the survey by Hommes (2005) covers the more analytic ones and those that can be handled by means of simple numerical simulations. Some of them are significative and deserve a special mention because of their role in HAM advancement. Challet and Zhang (1997) proposed a minority game model with N agents who have to choose between two alternatives: the goal is to be in the smallest group, that is the winner one. The model is interesting because it is quite simple and is accompanied by a numerical description and is suitable for analytical solutions. The minority game models share some characteristics with financial markets: The agents have limited resources and rationality, they learn from the performance of past choices, a good strategy today may become bad when others' behaviour changes, and these models can reproduce stylized facts (Challet et al.; 2001). The model by Lux (1997, 1999) and by Lux and Marchesi (1999, 2000) succeeded in explaining four stylized facts simultaneously: prices follow a near unit root process, there are fat tails in the distribution of short term returns, volatility clustering and no autocorrelation of raw returns. The model is based in three populations of agents that can switch strategy in consequence of contagion effects. They stressed the role of the market maker, that adjusts prices according to aggregate excess demand. They addressed the issue of *herding behaviour* in financial markets, that has been also recently studied by Cont and Bouchaud (2000) by means of random graphs and lattices.

Summing up, it is clear that the new heterogeneous paradigm is a growing field, that is providing answers to many issues of financial markets, and that goes beyond the limits of the fully rational representative agent. In a heterogeneous world the rational agent cannot survive because, as observed byArthur (1995) and Hommes (2001), if the world is heterogeneous, the rational agents have to know perfectly the beliefs of all other traders, but it seems quite unrealistic.

1.1.3 Artificial Financial Markets

The heterogeneous models discussed in Section 1.1.2 either are analytically tractable, or can by handled be means of a combination of analytical tools and simple numerical simulations. In order to derive tractable solutions, these models make use of many simplifying assumptions. For instance, they do not track the behaviour of each agent individually, but they group them into populations that can vary in size and behaviour. On the other hand, these simplifications lead to models that are not able to reproduce all the statistical features of financial time series, and the results cannot be convincing and lack of robustness.

The point is that the dynamics of financial markets are not simple, and they go beyond what can be handled analytically. In fact, the financial markets can be described as very large *complex systems*, whose dynamics depend on the behaviour and the interaction of a large ensemble of autonomous traders, on the market structure and organization. There systems exhibit *emergent properties*, that is, properties arising from the interactions of the components that are not properties of the individual units themselves (Tesfatsion; 2006).

In this context, the heterogeneity introduced in the previous section laid the basis for studying and explaining the complex characteristics of financial markets, but it is not enough. The heterogeneity of agents can be expressed in terms of wealth, strategies, learning capabilities, distribution of agents, interactions and so on, and it unavoidably leads to produce analytically intractable models that must be investigated numerically. Also the need to model the market microstructure and to develop realistic price formation mechanism contribute to add complexity to the models.

In order to address these issues, starting from the second half of the nineties some researchers started to develop heterogeneous agent models based on a fully computational approach. Since then, a number of computer-simulated market models have been built (LeBaron; 2006, presents a review of recent work in this field).

Fortunately, the power of computers has increased enormously during the last two decades, and this stimulated the study, the development and the analysis of complex heterogeneous agent models with boundedly rational agents that are based on a complete computational approach. It is worth noting that the new behavioural approaches, characterized by markets populated with bounded rational, heterogeneous agents using rule of thumb strategies, fit much better with agent-based simulation models, and computational and numerical methods have become an important tool of analysis. The fully computational approach expands the realm of investigation in finance, and proposes a methodology to face the various sources of complexity of financial markets. Microscopic simulation allows researchers to study models which take into account the heterogeneity of the agents, and to include the distinguishing features of each investor. As said by Levy et al. (2000), the strength of the microscopic simulation is that one is able to model any imaginable investor behaviour and market structure.

The research literature often refers to *agent-based models* to indicate the subset of heterogeneous agent models that make extensive use of computer simulations. The agent-based models consider financial markets as the result of boundedly-rational micro agents that interact and learn within the microstructure provided by the market rules. Generally speaking, the agent-based approach try to capture the emergence of phenomena from the micro level to the higher macro level represented by the aggregated whole system. For the sake of simplicity, we will use both the terms HAM and agent-based models interchangeably, making no distinction between them.

The Santa Fe Artificial Stock Market (SF-ASM) (Palmer et al.; 1994; Arthur et al.; 1997), is one of the earliest and most influential projects in this set of models. The original idea was to build a financial market with an ecology of trading strategies, and to determine which strategies will survive, and which will fail. The market was to be an evolving system, and new strategies could emerge from a soup of starting strategies, reinforce themselves and maybe survive. The SF-ASM agents are endowed with limited capabilities: they have a collection of rules that guide their behaviour on the basis of the market conditions. They have to test alternatives and to anticipate other agents' expectations, and are obliged to continually form individual, hypothetical, expectational models, test them, and trade trusting the ones that predict best. In other words, the SF-ASM includes a learning and a forecasting system, and agents build their behaviour on prices and dividends by matching specific forecasting rules and knowledge to current market conditions. One of the main goals was to proof that market complexity may be induced by the endogenous evolution of the system, rather than exogenous phenomena. In particular, one objective was to understand if the market converges to a tractable rational expectation equilibrium, and to understand what happens when the market does not converge. Another goal was to analyse the dynamics of learning and the effects on the market equilibrium. Arthur et al. (1997) showed that if the rate of exploration of alternative forecasts is high, the market exhibits a complex regime and a rich psychological behaviour emerges. Periods of technical trading regime appear, where fundamental strategies tend to be punished by the market. The SF-ASM platform has been also extended by other researchers, such as Joshi et al. (1998) who studied the interaction between technical and fundamental trading, and Tay and Linn (2001) who extended the set of classifiers of the SF-ASM by adding a fuzzy logic system. The SF-ASM is a pioneering work that has shown the way forward the creation of artificial financial markets with heterogeneous agent. Also, it suggested that simulated price series can be analysed to check for consistency with the stylized facts of real data. It is worth noting that, though the SF-ASM is able to replicate these facts qualitatively, no attempt is made to quantitatively line them up with results from real financial data.

The experience of the Santa Fe Artificial Stock Market stimulated the development of several other projects. For instance, Basu et al. (1998), at Sandia National Laboratories (SNL), developed an agent-based microeconomic simulation model of the US economy.

Recently, a project for developing an artificial financial market started at

the Artificial Intelligence Laboratory of the Massachusetts Institute of Technology (Chan; 2001). The project faces three key issues: the construction of artificial financial markets with adaptive trading and the analysis of the behaviour of market-making agents, the study of equilibrium conditions, and the development of an artificial market with web access. The Oxford Centre for Computational Finance (OCCF) is a recently funded research centre which has been investigating if and how game theory can be applied in artificial markets to help financial engineers better understanding and managing operational risk. For example, Johnson et al. (2001) developed an interesting application of the minority game to real financial time series.

Izumi and Ueda (1999a,b, 2001) proposed an exchange market model with artificial adaptive agents called A GEnetic-algorithm Double Auction SImulator of TOkyo Foreign exchange market (AGEDASI TOF). The model implements an interesting community of agents able to adapt their beliefs with time, on the basis of information coming from various sources of news. For each agent, the AGEDASI TOF iteratively execute five main steps: perception of prices, prediction of the future rate, submission of orders, rate determination and genetic adaptation of the prediction methods. The results show some interesting features about the emergence of clusters of strategies and of opinion trends.

There exists an impressive mass of scientific literature on the subject of agent-based computational economics: there are books (Levy et al.; 2000; Tesfatsion and Judd; 2006), editorials (Lux and Marchesi; 2002) and a number of works by Hommes (2005), by LeBaron (2000, 2006) and by Tesfatsion (2001a,b, 2002), among the others. This effort supports the idea that heterogeneous agent models, and the microsimulations approach, are a key tool to model financial markets and to reproduce the main statistical properties of financial time series.

1.2 How to develop an artificial financial market

Of course, developing an agent-based artificial financial market requires specialized knowledge and effort. One of the biggest challenges is to answer to a large number of design questions: what types of financial products will be traded? What kind of agents will be used? How agents will interact with each other? What kind of price clearing mechanism will be adopted? And what kind of simplifying assumptions will be necessary? The list could be long and, as observed by LeBaron (2006), there is no or little guidance in this matter. However, the design requirements can be grouped into some macro categories in order to make the decision process easier:

- Assets. Modern finance offers an huge number of financial products. Each of these has some peculiar characteristics, and they can be modeled in different ways. The dynamics of financial markets depend not only on the players who buy and sell, or on the market structure, but also on the assets that are exchanged. The main part of the literature focuses on some major products, such as stocks, bonds, some derivatives and currencies. A detailed description of these products can be found in Bouchaud and Potters (2004).
- Agents. Three typical trader types arising in many heterogeneous agent financial market models are fundamentalists, chartist or technical traders, and noisy traders. Fundamentalists base their investment decisions upon market fundamentals such as dividends, earnings, interest rates or growth indicators. In contrast, technical traders pay no attention to economic fundamentals but look for regular patterns

in past prices and base their investment decision upon simple trend following trading rules. Noise traders act randomly, regardless any specific information of the security. Their presence is sometimes necessary in order to provide liquidity to the market. The majority of the models proposed in the literature are less or more complex variants of these basic ideas.

- Evolution and learning mechanisms. Traders can organize their behaviour in many different ways, not only in terms of kinds of strategy but also in terms of learning processes and adaptive behaviour. For instance, they can share strategies and information, they can learn or imitate, or they can build a network of relationships with other participants of the market. The behaviour of traders is an example of the different sources of complexity of financial markets, as pointed out in the stimulating paper by Pellizzari (2005).
- Price determination. One of the most critical issues is the definition of the method for determining prices. Many methods exist in real markets, and most of them fall into one of four main categories: order book, market clearing, price adjustment and random matching (LeBaron; 2006). The fact that stock exchange markets use different price clearing mechanisms raises the question whether the market architecture significantly affects the price behaviour or not. Using an agent-based artificial exchange, LiCalzi and Pellizzari (2006) showed that different market protocols (batch auction, continuous double auction and dealership) lead to different results in terms of price stability and execution quality.
- Environment. As said by Pellizzari (2005), agents are not living in

a vacuum, but they act in a market environment. The environment establishes the rules of the game not only because it defines a price clearing mechanism, but also because it could levy taxes, regulate the kinds of allowable exchange orders and might have a major role in the process that produces the final aggregate results.

These remarks point out that there are many decisions that have to be taken in order to develop an artificial financial market model. Also, each component adds a degree of complexity to the overall result. The are some guidelines that could help to do the right choices. The first one is: think big, start small and scale fast. A good artificial financial market may become a framework that allows to study complex and large system, with many kinds of agents playing simultaneously in more than one market with different assets. However, the development of such a model is not an easy task, and one could easily get lost in it. The best solution is to outline the general architecture of the system, and then develop the simplest solution that could work, in agreement with the agile philosophy. For instance, one could start producing a model with one kind of assets only, one kind of agents and a simple price clearing mechanism. Then, it is possible to iteratively add complexity to the model. The second suggestion is to make use of a suitable programming language. The agent-based models fit perfectly with the object-oriented programming languages (Tveit; 2001; Gilbert and Bankes; 2002), which allow to model each agent as an instance of a particular class. Also, in order to scale fast is useful to adopt a proper software development process, such as an agile methodology (the most famous one is eXtreme Programming (XP) Beck; 1999; Beck and Andres; 2004), but also a powerful Integrated Development Environment (IDE), capable to assist the developers during the whole of software implementation. Finally, the price time series produced by the

artificial market have to exhibit the same statistical features of real markets, the so called *stylized facts*. But this is not a suggestion, it is the necessary condition that allows to validate the model: it is a must.

1.3 Stylized Facts

It is by now well known that the economic time series of almost all financial assets exhibit a number of non trivial statistical properties called *stylized empirical facts*. No completely satisfactory explanation of such features has yet been found in standard theories of financial markets, but more than fifty years of empirical studies confirm their presence. For a complete discussion about stylized facts and statistical issues see Pagan (1996); Cont (1997); Cont et al. (1997); Farmer (1999); Mantegna and Stanley (1999); Bouchaud (2000) and the interesting paper by Cont (2001). There is a set of stylized facts which appear to be the most important and common to a wide set of financial assets: unit root property, fat tails and volatility clustering.

1.3.1 Unit Root Property

A first order autoregressive process is a stochastic process of the form: $x(t) = \rho \cdot x(t-1) + \epsilon(t)$, where ρ is a coefficient, and $\epsilon(t)$ is a stationary stochastic increment. The term *autoregressive* indicates that the process defines a regression of x on its own past values. If $\rho = 1$ the process is called a *unit root process*. Although the term *unit root process* covers a wide range of processes, the most elementary form is the random walk with iid increments¹ In particular, if $\epsilon(t) \sim iid(0, \sigma^2)$, that is, ϵ is independently and identically

¹Here the terms random walk and unit root will be used interchangeably. With random walk I mean a non stationary process with a unit root.

distributed with mean zero and variance σ^2 , the process is a random walk².

Several statistical procedures have been proposed to test for the presence of unit roots, such as the original Dickey and Fuller (1979) test and the subsequent augmented Dickey-Fuller (ADF) test statistic (Dickey and Fuller; 1981). If $x(t) = \log [p(t)]$, where p(t) is the price of an asset at time t, one is usually unable to reject the null hypothesis $H_0: \rho = 1$ against the alternative hypothesis $H_1: \rho < 1$. If the logarithm of prices follows a random walk process, the future asset prices are unpredictable based on historical observations. Also, the volatility of prices can grow without limits in the long run. These findings fit very well with the efficient market view of asset price determination.

1.3.2 Fat Tails

Logarithmic returns are a measure of the relative fluctuations of prices. They are defined as $r(t) = \log [p(t)] - \log [p(t-1)]$, and are one of the most important variables in finance.

In recent years, considerable attention has been given to the distribution of asset returns. A lot of empirical data on prices and trading volumes is available since the 1990s, and the increased calculation power of modern computer technology has allowed researchers to conduct deep empirical analysis on financial data. The most important finding is that the distribution of returns is non Gaussian and heavy tailed. This result is at the odds with the efficient market hypothesis, that implies that the probability distribution of price returns follows a Gaussian distribution. In particular, the empirical studies generally concur that at weekly, daily and higher frequencies, return distributions consistently exhibit non Gaussian features. On the other

 $^{^{2}}$ If the distribution is a Normal one, then you have the simplest stochastic process: Gaussian white noise

hand, the distribution is usually normally distributed at monthly and longer horizons.

In the early literature, the fat tail phenomenon has been quantified by measuring the fourth moments (*kurtosis*) of the distribution. Kurtosis is the degree of "peakedness" of a distribution, and is defined such that its value is equal to zero for a Gaussian distribution. A distribution with positive kurtosis is called *leptokurtic* and shows a more acute *peak* around the mean and a fat tail. The distribution of the increments of asset prices is clearly leptokurtic, but this measure is not useful for identifying the distribution of price returns. Fortunately, recent literature established that the distribution of returns follows a power-law or Pareto-like tail, with a tail index which is finite, and in the range (2, 5) (often around 3).

1.3.3 Volatility Clustering

Volatility measures the amplitude of price fluctuation of a financial instrument within a specific time horizon. More broadly, it refers to the degree of (typically short-term) unpredictable change over time of a certain variable. Volatility is often estimated by calculating the standard deviation of the price values in a certain time window. In the time series of real stock prices, it is observed that the variance of returns or log-prices is high for extended periods and then low for subsequent extended periods: this phenomenon is called volatility clustering. This fact was observed by Mandelbrot (1963), who claimed that *large changes tend to be followed by large changes, of either* sign, and small changes tend to be followed by small changes. The clustering of volatility is also proved by the power-law decay of the autocorrelation function of the daily volatility, typically with a small exponent in the range $\gamma \in [0.1, 0.3]$ (Mantegna and Stanley; 1999; Liu et al.; 1999). Volatility clustering is strictly correlated with two more dependence properties of returns financial time series: the absence of linear autocorrelation and the presence of non linear autocorrelation.

Absence of autocorrelation in raw returns

The autocorrelation of raw returns is often insignificant, except for very small intraday time scales. It is well known that the autocorrelation decays to zero in less then fifteen minutes for all real price time series (Cont; 2001). It seems that this property could give support to the EMH, because one can consider returns as independent variables. However, the absence of linear autocorrelation is not sufficient to exclude that there is some time dependence in price returns.

Slow decay of autocorrelation in absolute returns

The autocorrelation of absolute returns and of their square, display a positive and slowly decaying autocorrelation, ranging from a few minutes to a several weeks (Cont; 2001). This phenomenon can be considered as a quantitative manifestation of the volatility clustering itself, and suggests that burst of volatility can persist for periods that range from hours to days, weeks or even months.

Chapter 2

A Framework for Financial Market Simulation

2.1 GASM Early History

The GASM was born in the early 2000's at the University of Genoa. The original project is described in Raberto et al. (2001), and the acronym means "Genoa Artificial Stock Market". The name was devoted to the project's birthplace, that in the Middle Ages was a major financial centre, where I.o.u. and the derivatives were invented (Briys and de Varenne; 2000).

The first release of GASM was an artificial financial market with heterogeneous agents that traded on a single asset. The agents had only limited financial resources and adopted a simple trading strategy: they were zero intelligence traders and issued random orders, constrained by their resources and past price volatility. The price formation process was a clearing house, a mechanism that determines the clearing price by crossing the demand and the supply curves given by the current limit orders. These ingredients were sufficient to build an artificial market able to reproduce the main stylized facts of financial markets: volatility clustering and fat tails in the distribution of price returns.

Since then, the GASM has been extended and a number of works has been published (see, e.g. Marchesi et al.; 2003; Raberto; 2003; Raberto et al.; 2003; Cincotti et al.; 2003, 2005). The project is being jointly developed by Genoa and Cagliari Universities since 2005, and the ultimate goal of our work is to develop a general framework for financial market simulation. First, we re-engineered the original model and the software system, and then we extended its features and functionalities in order to address some open issues in financial markets.

2.2 The Original Model

This section presents the main characteristics of the original GASM we used to develop the present release of the simulation framework. In the basic model, only one risky asset was traded in exchange for cash. The agents had limited resources and there were four different trading strategies. The price formation process was based on the intersection of the supply and demand curves. Note that the original GASM includes many more features than those described in this Section, but here are discussed only those that we used to lay the foundations of the new model.

2.2.1 Agents

Traders were segmented into four groups: random, fundamentalists, momentum and contrarian traders. At each simulation step t, the generic i - thtrader issues an order with probability $p_o = 0.02$. The orders are limit orders, whose limit price and size depend on the specific trader's strategy.
Random Traders

Random traders are characterized by the simplest trading strategy. They are traders with zero intelligence, and issue random orders. Random traders represent the bulk of traders who trade for reasons associated with their needs and not with market behaviour. Zero intelligence traders are described in many papers, following the pioneering work by Gode and Sunder (1993). If a random trader decides to issue an order, it may be a buy or sell order with probability 50%. The order amount is computed at random, but cannot exceed the trader's actual cash and stock availability. In particular, the limit price $l_i^b(t)$ of a generic buy limit order $b_i(t)$ issued by the i - th agent at step t, is computed multiplying the current price p(t) of the stock by a random number drawn from a Gaussian distribution $N(\mu, s_i)$, as shown by equation 2.1a. The limit price $l_i^s(t)$ of a sell order $s_i(t)$ is computed fairly symmetrically, as shown by equation 2.1b.

$$l_i^b(t) = p(t) \cdot N(\mu, s_i) \tag{2.1a}$$

$$l_i^s(t) = p(t)/N(\mu, s_i)$$
 (2.1b)

The mean μ is set at a value equal to 1.01 in order to have a spread between the limit prices of sell/buy orders (Raberto et al.; 2003). The standard deviation of this distribution, s_i depends on the historical market standard deviation, $\sigma_i(\tau_i)$, computed on a past price series whose length, τ_i , depends on each trader memory, according to equation 2.2:

$$s_i = k * \sigma(\tau_i), \tag{2.2}$$

where k is a constant that is usually set in the range between 3 and 4 and τ_i is randomly drawn for each trader from a uniform distribution of integers from 10 to 100 (Raberto et al.; 2003).

Fundamentalist traders

Fundamentalists strongly believe that each asset has got a fundamental price, p_f , related to factors external to the market and, sooner or later, the price will revert to that fundamental value. The fundamental price is the same for all fundamentalists. If a fundamentalist decides to trade, she places a buy (sell) order if the last price p(t-1) is lower (higher) than the fundamental price p_f . Fundamentalists' order limits are set exactly equal to p_f , and their size (in stocks for sell orders and in cash for buy orders) equals a random fraction of the current amount of stocks or cash owned by the trader.

Momentum Traders

Momentum traders are trend followers. They play the market following past price trends, and strictly rely on price information. Momentum traders buy (sell) when the price goes up (down). They represent, in a simplified way, traders following technical analysis rules and traders following a herd behaviour. A time window τ_i is assigned to each momentum trader at the beginning of the simulation through a random draw from a uniform distribution of integers in the range 10 to 50 days. If the momentum trader issues a limit order, the limit price $l_i(t)$ is set at the stock's price of the previous time step plus an increment (decrement) proportional to the price difference computed in the time window τ_i , as shown in equation 2.3.

$$l_i(t) = p(t) \cdot \left[1 + \frac{p(t) - p(t - \tau)}{\tau p(t - \tau)} \right]$$
(2.3)

If the momentum trader issues a sell order, the order size is a random fraction of the number of shares owned by the trader herself. In the case of a buy order, the trader employs a random fraction of her cash, and the number of demanded stocks is the ratio between that fraction and the limit price $l_i(t)$.

Contrarian traders

Contrarian traders are trend-followers too, but they speculate that, if the stock price is rising, it will stop rising soon and fall, so it is better to sell near the maximum, and vice versa. A time window (τ_i) is assigned to each contrarian trader at the beginning of the simulation in the same way as for momentum traders. The contrarian trader's order limit price and quantity are computed in the same fashion as the momentum traders, but in the opposite direction.

2.2.2 The Price Clearing Mechanism

The price formation process is based on the intersection of the demand and supply curves. The limit orders are all collected after each simulation step, and the market is cleared by crossing the supply and demand curves given by the current limit orders. The orders that are compatible with the new price are executed, while the ones that do not match the clearing price are discarded. The original algorithm, described in Raberto et al. (2001), is very simple and direct and can be summarized as following.

Let be U the number of buy orders and V the number of sell orders issued by the traders at a certain time step $t = t_h$. Also, let $\{a_u^b(t_h), b_u(t_h)\}$, u =1, ..., U, be the data associated to the U buy orders. In each pair, the quantity of stock to buy, $a_u^b(t_h)$, is associated with its limit price, $b_u(t_h)$. As regards the V selling orders, they are represented by the pairs: $\{a_v^s(t_h), s_v(t_h)\}$, v = 1, ..., V. Here the quantity to sell is $a_v^s(t_h)$, while its associated limit price is $s_v(t_h)$. The cleared price, p^* , is determined by intersecting the two functions:

$$f_{t_h}(p) = \sum_{u|b_u(t_h) \ge p} a_u^b(t_h) \qquad (Demand \ curve) \tag{2.4}$$

$$g_{t_h}(p) = \sum_{v \mid s_v(t_h) \le p} a_v^s(t_h) \qquad (Supply \ curve) \tag{2.5}$$

The orders matching the new price p^* , i.e. buy orders with maximum price lower than or equal to p^* , and selling orders with minimum price higher than or equal to p^* , are executed. Subsequently, the amount of cash and assets owned by each trader are updated.

Figure 2.1 shows the shape of the demand and of the supply curves in a case derived from a simulation. The resulting clearing price p^* is determined by the x-axis coordinate of the intersection point between the two curves. Note that in this example the unbalance towards buy orders causes an increase of price.



Figure 2.1: Price Clearing Mechanism. The new price p^* is determined by the intersection between the demand and supply curve. The figure is drawn from a simulation.

It is worth noting that in a closed market the number of shares sold must be equal to the number of shares bought. If $f_{t_h}(p^*) < g_{t_h}(p^*)$, only $f_{t_h}(p^*)$ stocks will be traded. In order to equilibrate the number of stocks exchanged, $g_{t_h}(p^*) - f_{t_h}(p^*)$ stocks offered for sale at a limit price p^* or more are randomly chosen and discarded from the corresponding sell orders. Symmetrically, if $f_{t_h}(p^*) > g_{t_h}(p^*)$, then $f_{t_h}(p^*) - g_{t_h}(p^*)$ stocks demanded for buying by traders at a limit price less or equal to p^* will be randomly discarded.

2.3 The Reengineering Process

The main goal of this research has been the development of a general framework for financial market simulation. The project made use of the experience gained with the original model, and improved its architecture and extended its functionalities in order to build a flexible and easily modifiable system, that could be rapidly adapted and extended to study, model and analyse the plenty of open issues of real financial markets.

The current version of the model includes both structural and behavioural assumptions. Structural assumptions are indicative of those trading mechanisms which define the market rules, while behavioural assumptions refer to trading strategies and the rules used by traders for making their decisions (LiCalzi and Pellizzari; 2002). The software system obtained is flexible and easily modifiable. The software framework is able to model the impact of transaction taxes on traders' behaviour and wealth, the effects of short selling and margin trading, the interplay of stock and option market, the interplay between stock markets in different currencies, with an exchange market in between. In fact, this framework has been developed in subsequent steps, each one aimed to extend and generalize the previous one:

- 1. Its first version was able to model and simulate a stock market populated by different kinds of autonomous heterogeneous agents. The agents have finite cash and stock amounts; they issue buy/sell limit orders basing on their behaviour and their constrained budget; both cash and asset initial endowments are obtained applying a given law, which can be uniform (all agents have the same initial endowment), or can be a Zipf's law (agent's initial endowments are distributed according to a power law, thus with big differences among traders). In this version of the artificial stock market there was one stock, traded in exchange for cash; the stock pays no dividend, and there are no transaction costs or taxes. The kinds of trader behaviour implemented in this version are: (i) random traders, who trade at random; (ii) fundamentalist traders, who pursue a fundamental value of the stock; (iii) chartist traders, who "follow the market", speculating that if prices are increasing they will continue to go up, and that if prices are decreasing, they will continue to drop; (iv) contrarian traders, who act in the opposite way than chartists. In this case, the proposed model exhibited the key stylized facts of financial time series and was able to simulate the long-run wealth distribution of the different population of the agents.
- 2. A second version of the framework added the possibility to introduce taxes on transactions, and to open the market, adding or subtracting cash to or from traders at given simulation steps, and with various possible strategies. It was used to study the introduction of transaction taxes both in a closed and in an open market (with cash inflow). Market dynamics and the traders' behaviour were studied, and in particular the distribution of wealth among different kinds of traders.
- 3. In the third version we added to traders the possibility to go short, both

in stock and cash. Also, both trade margin requirements and short sale restrictions were added. We used this version to study the effects of this kind of trading on daily price volatility and on traders'long-run wealth distribution.

- 4. A subsequent version of the framework had the ability to simulate the interactions between stock markets and a foreign exchange market. We enabled the framework to simulate two stock markets with different currencies, giving traders the option to operate in one stock market at a time, and to switch to another one if they chose to do so. A third FOREX market was also simulated, to manage currency exchange. In this case, the main purpose of the simulation was to analyse the interaction between two stock markets located in two different countries, and their influence on the FOREX market.
- 5. The last version of the framework was made able to simulate a stock option market, and the underlying stock market. The main goal is to analyse the impact of the stock option trading on the market of the underlying security. In this case the software system has been extended introducing a new kind of trades security, the option, and two new kinds of traders, one representing traders operating in the option market, and the other representing the issuer of options (a bank). The option we model is an European option, which gives the right to buy or sell another financial asset (underlying security) at a specified expiration date for a strike price. There are two types of options, call option and put option. The software framework allows the owner to exercise an option at its expiration date, of course only if it is in the money. The option trader can buy/sell stocks in the stock market, and can buy stock options in the option market. The bank issues options, and

cover itself in the stock market when option traders exercise their *in the money* options when they expire. The main goal of this version of the framework is to analyse whether the introduction of options have an impact on underlying asset volatility, and on wealth distribution of traders.

The first version of the model includes the main characteristics of the original GASM described in section 2.2. It was a proactive work, because we deeply analysed each component of the simulation model and we improved it by modifying some equations, by updating the trading process and re-tuning all the parameters. This work has been necessary to make the whole system more flexible and easily modifiable. The resulting model is more robust than the original one both in terms of changes in the parameters and in terms of changes in the composition of the population of traders. We validated the new model by showing that it exhibits the main stylized facts of financial markets, as found by using the original one. The second and the third version of the imposition of different rules and regulations on markets. The fourth and the fifth versions explore new kinds of financial markets, beyond the common case of the stock markets, and their mutual interaction.

Since that the current version of the framework is the result of an incremental and iterative process, we will introduce its features following the same approach. In particular, our research originated from some policy-oriented questions that require models with realistic behaviour, with agents capable of reacting to institutional changes, such as the imposition of a Tobin tax on the market. As said above, this topic leaded us to the development of the second version of the framework. In chapter 3 we introduce the details of the model and the results obtained with that version of the framework. For the sake of brevity, it will include the results we found with the first version of the model. Chapter 4 describes the improvements made to the model in order to explore another critical question in financial literature: the impact of margin requirements and of short-sale constraints on prices and volatility, and their relationship with stock market crashes. Finally, Chapter 5 is about a further extension of the model which allowed us to analyse the interaction between two stock markets located into two different countries, and their influence on the FOREX.

This structure has to main advantages: the first one is that we introduce the model details gradually, simplifying the treatment and the exposition of the results. The second one is that each Chapter could be read separately as a case study, without knowing anything of the details of the other Chapters.

Each one of these three Chapters is structured following the same pattern: the first section describes the historical background and the motivations that leaded to each version of the framework. Then there are details about the model itself, and the description of its extensions and modifications in comparison with the previous version. Finally, we discuss the simulation process and the results we found.

Note that this thesis describes the results we found with versions from one to four, while the last one will be skipped because it is in progress and is the result of a joint work with other researchers from our group. Details on the fifth version of the model have been presented in 2006 MDEF Conference (Ecca et al.; 2006), and then summarized in a paper submitted to a special issue of the Computational Economics Journal.

2.4 Simulation Software

In this section we give a brief account of the software engineering approach used to design and develop the simulator that implements the market model presented here. The framework is conceived to be a system in evolution, easy to modify and to extend. In order to achieve this goal, we used an Object Oriented language and adopted some practices from the Extreme Programming (XP) (Beck; 1999; Beck and Andres; 2004) software development methodology. XP is probably the most famous Agile Methodology for software development. It can be shortly described as a set of practices and values which encourage people to do the simplest things that could work, rejecting the complexity and ceremony of traditional approaches. XP is an incremental and iterative software development process, which enables to build the software system step by step and to release new features, when required, as soon possible. XP does not try to plan the development in advance, but is able to adapt to requirement changing. It is based mainly on coding, testing and refactoring, not on up-front analysis and design. A detailed description of XP lies outside the objective of this report, but you can find a broad literature on this topic.

A key feature of the software framework we developed is that all its modules are provided of automatic tests (*Unit Tests*), so each system class has a corresponding test class. All unit tests are grouped in a "*test suite*", and this is run very frequently and in an automatic fashion. In this way, every change made to the software, in order to extend its features, to fix a bug or to improve its structure (refactoring), is followed by running the tests. If the change introduced a bug, or undesired side effects, it is very likely that some test fails, immediately revealing the problem and allowing to fix it. This practice leads to less time spent in finding the code responsible for errors, and in a higher overall quality of the system.

The software framework has been developed using Smalltalk language, and specifically Cincom VisualWorks vr. 7.4, which is freely available for non-commercial applications. Smalltalk is a language fully Object Oriented, with terrific productivity, enabling to develop complex systems and to make substantial modifications to them very quickly, not jeopardizing quality. As regarding performance, Smalltalk is an interpreted language, and thus, like Java, less speedy than a compiled language like C or Fortran. However, it is enough for our purposes, and there is not any need to trade Smalltalk flexibility for further speed. In any case, translating the framework into a more popular OO language like Java would be straightforward, while it can be possible also to port it to C language.

2.4.1 Verification and Validation

One of the most critical issues in developing software simulation models is the verification and validation of the model itself. Unfortunately, there is no a well defined technique or process to certify the "correctness" of a model. Model verification usually refers to techniques used to ensure that the computer programming and implementation of the theoretical model are correct. The object-oriented design techniques and program modularity help to guarantee the correctness of the simulation software (Sargent; 1994). Our simulation software has been developed following some practices from XP (see section 2.4), and has a modular architecture that allowed us to group together functionally-similar classes into packages with clear behaviour and well defined interfaces. Also, each module is supplied with a corresponding test class that ensure the correct implementation of each operation.

Model validation consists in understanding if the computerized model

within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model (Sargent; 1994). The validation of economic models is a largely unexplored topic. Artificial financial markets can be characterized by a large number of parameters in order to fit any kind of real data, but this approach usually leads to complicated models, sometimes to the point to build intractable systems, or at least models extremely difficult to calibrate. On the other hand, if the framework is too simple, it becomes analytically tractable, such those proposed by traditional economic theories. In Chapter 1 we have already shown that these kinds of models cannot reproduce all the features of real financial markets. A rule of thumb is to keep the model as simple as possible, leaving out all unnecessary components.

The problem of validation can be addressed with the requirement that the model exhibits the main statistical properties of financial time series, both using different data from real markets and using various time horizons. In Section 1.3 we showed that there is a set of stylized facts that is common to a wide set of financial assets: unit root property, fat tails and volatility clustering. In the following Chapters we will show that our model exhibits all these statistical properties. A further kind of validation is the conceptual model validation. Its aim is to understand if the conceptual model is correct and reasonable for its purpose. We performed it by presenting the model and the results at many international conferences, and submitting papers to economics journals (see, e.g. Mannaro et al.; 2005; Ecca et al.; 2006; Mannaro et al.; 2006; Setzu and Marchesi; 2006). This activity allowed us to collect meaningful suggestions and feedback on the model and on its developments.

Chapter 3

Assessing the Impact of Tobin-like Transaction Taxes

Although the analysis of policy measures to curb speculative activity in financial market attracts public attention and raises intense policy debates, the literature on the effects of measures like the introduction of a Tobin tax is surprisingly sparse. Part of the silence of economic theory on this issue is due to the fact that it is not clear why volatility rises and falls, and policies directed at reducing it are unlikely to succeed and may also have harmful effects.

This chapter presents a study on the effects of a transaction tax on one and on two related markets. In each market it is possible to levy a transaction tax. In the case of two markets, each trader can choose the market where to trade, and an attraction function is defined which drives their choice, based on perceived profitability. For the sake of brevity, this chapter joins together the first two versions of the artificial financial market framework (see Section 2.3).

3.1 Motivations

The deep financial crises over the past decade, starting from the Mexican pesos crisis in 1994 to the Argentina one in 2001, raised serious doubts as to the ability of free markets to reflect the "true" value of a specific currency. In fact, too many speculative activities can produce a strong bias in exchange rates and create a monetary crisis, or at least amplify its effects. Many observers claim that a tax on currency transactions may prove a powerful tool for penalizing speculators and stabilizing markets. For these reasons, in recent years there has been an ongoing interest in the idea advanced by some economists (among whom the most famous is James Tobin; 1978) to levy a small tax on currency transactions.

Over the last thirty years the volume of foreign exchange trading has increased hugely. In 1973, daily trading volume averaged around \$15 billion; today, it averages \$ 1.9 trillion (BIS; 2005). Moreover, 90% of the trading volume concerns short-term transactions. In general, economists believe that most short-term transactions are of a speculative nature, and many considered them to be a source of market volatility and instability. Instead, medium or long term transactions are usually related to real investments.

In 1936, Keynes in *The General Theory of Employment, Interest, and Money* (Keynes; 1936) asserted that the levy of a small tax on all stock exchange transactions should contribute to reducing instability in domestic stock markets. According to Keynes, this tax should discourage speculators from trading, resulting in lower price volatility of the taxed asset.

In 1978, the Nobel Prize Laureate in Economics James Tobin (1978) proposed the levy of a small tax (0.1%) on all foreign exchange transactions. This would penalize short-term speculators but not long-term investors, favoring market stability. Later, several authors (see, e.g. Palley; 1999; Baker; 2000; Felix and Sau; 1996; Frankel; 1996; Kupiec; 1995) proposed a similar solution for other kinds of securities.

On the other hand, some economists disagree with Keynes and Tobin's views. Friedman (1953) challenged these theories arguing that speculative trading could stabilize prices.

There are only a few empirical analyses on the effects of transaction taxes on price volatility. Umlauf (1993) studied Swedish stock market data and showed that the introduction of a Swedish tax increased the volatility of stock prices. Its worth noting that the tax level was set at 1% in 1984 and at 2% in 1986: such values are far too high compared with the percentage proposed by Tobin.

Habermeier and Kirilenko (2001) analysed the effects of transaction costs and of capital controls on markets, and showed that they can have negative effects on price discovery, volatility and liquidity, reducing market efficiency. They produced evidence that the Tobin tax increases market volatility by discouraging transacting, thereby reducing market liquidity.

(Palley; 2003) argues that the Tobin tax is good for financial stability, and that total transaction costs are not necessarily increased by its imposition. Actually, transaction costs could change the composition of traders, precluding short-term investors from the market. It leads to a reduction in volatility and consequently in total transaction costs.

Aliber et al. (2003) demonstrated that a Tobin tax on Foreign Exchange Transactions may increase volatility. They constructed the time series of monthly transaction costs estimates, volatility and volume, for four currencies (the British Pound, the Deutsche Mark, the Japanese Yen and the Swiss Franc) for the period 1977 to 1999. They showed that volatility is positively correlated with the level of transaction costs, while trading volume is negatively correlated. Their results suggest that an increase in transaction costs leads to a decrease in trading volume. Therefore, the effect of the tax on volatility is exactly the opposite of what the proponents of the Tobin tax would like to have seen. On the other hand, the findings of Aliber et al. (2003) were strongly criticized by Werner (2003), who argued that the direction of causality between tax and volatility/volumes may be just the opposite.

In The effectiveness of Keynes-Tobin transaction taxes when heterogeneous agents can trade in different markets: a behavioural finance approach Westerhoff (2004a) developed a model in which rational agents apply technical and fundamental analyses for trading in two different markets. Their model shows that, if a transaction tax is imposed on one market, speculators leave this market, making it less volatile. Therefore, their model confirms Tobin's hypothesis.

3.2 Model Description

The model is made up of an economy with two stock markets, each trading an asset with similar characteristics, as regards prices dynamics and traders' behaviour.

Each trader is modeled as an autonomous agent, and each is given a given amount of cash and assets. The simulation software (see Section 2.4) enables to track the traders' portfolio, the price series history and the orders issued by each trader for each time step. A time step is conventionally one day in duration.

First we examined the dynamics of a single market, both without and with a transaction tax of 0.05% to 0.5%. Then we considered the case of two markets, examining market trend without tax, and then the effects of

introducing the tax first in one market, and lastly in both markets.

At each time step, each trader trades only within one market. Before trading takes place, each trader, in accordance with an attraction function based on expected gain, may decide to leave one market, switching to the other.

The trader model defines the basic behavioural rules for each kind of trader. Each kind of trader is tuned setting the values of some parameters, in such a way that the resulting price series show the well-known "stylized facts", and price volatility is similar to that found in real markets. Each kind of trader is provided with an "activity" parameter that roughly controls the activity of the trader, and her reactivity to the markets, thus influencing the trader's contribution to price volatility. After many trials, we were able to introduce a parameter k common to all kinds of traders – an increase in k leads to an increase in volatility and in volumes.

We concentrated our study on the effects of different compositions of the populations behaviour on taxed (and non taxed) markets. The price clearing mechanism we used is the same in all simulations, and is "neutral" under this respect. Other works analysed market dynamics using different market mechanisms and different trade behavioural rules in terms of stylized facts and of allocative efficiency (see, e.g. Bottazzi et al.; 2005).

We studied the case of a single market, to assess the impact of a transaction tax on price volatility and traders' wealth. Then, we studied two related stock markets, to assess the impact of levying a tax on one of them, and then on both.

3.2.1 The decision making process

The proposed model includes N traders having four different kinds of behaviour: random, fundamentalist, momentum and contrarian. At each simulation step, a trader can issue orders with a given probability, which we usually set at 10% for every trader. In the case of two markets, each trader chooses the most attractive market, according to her attraction function. The behaviour of the agents is based on the equations described in Section 2.2. The main limit of the original model was that in order to obtain a good price process the number of non random traders has to be kept very small. As described in Raberto et al. (2003), random traders are a "thermal bath", and the number of chartist and fundamentalist traders is always less than 1%. Also, the probability that an agent will issue an order is small and equal to 2%. We upgraded the and improved the agents' model so that the total number of agents which use a different trading strategy from the random one can reach the 30% of the total number N of agents, without influencing the overall price process. Also the probability that an agent issues orders can be increased over 5-fold. This result allowed us to deeply analyse the interactions among the various kinds of populations, in a more realistic way.

Random traders

Random traders (type R) are zero intelligence traders. We modeled them using the equations described in Section 2.2, but we performed a sensitivity analysis of each parameter to calibrate them. We set the window length τ_i used for random traders to a value randomly chosen for each trader between 2 and 5, while the value of k was set at 1.9. These values differ somewhat from those typically used in past simulations with the original model, that had a longer time window, and an higher value of k. By so doing, we increased the feedback of price volatility on trader's behaviour. In this way, we obtained more realistic price statistical behaviour in terms of stylized facts, varying the trader population and levying various tax percentages.

Fundamentalist traders

Fundamentalists (type F) strongly believe that each asset has got an intrinsic fundamental value p_f . Fundamentalists' order limits are set toward p_f , and their size (in stocks for sell orders and in cash for buy orders) equals a fraction of the current amount of stocks or cash owned by the trader. This size is proportional to a term q shown in equation 3.1, where k is the same k used for random traders in equation 2.2.

$$q = k \cdot \frac{|p(t) - p_f|}{p_f} \tag{3.1}$$

When a transaction tax is levied, these computations are performed increasing (or decreasing) the current price of the tax value.

Momentum traders

Momentum traders (type M) are trend-followers. If the momentum trader issues a buy (sell) order, the limit price l_i is calculated as in the original model, as shown in equation 3.2. The time window τ_i is draw from a uniform distribution of integers in the range 2 to 10 days. The expected increment (or decrement) of the price is divided by the window length, and then multiplied by the same k used for random traders in equation 2.2. In this way, the trend is always computed proportionally to an estimate of the derivative of prices.

$$l_i = p(t) \cdot \left[1 + k \cdot \frac{p(t) - p(t - \tau)}{\tau p(t - \tau)} \right]$$
(3.2)

If a transaction tax is levied, the current price p(t-1) is adjusted adding (or subtracting) the tax to (from) it, to account for the tax effect.

If a momentum trader decides to sell the quantity of assets that s/he can sell q_i^s cannot exceed the amount of assets $a_i(t)$ owned by the trader *i*. If a momentum trader decides to buy, the maximum purchasable quantity q_i^b is limited by the cash $c_i(t)$. Both q_i^s and q_i^b are computed proportionally to the absolute value of an estimate of the derivative of prices, as shown in equations 3.3 and 3.4

$$q_i^s = a_i(t) \cdot U(0,1) \cdot \left[1 + k \cdot \frac{|p(t) - p(t-\tau)|}{\tau p(t-\tau)} \right]$$
(3.3)

$$q_i^b = \frac{c_i(t)}{p_i(t)} \cdot U(0,1) \cdot \left[1 + k \cdot \frac{|p(t) - p(t-\tau)|}{\tau p(t-\tau)} \right]$$
(3.4)

where U(0,1) is a random draw from a Uniform Distribution between 0 and 1.

Contrarian traders

The contrarian (type C) trader's order limit price and quantity are computed in the same fashion as the momentum traders, but in the opposite direction. The transaction tax is dealt with in the same way as for momentum traders.

Attraction functions

In the case study of two markets, at each simulation step (t), the trader decides in which market she prefers to trade by evaluating an attraction function for both markets.

Let $A_1^{T,i}(t)$ and $A_2^{T,i}(t)$ be the attraction functions for the i-th generic trader of type T for the first and the second market, respectively. At each

simulation step t, the i-th trader chooses SM_1 with probability given by equation 3.5a, and SM_2 with probability given by equation 3.5b.

$$\pi_1^i(t) = \frac{A_1^{T,i}(t)}{A_1^{T,i}(t) + A_2^{T,i}(t)}$$
(3.5a)

$$\pi_2^i(t) = 1 - \pi_1^i(t) \tag{3.5b}$$

The attraction functions have been designed taking into account the characteristics of each sub-population of traders.

In most simulations, the trader populations of two markets do not differ significantly – no more than a few percentage points. However, in about 1-2% cases, it may happen that one market becomes too attractive compared to the other, triggering an avalanche of traders and leaving empty – or almost empty – the other market. To avoid this divergent behaviour, we constrained the values of the probability function $\pi_1^i(t)$ to a minimum set at 0.3. This value is somewhat arbitrary, but it is sufficient to obviate the problem completely, without introducing any significant side-effect in the simulations.

As regards attraction functions, they have been designed taking into account the specific characteristics of various kinds of traders.

Random traders represent the bulk of traders operating in the market for personal reasons, or with no specific trading strategy. When faced with the possibility of operating in one of two markets, they naturally tend to prefer the less volatile one. Moreover, they also tend to avoid the market with higher tax rate. In our model, at each simulation step random traders choose randomly to buy or sell, with equal probability. If a random trader decides to sell, her attraction function reflects the considerations made above, and is shown in equation 3.6.

$$A_{j,sell}^{R,i} = e^{\sigma_j^2(\tau_i)} (1 - tax_j)$$
(3.6)

The superscript R denotes the random trader, j denotes the j-th asset, and $\sigma_j^2(\tau_i)$ represents the volatility of the returns computed in the time window τ_i specific for each trader. The exponential term ensures that random traders prefer to sell in a volatile market. The $(1 - tax_j)$ term reduces the attraction of a taxed market, being tax_j the transaction tax imposed in j-thmarket. For instance, if the tax is 1% in market j, the term tax_j is set at 0.01.

If a random trader decides to buy, she performs this action in a less volatile market with a higher probability. So, the probability that a random trader buys in a less volatile market is equal to the probability that a random trader sells in a more volatile one. The attraction function is given by equation 3.7.

$$A_{j,buy}^{R,i} = e^{-\sigma_j^2(\tau_i)} (1 - tax_j)$$
(3.7)

Fundamental analysis requires a deep knowledge of the market. Fundamentalists thus tend to concentrate on a limited number of markets (Westerhoff; 2004b). In our model, each fundamentalist issues orders in one market only, where she is more knowledgeable, so for each of them the attraction function of one market is one, and that of the other market is zero. The fundamentalist traders' population is equally divided between the two markets, as well as their total initial wealth.

Momentum and contrarian traders are trend-followers, so they choose the market depending on the trend of past prices. Basically they prefer the market with the highest trend, computed in their time window τ_i . They also take into account the transaction tax rate, in the same way as random traders. These choices are reflected in the attraction function reported in 3.8.

$$A_j^{M,i} = A_j^{C,i} = e^{\frac{|p_j(t) - p_j(t - \tau_i)|}{\tau_i p_j(t - \tau_i)}} (1 - tax_j)$$
(3.8)

The exponential term ensures that the attraction functions will be always ≥ 1 so that equations 3.5a and 3.5b will not diverge.

3.2.2 Price clearing mechanism

The price clearing mechanism of each market is based on the intersection of the demand-supply curve. We adopted the original algorithm described in Chapter 2.

3.2.3 Financial Resources

Each agent owns a finite amount of financial resources, that is cash and stocks. The simulation software is able to keep track of the traders' portfolio, and the decisions of the individual are influenced by their limited budget.

Traders' initial endowment, both in cash and in stocks, follows a Zipf's law. This law usually refers to the frequency of an event relative to it's rank. George Kingsley Zipf (1949) found that the frequency of use of the English words in texts decays as a power law of its rank. The frequency f(i) of the i-th most common word is given by $f(i) \sim i^{-\beta}$, where $\beta \simeq 1$. A power law $y = Cx^{-a}$ can be expressed by the formula: $log(y) = log(C) - a \cdot log(x)$, that is a straight line with slope -a on a log-log plot. A power law decay means that small occurrences are very common, but large ones are extremely rare. It is worth noting that this regularity is sometimes also referred to as Pareto. Pareto was interested in the distribution of income. Let be X a random variable, X is said to follow a Pareto law if $P(X \ge x) = 1/x^{\alpha}$, where α is a positive constant (Pareto; 1897). In other words, his law means that there are a few millionaires and many people who make modest income. Note that a power law distribution gives the number of people whose income is x, and not how many people have an income greater than x. It means that the power law gives the probability distribution function (PDF) associated with the cumulative distribution function (CDF) given by Pareto's law. The three terms: power-law, Zipf and Pareto can refer to the same thing and, in the case of $\beta = 1$ (or $\alpha = 1$) the power-law exponent a = 2 (see, e.g. Adamic; 2000). This kind of law can be applied to many real phenomena, and holds also for wealth (Dragulescu and Yakovenko; 2002).

The initial traders' endowment, both in cash and stocks, was obtained by dividing agents into groups of 20 traders, and applying Zipf's law to each group. We found that an unequal initial endowment increases trading volumes and generates logarithmic returns with fatter tails. In the simplest case of a market with one stock and one currency, the distribution of wealth among traders is calculated as follows.

Let be C(0) the aggregate amount of cash at the beginning of the simulation $C(0) = \sum_i c_i(0)$, and A(0) the aggregate amount of stocks $A(0) = \sum_i a_i(0)$. Also, let \overline{p} be the average price at which the aggregate value of stocks equals the total value of cash: $\overline{p} = C(0)/A(0)$, and let N be the number of traders. At the beginning of each simulation, the i-th agent is endowed with an amount of cash $c_i(0) = \hat{C}/i$ and with an amount of shares $a_i(0) = \hat{A}/i$, where \hat{C} and \hat{A} are two positive constants, such that the average amount of cash owned by the agents of each group $\sum_{k=1}^{20} c_k(0)$ is equal to $C(0)/N = \sum_{i=1}^{N} c_i(0)/N = \overline{c_i(0)}$, and the average number of stocks $\sum_{k=1}^{20} a_k(0)$ is equal to $A(0)/N = \sum_{i=1}^{N} a_i(0)/N = \overline{a_i(0)}$. We usually set $\overline{c_i(0)} = 50,000$ and $\overline{a_i(0)} = 1,000$ stocks.

It is worth noting that \overline{p} is the "equilibrium" price at which the aggregate value of stocks equals the total value of cash value for markets with only random traders Raberto et al. (2003). It is the equilibrium price for a closed market, without external inflows or outflows of cash. It is due to the budget constraints, that oblige the price p(t) to oscillate around the equilibrium value set at C(0)/A(0). Its value is linked with the mean-reverting behaviour of the simulator, and we selected it as best unbiased fundamental price p_f used by fundamentalists.

In the case of two markets, we found that the "equilibrium" price depends on the square root of the number of markets: $\overline{p} \simeq \sqrt{m} \cdot C(0) / \sum_{i=1}^{m} A_m(0)$, where *m* is the number of markets. In this case each trader is given with an average 1,000 stocks per market, and \$70,500.

3.3 Results

In this section we describe the results of the computational experiments we performed. We studied the effectiveness of the Tobin tax in two steps. In section 3.3.1 we discuss one market only, first without the tax, then levying a tax rate between 0.05% and 0.5% on each transaction. In section 3.3.2 we discuss two markets, first with no tax applied, and then applying the tax to the first.

We performed numerous simulations for all cases. Each simulation is usually run with 4,000 time steps (corresponding to a time span of 16 years), and with 400 agents, each with a probability p = 0.1 to trade at each time step. We also performed some simulation runs with 4000 agents, each with a probability p = 0.01 to trade at each time step. In this way, the average number of market transactions is the same as the previous case, but each trader places on average ten times less transactions, thus maintaining virtually unchanged her wealth, irrespectively of the trading strategy and of the tax. For each trader configuration we performed 20 runs. In some cases, we also performed 50 runs, but we never found results to differ significantly from those obtained with 20 runs.

3.3.1 One market

As described in the following three Sections, we first tested the overall behaviour of our model, varying the percentage of fundamentalists from zero to 30%, in steps of 10, and the percentage of chartists from zero to 30% in a similar fashion. Note that chartists always comprise the same percentage of momentum and contrarian traders. Then we tested our model keeping the percentage of fundamentalists (20%) and of chartists (20%) unchanged but varying the percentage of momentum versus contrarian traders in 5% steps.

Stylized Facts

First, we tested for the presence of the stylized facts broadly explained in Section 1.3. The results are in agreement with those of the original version of the GASM (see Raberto; 2003), so this Section provides just concise summary of the main findings. For the sake of brevity, the following Chapters will not report details on the stylized facts, except for those cases which deserve a special mention. Price series show the usual stylized facts, with fat tails of returns and volatility clustering. Note that, as discussed in section 3.2, the trader models are not the same as previous reported simulations, but now all depend on the same coefficient k, able to control traders' reaction to price trend, and thus to tune market volatility. After many test runs, we set the value of k at 1.9, which guarantees the appearance of the price series stylized facts for virtually every trader composition used.

In Figure 3.1 we plot the histogram of daily log-returns. A best-fit normal

distribution is superimposed; its narrow peak is well defined and is typical of all simulations we ran. Figure 3.2 shows the survival probability distribution of the standardized logarithmic return. The solid line represents the survival probability distribution of the best Gaussian fit and the bold stars that of the returns. The deviation from Gaussian distribution shows again a leptokurtic behaviour in the returns tail, with a very well defined power-law behaviour for high values of returns. We found that the tail of the empirical survival probability distribution follows a power law, with a slope that is always in the interval [3, 5].

Figure 3.3 shows the simulated stock price path (top) and logarithmic returns (bottom) of a typical simulation. This figure emphasizes the volatility clustering phenomenon, and the mean-reverting behaviour of the price path in the long-run. We tested for the presence of a unit root, according to the Augmented Dickey-Fuller test (see Dickey and Fuller; 1979, 1981). The null hypothesis of a unit root is rejected at the significance level of 1%. This result is in agreement with the results found by Raberto (2003), and is due to the fact that we analysed a closed market, with a strong mean-reverting behaviour. A real market cannot be considered a closed system, but it heavily interacts with the economy, interest rates external events and so on. It is worth noting that the unit root hypothesis is recovered in the case of an open market. In the case of an open market the null hypothesis of a unit root cannot be rejected. These results have been explained in Raberto (2003) and, for the sake of brevity, they are not reported here. Finally, figure 3.4 shows the autocorrelation function of raw returns and of the absolute values of log-returns: the first one quickly decays to zero, but the second one exhibits a slow decay and the presence of long-range correlations.



Figure 3.1: Histogram of the distribution of daily log-returns. The figure shows the data related to a simulation superimposed on the best normal fit.



Figure 3.2: Survival probability distribution of logarithmic returns. The figure shows the data related to a simulation superimposed on the best normal fit.



Figure 3.3: Daily time series for prices (top) and returns (bottom) in the case of a single-stock closed market.



Figure 3.4: Estimate of the autocorrelation function of logarithmic returns (top) and of the autocorrelation of absolute returns (bottom).

One market with no transaction tax

When the tax is not levied in a closed market, we obtained results similar to those reported in Raberto et al. (2003), with fundamentalists and contrarian

traders gaining wealth with time, at the expense of momentum traders and, to a lesser extent, of random traders.

Here we describe the results of several simulations performed varying the percentage of fundamental and chartist traders from 0% to 30% in steps of 10%, the remainder being random traders. Table 3.1 shows the mean and standard error of price volatility, computed for the case of no Tobin tax. Volatility of the returns was computed as the variance during period T:

$$\sigma_r^2 = \frac{1}{T-1} \sum_{t=1}^T (r_t - \bar{r})^2 \tag{3.9}$$

where $r_t = ln(p(t)) - ln(p(t-1))$ represents the logarithmic returns at the instant t. We always omitted to include the first 250 simulation steps in price volatility computation, to accommodate possible initial transient effects on price volatility. Note that all values reported in tables from 3.1 to 3.13 are multiplied by 10^3 .

As regards the width of time window T, we performed various tests, varying T between 5 and 50 time steps. In all cases, irrespectively of trader composition and tax value, we found very stable average price variance values, slowly decreasing with T. The percentage difference of average price volatility between T = 5 and T = 50 is always below 12%. We then decided to use the value T = 10, which guarantees the best trade-off between low and high values of the time window used to compute price volatility.

The results of the simulations are reported in Table 3.1. In these runs, price volatility decreases as the percentage of chartists increases, and increases as the percentage of fundamentalists decreases. These results are fairly robust and repeatable, because the presented figures are each averaged over 20 runs, and their standard error is usually much lower than volatility itself. Since they are not obvious, we will discuss them in detail.

Tabl	e 3.1:	Mean	and	standard	error	of	volatility	in	a	single	market	with	no
tax.	The r	esults a	are n	nultiplied	$by \ 10^{3}$	3.							

	Chartist						
Fundamentalist	0%	10%	20%	30%			
0%	1.61 (0.21)	0.78 (0.10)	0.46 (0.07)	0.19 (0.01)			
10%	4.33 (0.44)	1.42(0.16)	0.49 (0.03)	$0.22 \ (0.01)$			
20%	7.31 (0.62)	2.41 (0.31)	$0.60 \ (0.06)$	$0.27 \ (0.04)$			
30%	15.17 (0.99)	2.94(0.33)	1.00 (0.10)	0.26 (0.02)			

In the presented model, random traders alone are able to create consistent price volatility, for two main reasons:

- their wealth is distributed according to Zipf's law, so from time to time the wealthiest traders place large orders, that are able to generate significant price variations;
- the limit price of orders is randomly chosen, according to a Gaussian distribution with variance depending on past price volatility; this introduces a GARCH-like effect, able to yield volatility clustering and to increase overall price volatility.

On the other hand, chartists are composed equally of momentum and contrarian traders. While momentum traders can destabilize the market, and thus increase its volatility, contrarian traders tend to stabilize it, and basically counteract the effect of momentum traders. The joint behaviour of both populations tends to stabilize the market, with respect to the effect of random traders, whose number decreases as the total number of chartists increases. As regards fundamentalists, price volatility increases sharply as their percentage increases. This phenomenon is related with the prompt intervention of fundamentalists when prices diverge from their fundamental value. In practice, occasional major price variations caused by large orders placed by random traders, with a limit price that differs substantially from the current price, are immediately counter-acted by fundamentalists, strength being proportional to the original variation. This behaviour drives prices toward the fundamental value, thus adding volatility to the system. Indeed, fundamentalists can be seen as "short memory" traders, since they only look at the last price realized. The destabilizing behaviour of "short memory" traders is in line with the findings of other agent-based investigations.

In Fig. 3.5 we report the dynamics of wealth of the four populations of traders for a simulation of 2000 steps. Here both fundamentalists and chartists account for 10% of total trader population. Fundamentalists and contrarian traders tend to increase their wealth at the expense of momentum traders and, to a lesser extent, of random traders, confirming again the results reported in Raberto et al. (2003) The decreasing wealth of momentum traders, however, does not substantially affects their behaviour and their effect on the price.

One market with transaction tax

Here we show the effects of the tax on a single market. We performed numerous runs, varying the percentages of trader populations and tax rate. The tax rate is set at 0.1% and 0.5%. The former figure is within the range usually proposed and discussed by supporters and detractors of a transaction tax, while the latter figure is much higher, and is used to analyse the effects of an amplified tax. Note that, since both buyer and seller pay the tax, all



Figure 3.5: Dynamics of wealth of the four populations of traders for a simulation of 2000 steps.

these figures should be doubled.

Introducing the tax in one market made it possible to analyse a number of issues debated among its supporters and detractors. Namely, whether the taxed market becomes stabler or not, and how volatility and trading volumes change. In all reported cases, the simulations show the stylized facts of price series (return autocorrelation, fat tails, volatility clustering).

The results show the price volatility, and its standard error, averaged over 20 runs each. The simulations were performed using 400 traders and 2000 time steps, varying the percentages of trader populations as in the previous section.

Since our market model has finite resources, levying a tax leads to a reduction of total traders' wealth with time, that may be significant for the highest tax rates. To compensate for this effect, we computed the cash outflow due to the tax every 100 steps, and gave randomly chosen traders small amounts of cash totalling their cash outflow.

Table 3.2 shows mean and standard error of price volatility, computed for a Tobin tax of 0.1%. In this case, for low (10%) or zero chartist percentage, there are no significant differences compared to the no tax market. As the chartist percentage increases, however, volatility increases substantially except when no fundamentalists are included. This increase may be as much as 80%, for the largest percentage of fundamentalist and chartist traders.

Table 3.2: Mean and standard error of volatility in a single market with 0.1% tax. The results are multiplied by 10^3 .

	Chartist						
Fundamentalist	0%	10%	20%	30%			
0	1.62 (0.14)	0.70 (0.04)	0.45 (0.04)	0.28 (0.03)			
10%	3.71 (0.33)	1.35(0.14)	0.60 (0.07)	0.31 (0.01)			
20%	7.78 (0.79)	2.44(0.25)	0.74 (0.08)	0.36 (0.03)			
30%	14.44 (1.35)	2.99(0.28)	1.32 (0.16)	0.56 (0.04)			

Table 3.3 shows mean and standard error of price volatility, computed for a Tobin tax of 0.5%.

For markets with random and fundamentalists traders alone, levying the tax produces a small reduction in volatility, which varies with fundamentalist percentage, attaining 5-8% (tax = 0.1%), and as much as 30-40% (tax = 0.5%).

When chartists are taken into account, the tax systematically leads to an increase in volatility, of up to 80% for tax = 0.1%, and of up to 7-fold for tax = 0.5%, for the highest percentage of chartists. This effect is evident for a chartist percentage of 20% or 30%. When chartists only account for 10% of the entire trader population, the increase in volatility is significant only

Table 3.3: Mean and standard error of volatility in a single market with 0.5% tax. The results are multiplied by 10^3 .

Ш

	Chartist							
${\it Fundamentalist}$	0%	10%	20%	30%				
0	1.06 (0.16)	$0.85\ (0.09)$	0.66 (0.03)	0.71 (0.04)				
10%	3.06 (0.41)	1.36(0.12)	1.15 (0.10)	$1.04 \ (0.09)$				
20%	5.74 (0.56)	$2.73\ (0.25)$	1.89 (0.15)	1.55 (0.11)				
30%	11.05 (0.64)	5.51(0.28)	3.12 (0.24)	2.25 (0.12)				

for 30% fundamentalists. In general, when chartist are included, increasing the percentage of fundamentalists further increases the volatility when the tax is levied.

To gain more insight into this behaviour, we performed several runs with a market model composed of 10% fundamentalists, 5% momentum traders and 5% contrarian traders, varying the tax from 0 to 1% with steps of 0.025%. Each tax percentage has been simulated 10 times, and the resulting price variances averaged. The results shown in Figure 3.6 clearly indicate the steady increase in volatility, despite the noise in measurements, confirming the results reported above.

Using this model, price volatility increases steadily with tax rate, provided that the percentage of chartists is sufficiently large. When chartists are excluded, price volatility tends to decrease slowly with increasing tax rate. These results are in agreement with the empirical findings reported by Umlauf (1993) and by Aliber et al. (2003), who observed a price volatility increase with tax rate (or, better, with transaction costs).

We conducted further tests keeping the percentage of fundamentalists (20%) and of chartists (20%) unchanged, but varying that of momentum



Figure 3.6: Price variance as a function of tax rate for 10% fundamentalist and 10% chartist traders.

versus contrarian traders in 5% steps.

The results of these simulations are shown in Table 3.4. We found volatility increased with tax rate in all cases, but when contrarian traders were omitted. With this model, an "extreme" chartist composition, made up of contrarian or momentum traders alone, seems to increase volatility, while more balanced compositions result in lower volatility. Momentum traders tend to increase volatility more, especially when no tax, or a small tax, is levied.

Figure 3.7 shows the daily time series of logarithmic price returns for a typical simulation having a trader population composed of 20% fundamentalist, 10% momentum, 10% contrarian, for a 0.05% tax rate. Note that prices always oscillate around 50. This is due to the fact that the finite amount of cash and stocks induces mean-reversion on prices around a constant long-run mean which depends on the ratio of the total amount of cash
Table 3.4: Mean and standard error of volatility computed for different contrarian traders percentages, p_c . The total percentage of chartists is always 20%. All values are multiplied by 10^3 .

Tax	$p_c = 20\%$	$p_c = 15\%$	$p_c = 10\%$	$p_c = 5\%$	$p_c = 0\%$
rate %	Mean (stEr)	Mean $(st Er)$	Mean (stEr)	Mean $(st Er)$	Mean (stEr)
0.0	1.83 (0.13)	0.62 (0.07)	0.58 (0.06)	1.15 (0.11)	4.88 (0.46)
0.1	1.96 (0.28)	$0.95 \ (0.08)$	0.71(0.07)	$1.25 \ (0.09)$	4.31 (0.26)
0.5	4.70 (0.31)	2.26(0.14)	1.84 (0.14)	$2.20 \ (0.19)$	4.78(0.26)

to the total number of shares (Raberto et al.; 2003). Moreover, the fundamental price p(f) of fundamentalist traders is set to 50 to be consistent with the mean reverting behaviour. The increase of volatility could be explained by the reduction of orders when a tax is applied. Note that we have 400 agents, and on average 40 active agents at each time step, resulting in an average of 20 sell orders and 20 buy orders of different sizes. If transaction taxes reduce these numbers, and also the agents' trading amount, the demand and supply curves from which the price is derived becomes much fuzzier, magnifying price variations. The relationship between transaction taxes, market depth and price volatility is also explored by Ehrenstein et al. (2005), yielding similar results.

In Tables 3.5, 3.6 and 3.7, we report the average daily volumes for the cases studied. The traded volumes do not change as much as volatility as trader composition and tax rate are varied. However, note the strong anti-correlation between volatility and volumes in many cases.

Westerhoff (2003), using a different model with unlimited resources observed a different behaviour - a reduction in volatility for low tax rates, increasing as rates are increased. These results confirm how difficult it is to



Figure 3.7: Daily time series for prices (top) and returns (bottom).

Table 3.5: Mean and standard error of daily volumes in a single market with 0.0% tax. The results are multiplied by 10^3 .

	Chartist				
Fundamentalist	0%	10%	20%	30%	
0%	9.51 (0.91)	10.04 (0.96)	10.63 (1.00)	11.17 (1.06)	
10%	8.02 (0.75)	8.69 (0.83)	9.28 (0.90)	9.94 (0.97)	
20%	6.92(0.65)	7.54 (0.74)	8.19 (0.82)	8.76 (0.89)	
30%	5.80 (0.56)	6.43 (0.66)	7.12 (0.75)	7.69 (0.83)	

assess the impact of a change in market regulation using a theoretical model, suggesting further studies are warranted to gain a greater insight into market behaviour.

Table 3.6: Mean and standard error of daily volumes in a single market with 0.1% tax. The results are multiplied by 10^3 .

Ш

...

	Chartist					
${\it Fundamentalist}$	0%	10%	20%	30%		
0%	9.51 (0.91)	9.82 (0.93)	10.07 (0.96)	10.18 (0.99)		
10%	8.06(0.75)	8.41 (0.80)	8.75~(0.85)	8.91 (0.90)		
20%	6.85(0.64)	7.27 (0.71)	7.57 (0.77)	7.70(0.82)		
30%	5.76(0.55)	6.16 (0.64)	$6.44 \ (0.69)$	6.57 (0.74)		

Table 3.7: Mean and standard error of daily volumes in a single market with 0.5% tax. The results are multiplied by 10^3 .

	Chartist					
Fundamentalist	0%	10%	20%	30%		
0%	9.87 (0.88)	9.30 (0.85)	8.82 (0.85)	8.41 (0.88)		
10%	8.16 (0.73)	7.78 (0.72)	7.35 (0.73)	7.01 (0.76)		
20%	6.88 (0.63)	6.57(0.64)	6.19 (0.65)	5.88(0.67)		
30%	5.73 (0.55)	5.44(0.56)	5.15 (0.58)	4.87 (0.61)		

3.3.2 Two markets

In this subsection we discuss the second part of our experiment. We analysed two markets first levying no transaction tax and then introducing the tax in one market, leaving the other untaxed. The rules enabling traders to switch from one market to the other are described in Section 3.2.1. We recall that fundamentalist traders do not switch between markets.

Traders' initial cash and stock endowment was chosen with the constraint that wealth was balanced in the two markets. We performed a number of simulations varying fundamentalist and chartist percentages between 0 and 20, maintaining the same percentage of momentum and contrarian traders. In this way, we had a "thermal bath" of random traders, and at the same time a sufficiently large number of other kinds of traders to make their influence felt in the price dynamics.

The number of traders used in each simulation was 4000, with a 2% probability of trading at each time step. In this way, we were able to generate for each market the same average number of orders as the simulations for a single market. Note that we had 400 traders in the single market simulations, with a 10% probability of trading at each time step. The much larger number of traders was chosen so as to minimize any possible side-effects caused by a wealth increase or reduction for specific trader kinds.

Each configuration was simulated 20 times, using 2000 time steps each, and we computed the average price variance (volatility) and the standard error of this variance, to assess price volatility data consistency. The variance has been computed for intervals of 10 time steps, discarding the first 250 steps, as in previous runs.

Two markets: no transaction tax

Here we examine the dynamics of two markets with no transaction tax. When no tax is levied in either of the closed markets, we obtain the classical stylized facts (returns autocorrelation, fat tails, volatility clustering), observed for the single market. For the sake of brevity, these results are not reported here.

Tables 3.8 and 3.9 give the mean and the standard error of price volatility, computed for the case of no Tobin tax, varying the percentages of fundamentalists and chartists.

The price volatility values confirm that both markets behave in the same way. Moreover, in this case the number of traders switching from one market to another, and vice-versa, are balanced in both markets. Volatility is somewhat higher than in the corresponding Table 3.1, denoting that the presence of two markets, with traders switching between them, tends to increase price volatility. Again, price volatility decreases with increasing percentages of chartists, and increases with increasing percentages of fundamentalists. These experiments confirm the findings obtained for single markets and two markets with no tax. The first observation is that having two markets leads to a substantial increase in price volatility. This phenomenon might be explained by the imbalance between cash and stocks with respect to a single market. However, we performed a number of simulations with one market, varying traders' initial cash endowment, while leaving their stock endowment unchanged, but did not notice any change in volatility. Note that volatility increases 3-4 fold even for markets with just random traders, who switch from one to the other trying to reduce their risk. They tend to sell in the more volatile market, and buy in the less volatile one, as shown in equations 3.6 and 3.7 (section 3.2.1). Probably, this behaviour creates an imbalance in orders resulting in an overall increase in volatility. The intrinsic mean reversion mechanism due to limited trader resources avoids long-term imbalance between the two markets. The presence of fundamentalists seems to reduce the increase in volatility, while the combined presence of high percentages of chartists and fundamentalists magnifies it - for 20% fundamentalists and 20% chartists, we observed a 8-9 fold increase (see Tables 3.1, 3.8 and 3.9).

In Fig. 3.8 we show the wealth dynamics for the four trader populations in both markets, for a simulation of 2000 steps. Both fundamentalists and chartists account for 10% of total trader population. For the two markets,

Table 3.8: Mean and standard error of volatility in market one. The results are multiplied by 10^3 .

	Chartist					
Fundamentalist	0%	5%	10%	20%		
0%	5.62 (0.58)	3.72(0.50)	2.04 (0.25)	0.67 (0.08)		
5%	8.15 (1.34)	4.20(0.61)	1.89(0.28)	0.79 (0.15)		
10%	12.18 (1.87)	4.88(0.79)	3.92(0.71)	$1.07 \ (0.19)$		
20%	37.22 (3.42)	19.38 (2.15)	7.49 (0.88)	2.25 (0.36)		

Table 3.9: Mean and standard error of volatility in market two. The results are multiplied by 10^3 .

	Chartist				
Fundamentalist	0%	5%	10%	20%	
0%	4.90 (0.73)	3.87(0.84)	1.95 (0.27)	0.67 (0.09)	
5%	8.12 (1.53)	4.07(0.66)	1.88(0.29)	0.74 (0.09)	
10%	7.99 (1.14)	5.08(0.76)	2.26 (0.27)	1.01 (0.18)	
20%	35.65 (3.22)	17.23 (1.87)	7.78 (1.02)	2.04 (0.48)	

fundamentalists and contrarian traders tend to increase their wealth at the expense of momentum traders and random traders. The differences in wealth are less pronounced for the single market, but we should recall that in this case the number of traders rises to 4000, and they tend to trade five times less than for the single market.



Figure 3.8: Wealth dynamics of the four trader populations for a simulation of 2000 steps, for two markets.

Two markets: transaction tax in one

Here we discuss the dynamics of two markets, levying the tax in just one of them (Market 1). When levying a tax on Market 1 transactions, we obviously found total traders' wealth to decrease over time, because our market model has limited resources. This decrease affects both cash – because the tax is paid in cash – and prices – because a cash shortage affects prices. If the fundamental price (p_f) is not adjusted according to the cash reduction, in a closed market after a while fundamentalists wealth will also diminish, because they tend to push prices towards their fundamental value which eventually becomes unsustainable. They buy all the stocks they can and then stay still, while the value of their stocks slowly diminishes. However, in our simulations the cash drain of tax payment is negligible, because the tax rate is low, the number of transactions made by each trader is also low, and the number of simulated time steps is limited. Thus, the reported results are not affected by any cash drain.

Tables 3.10 and 3.11 give the mean and standard error of price volatility in markets 1 and 2 respectively, computed for a tax rate of 0.1%, varying the percentage of fundamentalists and chartists. Tables 3.12 and 3.13 show the same configurations and data, this time computed for a tax rate of 0.5%.

Table 3.10: Mean and standard error of volatility in market one, with 0.1% transaction tax. The results are multiplied by 10^3 .

	Chartist				
Fundamentalist	0%	5%	10%	20%	
0%	7.27 (0.99)	3.88 (0.59)	2.96(0.48)	1.16 (0.17)	
5%	5.88(0.76)	4.38(0.53)	2.23(0.44)	1.36 (0.26)	
10%	11.55 (1.26)	7.16 (1.02)	$3.39\ (0.52)$	1.28 (0.17)	
20%	43.12 (4.44)	16.71 (1.87)	10.86 (1.30)	2.71 (0.52)	

Table 3.11: Mean and standard error of volatility in market two, with 0.1% transaction tax. The results are multiplied by 10^3 .

	Chartist					
Fundamentalist	0%	5%	10%	20%		
0%	6.16 (0.81)	3.19 (0.47)	2.45(0.43)	0.85 (0.08)		
5%	$5.77 \ (0.69)$	$3.89\ (0.67)$	2.80(0.38)	$0.68 \ (0.09)$		
10%	11.48 (1.83)	7.44 (1.05)	2.10 (0.31)	1.33 (0.24)		
20%	36.92 (4.61)	17.62 (2.29)	7.05 (1.05)	1.56 (0.19)		

The effects of the tax observed for a single market (no substantial differences for random and fundamentalist traders alone, volatility increase in the presence of chartists) are fully confirmed in the case of a tax levied on

Table 3.12: Mean and standard error of volatility in market one, with 0.5% transaction tax. The results are multiplied by 10^3 .

п

	Chartist					
${\it Fundamentalist}$	0%	5%	10%	20%		
0%	6.76 (0.92)	5.06(0.65)	3.14 (0.44)	2.05 (0.26)		
5%	5.28(0.67)	4.82(0.93)	$3.95\ (0.73)$	2.24 (0.28)		
10%	11.09 (1.57)	6.61(0.83)	5.77(0.88)	3.81 (0.66)		
20%	36.10 (3.64)	29.09 (2.48)	16.26 (1.69)	7.28 (0.72)		

Table 3.13: Mean and standard error of volatility in market two, with 0.5% transaction tax. The results are multiplied by 10^3 .

	Chartist					
Fundamentalist	0%	5%	10%	20%		
0%	5.58 (0.61)	4.54(0.79)	1.98 (0.29)	0.89 (0.15)		
5%	6.98 (1.11)	3.52 (0.7)	$1.91 \ (0.35)$	$0.72 \ (0.10)$		
10%	12.09 (1.84)	3.74(0.60)	4.05(0.74)	$1.02 \ (0.24)$		
20%	43.63 (3.79)	18.06 (2.10)	5.92(0.86)	1.10 (0.12)		

one market, linked to a second, untaxed market. The increase in volatility is however less pronounced, maybe because the market volatility is already very high.

The most significant effect we found is that the taxed market presents a greater volatility than the linked untaxed market, in all those cases where levying the tax has a major effect, i.e. in the presence of chartists. For a highly speculative market, i.e. for the higher percentages of chartists, the untaxed market shows a reduction in volatility with respect to the case of no tax market, while the taxed market shows a strong increase, thus "adsorbing" to some extent additional volatility from the former.

Table 3.14 shows average daily trading volumes in both markets, for different trader compositions and tax rates. The taxed market is, as always, Market 1. The values are averaged over 20 simulations, and standard errors are also given.

As expected, when no tax is levied, average trading volumes do not differ significantly from one market to the other. The introduction of the tax leads to smaller volumes in the taxed market, in all those cases where it results in a price volatility increase. The difference in trading volumes is not as large as the difference in volatility, at the most in the order of 20%. This finding confirms, however, that traders tend to shun the taxed market, and that a lower volume triggers an increase in volatility, as discussed in Section 3.3.1.

Pop. (%) No Tax		0.1% Tax		0.5% Tax			
F.	C.	Mkt1	Mkt2	Mkt1	Mkt2	Mkt1	Mkt2
0	0	9.49 (1.05)	9.48 (1.04)	9.55 (1.07)	9.46 (1.05)	9.47 (1.07)	9.57 (1.07)
0	10	9.97 (1.05)	10.01 (1.05)	9.81 (1.04)	9.91 (1.05)	9.37 (1.02)	9.85 (1.04)
0	20	10.55 (1.07)	10.63 (1.08)	10.13 (1.05)	10.54 (1.08)	9.26 (1.03)	10.31 (1.05)
10	0	8.61 (0.94)	8.60 (0.94)	8.56 (0.94)	8.61 (0.94)	8.57 (0.94)	8.68 (0.94)
10	10	9.24 (0.99)	9.29 (0.98)	9.05 (0.97)	9.25 (0.99)	8.51 (0.94)	9.25 (0.99)
10	20	10.01 (1.05)	9.95 (1.06)	9.44 (1.01)	9,99 (1.05)	8.40 (0.96)	9.96 (1.04)
20	0	7.91 (0.90)	7.87 (0.90)	7.85 (0.90)	7.89 (0.89)	7.82 (0.89)	7.91 (0.90)
20	10	8.50 (0.94)	8.47 (0.95)	8.30 (0.93)	8.50 (0.94)	7.78 (0.88)	8.58 (0.95)
20	20	9.17 (1.00)	9.12 (0.99)	8.76 (0.98)	9.21 (1.01)	7.77 (0.92)	9.27 (1.01)

Table 3.14: Average daily volumes. Tax levyed on market one only. The results are divided by 10^3

Chapter 4

Short Selling and Margin Trading

In this Chapter, we discuss the effects of introducing and removing shortselling restrictions and margin requirements on a stock market. Our aim was to study whether and how stock prices, volatility and long-run wealth distribution are influenced by these kinds of restrictions. The introduction and the removal of constraints enabled us to analyse some interesting issues: effects of restrictions on volatility, long-run agents' wealth distribution and the relationship between price shocks and *in debt positions*.

4.1 Motivations

After the great stock market crash of 1929, some restrictions were implemented to ensure the market does not crash again. On one hand, when prices declined, many investors who had bought stocks on margin tried to sell their shares disrupting the market. On the other hand, short sellers were pointed out as one of the main causes of the crash. The U.S. stock market reacted restricting short-selling and setting margin requirements. In 1934 the U.S. Congress gave the Federal Reserve Board the power to set initial, maintenance and short sale margin requirements on stock markets. Margin requirements were set in order to reduce excessive volatility of stock prices, protect investors from losses due to speculative activities, and reduce loans by banks to stockholders, moving credit toward more productive assets.

The 1987 stock market crash renewed both political and academic interest on the effectiveness of restriction policies for stocks and derivative products. Since then, a wide debate on these solutions started, and studies were performed on the effects of such impositions.

In April 2005, the China Securities Regulatory Commission (CSRC) issued a new plan for state share reform. As reported by Bloomberg News¹, "China plans to allow investors to take out loans to buy shares and to sell borrowed stock for the first time, moves aimed at tapping the country's \$4 trillion of bank deposits and boosting trading. The China Securities Regulatory Commission may select five brokerages to start margin-lending and short-selling services this year". This event will surely renew the interest on short-selling and margin requirements regulations.

Buying on margin means to borrow money from a bank or a brokerdealer to buy securities. The margin requirements set the maximum legal amount that an investor may borrow to increase her purchasing power, so she can buy securities without fully paying for them. For instance, if the initial margin requirement is set at 20 percent, an investor can borrow up to 80 percent of the current value of the owned securities.

There has been an heated debate on the effectiveness of margin regulations and on their influence on asset prices. The central issue is the claim that margin requirements have an influence on stock price volatility.

¹For more details see http://www.bloomberg.com

In late eighties, some studies by Hardouvelis (1988, 1990), claimed that there is evidence of a negative relationship between stock volatility and margin requirements. Moreover, he asserted that changes in margins level can influence monthly stock return volatility. These conclusions support the opinion that margin requirements could be used to control price volatility.

On the other hand, previous literature disagree with Hordouvelis' findings. Moore (1966) stated that margin requirements fail to fulfil their objectives. Largay and West (1973) and Officer (1973) also concluded that changes in margin requirements had little or no effect on stock price volatility.

Owing to the results of Hardouvelis, the debate on margin requirement effectiveness has become very heated. Many authors, including Salinger (1989), Ferris and Chance (1988), Schwert (1989) and Hsieh et al. (1990), re-examined the connection between margin and volatility. These authors examined the issue from different points of view using different econometric techniques, but they uniformly concluded that there is no evidence of a relationship margin-volatility. Kim (2002) tested whether margin requirements affect individual wealth-constrained speculators. To test this possibility, they examined the stock market reaction to changes in the initial margin requirement of the Tokio Stock Exchange. The analysed the volatility of the stocks with the highest percentage of individual ownership, but they found, finding that changes in initial margin requirement don't have much effect on the volatility of those securities.

Another critical question in financial literature, that is fairly symmetrical to margin trading is whether and how short-sale constraints affect the tendency of the stock markets. Short selling is a technique used by investors who try to profit from the falling price of a stock. They borrow the shares from someone else and sell them. When the price falls, they will cover their position by buying back the shares. If their prediction was right, short sellers gain a profit.

After the stock market crash of October 1929, many short-sale restrictions were imposed on short-selling in the United States. Short sellers were immediately pointed out as the cause of the collapse, so three regulatory changes were decided in order to reduce short-selling².

Short-selling advocates claim that it increases liquidity, favours risk sharing and increases informational efficiency. On the other hand, opponents of short-selling claim that it causes high volatility, favors market crashes and panic selling.

Miller (1977) observed that, if short-selling is restricted and investors have heterogeneous beliefs, the observed price of a security does not reflect the beliefs of all potential investors, but only the opinion of the optimistic ones. The implication of his idea was that stocks may be overpriced because of short-selling restrictions. Miller's hypothesis implies a negative relationship between short interest and returns. In recent years, empirical evidence on this relationship has been pointed out by several studies, among them we cite Jones and Lamont (2002), Ofek and Richardson (2001) and Chen et al. (2002).

King et al. (1993) studied the effect of short selling on asset market bubbles in an experimental laboratory environment. found that short selling does not influence market bubbles. Ackert et al. (2002) conducted experiments on two asset markets and stated that short selling eliminates the bubble-andcrash phenomenon. Haruvy and Noussair (2006) studied the relationship between short-selling constraints and assets prices using a simulation model based on the work by De Long et al. (1990b). They found that short selling

 $^{^{2}}$ See http://www.prudentbear.com/press_room_short_selling_history.html

reduces prices to levels below fundamental values and that the reduction of the bubble-and-crash phenomenon is the consequence of such a trend rather than of the effectiveness of short selling restrictions.

Some studies examine the relationship between return volatility and short-sale constraints. Ho (1996) produced evidence that volatility increased when short-selling prohibition was lifted during the Pan Electric crisis of 1985. Kraus and Rubin (2003) developed a model to predict the effect of index options introduction on volatility of stock returns. Since short-selling the stock was restricted, the option was considered as a form of reduction of this constraint. The model is highly stylized, and it predicts that volatility can either increase or decrease, depending on model parameters.

Diamond and Verrecchia (1987) asserted that short-sale restrictions can slow down the response of prices to new information: some investors who want buy or sell cannot take part in the market bringing a decline in liquidity. In other words, if short-selling is possible, there is greater liquidity.

4.2 The Extended Model

In this Section we introduce the major improvements to the model we developed in order to assess the impact of short selling restrictions and margin requirements.

Both margins and short-selling restrictions are implemented in a simplified manner. We don't distinguish between initial margin requirements and maintenance requirements. There are no transaction costs or taxes, so agents can borrow money/stocks without paying any interest for them. Moreover, margin and short-selling requirements are kept symmetrical, in the sense that their maximum allowed percentages are the same.

The population of traders is made up of two main categories: the first one

consists of agents that can issue orders using their available limited resources. They are forbidden to sell if they do not have any stock to sell, and they cannot buy if they do not have enough money to do so. The second one is made up of traders that are allowed to buy/sell stocks in debt. The agents can sell stocks without owing them (we will say that they can issue *in debt selling orders*) and buy shares without owing enough money to pay them (*in debt buying orders*). We will name agents belonging to the second group "Debt Prone Trader" (DPT), and we will call agents from the first group "non-Debt Prone Trader" (non – DPT). Both DPT and non – DPT belong to one of the four categories of traders described in Chapter 3. For the sake of brevity we will mark DPT traders with a star (for instance Random^{*} means "Debt Prone Traders of type Random").

It is worth noting that in this case we slightly modified the strategy of fundamentalist traders. We improved their model by transforming the trading probability from a constant to a function depending on the current price p(t) and on the fundamental price p_f . At each time step, they decide whether or not to trade with a probability p depending on the ratio between p_f and p(t). If $p(t) = p_f$, the probability p will be equal to 0.0, and it will increase as a squared function of the ratio $max(\frac{p_f}{p(t)}, \frac{p(t)}{p_f})$. The maximum value of p is set at 0.1. Note that, if the price of the asset is close to p_f , the trading activity of fundamentalists is low, because the market is not very attractive for them.

4.2.1 non-Debt Prone Traders

non-DPT are risk-adverse agents, so they trade using their limited resources without issuing *in debt orders*. If a *non debt prone* trader issues a buy (sell) limit order, the order amount and the limit price are computed as described in Chapter 2. Note that here the parameter k is set at a value equal to 1.4. This choice brings to smaller values of volatility in comparison with those found in Chapter 3, but these values are still in agreement with those of real financial data.

4.2.2 Debt Prone Traders

DPT are risk-prone agents. They can borrow money (or stocks) without paying any interest on it (there are no transaction costs or taxes), but in debt transactions must be guaranteed by the agents' total wealth. The debt level of each DPT cannot exceed a certain threshold called safety margin (m). If a trader exceeds the *safety margin* she is forced to cover her position and repay her debts. If an agent has negative wealth $w_i(t)$, she goes bankrupt and is obliged to leave the market. The wealth $w_i(t)$ of the generic i - thtrader at time step t is defined as $w_i(t) = c_i(t) + a_i(t) \cdot p(t)$, where $c_i(t)$ is the amount of cash and $a_i(t)$ the amount of stocks that the agent holds at time t. The safety margin is a constraint that can be moved up or down in order to allow agents to borrow more or less money (stocks), setting the debt limit. In our tests, the value of m varies from 0.0 to 0.9. If m = 0.0, it means that both short selling and margin trading are forbidden. If m > 0.0, it means that short selling and margin trading are allowed. For instance, if m is set at the maximum value (0.9), it means that margins are set at 10%and a debt prone trader can borrow stocks (to sell short) or cash (to buy on margin) up to 90% of her cash (stock value). Each debt prone trader decides whether to buy or sell first on the basis of her strategy, then she has two choices: to trade using her limited resources or to trade borrowing stocks or money. These choices have equal probability.

If the i - th agent decides to issue an *in debt order*, the order size has

an upper limit. If the agent issues a buy order, the amount of stocks to purchase cannot exceed the quantity $\hat{a}_i^b(t)$ (see equation 4.1). In debt selling orders are generated fairly symmetrically relative to in debt buying orders, the maximum quantity on sale is $\hat{a}_i^s(t)$ (see equation 4.2).

$$\hat{a}_i^b(t) = m \cdot a_i(t) + \lfloor \frac{c_i(t)}{p(t)} \rfloor$$
(4.1)

$$\hat{a}_i^s(t) = a_i(t) + \lfloor m \frac{c_i(t)}{p(t)} \rfloor$$
(4.2)

If an agent exceeds her safety margin, she is obliged to cover her position. In particular, if she holds an amount of assets $a_i(t) < 0$ and $\hat{a}_i^s(t) < 0$, she is forced to buy the amount of stocks equal to the quantity expressed in equation 4.3. Symmetrically, if a trader holds an amount of cash $c_i(t) < 0$ and $\hat{a}_i^b(t) < 0$, she is forced to sell an amount of stocks equal to the quantity expressed in equation 4.4.

$$a_m^b = \left\lceil -\frac{a_i(t) + m\frac{c_i(t)}{p(t)}}{1 - m} \right\rceil$$

$$(4.3)$$

$$a_m^s = \left\lceil -\frac{m \cdot a_i(t) + \frac{c_i(t)}{p(t)}}{1 - m} \right\rceil$$

$$(4.4)$$

4.3 Results

First, we performed several computational experiments on a closed market, i.e. a market with no cash or stock inflow or outflow. We performed several tests varying some parameters of the model such as the percentage of DPTand non - DPT agents, the percentage of the four population types, the safety margin and the probability that DPT traders issue in debt orders. We considered a large number of experimental cases, performing 20 runs for each case. Each simulation is usually run with 4000 time steps (corresponding to a time span of 20 years) and with 400 agents. Then, we opened the market by varying the cash of the traders, in order to understand how external shocks influence volatility, both with and without *DPTs*.

4.3.1 Closed Market

Random Traders

We first explored market behaviour when only random traders are present. We studied volatility trend varying some parameters of the model. Volatility is defined as the standard deviation of prices in a time window 50 steps long. We set the Safety Margin at 0.8 and varied the percentage of Random* traders from 0% to 100% in steps of 25%. The results showed that an increase in the percentage of random traders able to trade in debt brings a very slight increase in volatility, as shown in figure 4.1.

We also explored return volatility varying the value of m parameter from 0.1 to 0.9. We observed that volatility looks not affected by m, but for the highest values of m. This result does not depend on the percentage of DPTs. In figure 4.2 we report this behaviour for simulations with 50% of debt prone traders. When m = 0.9, there is an increase in volatility, but this phenomenon is due to the bankrupt of some traders, which makes the market unstable. When a trader goes bankrupt, she is forced to cover her position as far as possible, and then she leaves the market. On one hand, this fact implies that an amount of stocks are sold or bought at limit prices low or high enough to have a high probability to be fulfilled, and on the other hand it lowers the number of traders. Both effects tend to increase market volatility.

We found similar results by varying the probability that DPTs issue



Figure 4.1: Mean and standard deviation of price variance as a function of Random*. The percentage of DPT was varied from 0% to 100% in steps of 25%, with m = 0.8.



Figure 4.2: Mean and standard deviation of price variance as a function of m.

in debt orders, as shown in figure 4.3. This behaviour is not unexpected, because increasing this probability is equivalent to increase the percentage

of in debt orders; with a high margin equal to 0.8, this yields many bankrupts, with consequent volatility increase.



Figure 4.3: Mean and standard deviation of price variance as a function of P(in-debt), with the percentage of DPT random traders set at 50% and m = 0.8.

We also studied the dynamics of wealth of the populations of traders. We found that the wealth of both random and Random^{*} traders remains approximately the same during the whole simulation. Figure 4.4 shows the average wealth $1/N \sum_i w_i(t)$ of the two populations for a typical simulation 4000 steps long with 50% random and 50% Random^{*} traders. The total traders' wealth varies, depending on the stock price variations, but the wealth of both populations does not differ significantly for the whole simulation.

Trend Followers

Next, we investigated how volatility is influenced by the presence of trend followers (momentum and contrarian) debt prone traders. We performed six groups of tests: Random and Momentum; Random and Momentum*; Ran-



Figure 4.4: Dynamics of wealth of Random and Random^{*} for a typical simulation with m = 0.8 and P(in - debt) = 50%.

dom and Contrarian; Random and Contrarian^{*}; Random, Momentum and Contrarian; Random, Momentum^{*} and Contrarian^{*}. The total percentage of trend followers has been set to 0, 10% and 20%. When there are both momentum and contrarian traders, each kind accounts for one half of the total percentage. Each value shown is the mean of 20 runs. The standard deviation of market volatility in these runs is shown in parenthesis.

Table 4.1 reports the results for the various kinds of trend followers. First, we found that the presence of a small percentage of momentum traders alone (up to about 10 - 15%) does not tend to increase volatility, that increases only for higher percentages. This is probably due to the limited amount of traders' resources, and to the different time scales the momentum traders use to compute the trend. This behaviour is similar with Momentum*, but when they reach 20%, volatility sudden increases. This is due to a not negligible number of traders who declare bankrupt, with consequent increase in volatility.

In the performed simulations, the presence of contrarian traders alone tend to slightly increase volatility. This phenomenon is due to the "hits" to the price in the opposite direction of the current price trend. This phenomenon is not affected by limited traders' resources, because it is in accord with the intrinsic mean reversion behaviour of prices. The presence of debt prone contrarian traders obviously increases this behaviour. When debt prone contrarian traders reach 20%, there are very few bankrupts, that further slightly increase volatility.

When both kinds of trend followers play together, the situation stabilizes, irrespectively of their debt inclination. Market volatility tends to be constant, and in this case we did not observe any bankrupt.

	0%	10%	20%
Momentum	0.27(0.04)	0.26 (0.04)	$0.30\ (0.04)$
Momentum*	0.27(0.04)	0.29 (0.04)	0.94(0.41)
Contrarian	0.27 (0.04)	0.31 (0.04)	$0.45\ (0.06)$
Contrarian*	0.27 (0.04)	0.35 (0.06)	0.59 (0.11)
Momentum and Contrarian	0.27 (0.04)	0.27 (0.02)	$0.25\ (0.03)$
Momentum [*] and Contrarian [*]	0.27 (0.04)	0.28 (0.03)	0.27(0.03)

Table 4.1: Mean and standard error of volatility with trend followers and random traders. The results are multiplied by 10^3 .

We then analysed the effects of the dynamics of wealth with the trend follower traders. We investigated if and how DPT agents influence this behaviour. We found that debt prone traders show the same dynamics of non - DPT traders, but the effects are amplified. Contrarian* traders gain more than Contrarian traders, Momentum* lose more than Momentum traders, as shown in figure 4.5.



Figure 4.5: Dynamics of wealth with trend followers for a typical simulation with m = 0.8 and P(in - debt) = 50%.

Fundamentalists

When studying volatility behaviour using random traders and fundamentalist traders, we found that allowing fundamentalists to short sell and to buy on margin volatility increases. Table 4.2 shows the market volatility (and its standard deviation, related to 20 different runs), setting the total percentage of fundamentalists to 0, 10% and 20%. Note that no trader declares bankrupt during all simulations. In all cases, the total wealth of both fundamentalists and debt prone fundamentalists tend to increase at the expenses of random traders' wealth.

Figure 4.6 shows that volatility slightly increases with the increase of the safety margin m and of the probability that debt prone traders issue a debt order P(in - debt).

/olatility

Table 4.2: Mean and standard error of volatility with fundamentalists and random traders. The results are multiplied by 10^3 .

		0%	10%	20%	
	${f Fundamentalist}$	$0.27 \ (0.04)$	0.33 (0.04)	0.51 (0.10)	
	$Fundamentalist^*$	0.27 (0.04)	0.36 (0.04)	0.55(0.11)	
4.2 × 10 ⁻⁴		4.1	8×10 ⁻⁴		
4-		41	6 -		
3.6	/	نه م	4		
3.4 -	0 0	rstity Vdaality 15	8		/
3-		3.	4 0		
2.8			3	1111 C	

(a) Volatility as a function of the(b) Volatility as a function of P(in-Safety Margin.debt).

Figure 4.6: Volatility with a population made of 10% of DPT fundamentalists and of 90% random traders.

All Kinds of Traders

In this section we report the results of tests we conducted using all trader populations. The main goal was to understand whether or not the results were merely the sum of the effects of each population.

First, we used momentum, contrarian and fundamentalist traders, setting the same percentage of agents for each kind of strategy. We found that DPTtraders slightly increase volatility, as shown in table 4.3. In this table, the reported percentages refer to each kind of traders. So, a percentage of 5% means that there are 5% of fundamentalists, 5% of momentum and 5% of contrarian traders.

Table 4.3: Mean and standard error of volatility with fundamentalists trend followers and random traders. The results are multiplied by 10^3 .

	0%	5%	10%
Fundamentalist, Momentum, Contrarian	$0.27 \ (0.04)$	0.28 (0.04)	$0.34\ (0.06)$
Fundamentalist*, Momentum*, Contrarian*	$0.27 \ (0.04)$	0.29 (0.04)	$0.38\ (0.07)$

Note that the increase in volatility is not due to failures of traders, because no trader fails during any of these tests. The sensitivity analysis both of the m parameter and of the probability that debt prone traders issue a debt order show results similar to those presented in previous sections. The findings are shown in figure 4.7.



Figure 4.7: Volatility with a population made of 5% of DPT fundamentalists, 5% of DPT momentum and 5% of DPT contrarian traders.

Finally, we performed a group of tests using all kinds of traders, with and without PDT. We set the percentage of both DPT and non - DPTfundamentalist, momentum and contrarian traders at 5%, and the percentage of both DPT and non - DPT random traders at 35%. We chose to equally divide the *thermal bath* of agents of type random into 2 populations of the same size of DPT and non - DPT agents in order to avoid any kind of asymmetry in the results. The resulting volatility was 0.31, with a standard deviation of 0.04. This figure has to be compared with the case of a market with no debt prone trader, but with the same percentage of fundamentalist, momentum and contrarian traders with respect to random ones. In this latter case, we had a volatility of 0.28, with a standard deviation of 0.04. In both cases, there is no trader declaring bankrupt.

The sensitivity analysis referring to this case is shown in figure 4.8. Here volatility looks to slowly decrease with m, except for the highest values. The most interesting result is that, with all kinds of traders playing the market, volatility clearly decreases with the probability that debt prone traders place in debt orders. This result is due to the interplay of all kinds of traders, and we don't have at the moment an explanation for it.

Figure 4.9 presents the wealth dynamics for a typical simulation in which all eight populations are taken into account. Note that fundamentalists and contrarians gain wealth, while momentum traders and, to a lesser extent, random traders, lose wealth. This behaviour is due to the relationship between the strategies of each type of trader and the mean reverting behaviour of the market Raberto et al. (2003). The new finding is that debt prone traders present the same behaviour of non - DPT, but they amplifies the effects obtained without them. Actually, fundamentalist and contrarian DPTs gain more than fundamentalist and contrarian non - DPTs, while momentum and random DPTs lose more than non - DPTs of the same kind.



Figure 4.8: Volatility with a population made of all types of traders, both DPT and non - DPT.



Figure 4.9: Dynamics of wealth with all eight types of traders for a typical simulation with m = 0.8 and P(in - debt) = 50%.

4.3.2 Open Market

We opened the market by varying the cash of the traders. The main goal is to understand how external shocks influence volatility, both with and without DPTs. The cash variation $\Delta c_i(t)$ follows the law expressed in equation 4.5. $\Delta c_i(t)$ is proportional to each trader's wealth and its level depends on the σ parameter.

$$\Delta c_i(t) = w_i(t) \cdot [e^{N(0,\sigma)} - 1]$$
(4.5)

where $N(0, \sigma)$ is a random draw from a Gaussian distribution with average 0 and standard deviation σ , and σ is a parameter. These inflows and outflows of cash can be considered as external factors able to influence the market. We varied the amount of cash 10 steps apart, by adding to each trader's cash the term $\Delta c_i(t)$ defined in equation 4.5.

We performed many runs changing the population of traders. For the sake of brevity, we report here just two examples: the first one with random traders and the second one with all kinds of traders. The findings are similar to those obtained using other combinations of traders.

The main result is that changes to traders' cash increase volatility. The increase is patient both with and without DPT traders. Table 4.4 shows this finding for tests conducted with non - DPT random traders and with a population made up of 50% non - DPT and of 50% DPT random traders. Note that if DPTs are present, volatility will increase more than without them. Also, if the value of σ is too high, volatility will suddenly increase. We studied the last case (reported in the last column of table 4.4) more deeply, and we found that the excessive increment in volatility is due to a sudden increase in the number of traders who fail.

The quantity of traders who declare bankrupt can be inferred from figure 4.10, which shows the total number of traders active in the market versus simulation steps. An excessive cash inflow can destabilize the market. Even if DPTs are not present, a large number of agents can fail and leave the market, because of negative cash inflows. This phenomenon yields an increase in price

returns and in volatility. Figure 4.10 shows the population size superimposed on prices and the population size superimposed on logarithmic returns for a simulation with random traders alone, and $\sigma = 10^{-4}$. These figure show a correspondence between the steps where traders' failures happen, and daily return variations, which look very high during these steps.

Table 4.4: Mean and standard error of Volatility with random traders. The results are multiplied by 10^3

Popu	lation	σ		
Random	Random*	0.0	10^{-5}	10^{-4}
100%	0%	$0.27 \ (0.04)$	$0.29\ (0.05)$	1.99 (1.48)
50%	50%	$0.27 \ (0.03)$	0.29 (0.06)	6.19 (4.53)



Figure 4.10: Daily time series for prices (a) and returns (b) with random traders and $\sigma = 10^{-4}$. The dotted line represents the population size.

If DPTs are present, the number of agents who declare bankrupt increase. Also, these traders tend to fail sooner. We calculated the number of failed traders after the end of the simulations, and we found an average of 150.95 bankrupts in the case of random traders alone (with standard devi-

ation 71.00). If random DPTs are taken into account, the average number of traders who leave the market increase to 239.10 (with standard deviation 107.78). Figure 4.11 shows the relationship between traders' bankrupts, prices and returns. In order to remark that debt prone traders tend to fail more than non - DPTs, figure 4.11 reports the population size of both random and random* traders. Note that, if the number of failures is too high (over 50%), the market will become unsteady. Moreover, in the case of open market, the results are robust to changes in the values of the safety margin and of P(in - debt).

We conducted further experiments using different traders' populations, as the ones described in section 4.3.1. We found similar results to those obtained with only random traders – cash inflows and outflows increase volatility, debt prone traders tend to declare bankrupt more frequently than non - DPTs of the same kind, simulations are robust to changes in m and in P(in - debt).



(a) Population size superimposed on prices.



(b) Population size superimposed on returns.

Figure 4.11: Daily time series for prices (a) and returns (b) with 50% non – DPT and 50% DPT random traders, with $\sigma = 10^{-4}$. The dotted line represents the population size.

Chapter 5

The Interplay Among Two Stock Markets and the FOREX

It is well known that the recent financial crises, starting from the Mexican pesos crisis in 1994 to the Argentina's in 2001, have been accompanied by episodes of financial markets *contagion*, that is, many countries have experienced increases in the volatility and comovements of their financial asset markets.

Although changes in the statistical properties of prices are predictable in countries experiencing financial and exchange rate crises, the patterns of comovement and of contagion of crises across countries are still not fully understood. The definition itself of contagion, in fact, varies widely across literature. A large number of tests have been proposed for assessing the presence and the level of international contagion, but the results are often conflicting.

The main goal of this Chapter is to analyse the interaction between two stock markets in two different countries, both during tranquil periods and during a monetary crisis. We developed a multi-agent model with two artificial stock markets and two different currencies, by extending the general framework described in previous chapters. The starting point of this study was the development of a foreign exchange market (FOREX), that provides a link between the two stock markets and sets the current exchange rate.

5.1 Contagion and interdependence

In recent years, much attention has been given to transmission of financial crises from a country to other countries, and usually this topic gains interest after an international crisis.

Generally speaking, the spread of crises from one market to other markets depends on the links existing among those markets. One of the most important issues is that there is still ambiguity on the definition and on the meaning of the term "contagion", and therefore a large number of methodologies and of tests have been proposed to measure it.

There are papers that attempt to detect and measure the factors that favour financial crises, such as those by Forbes (2004) and by Eichengreen et al. (1996). Also, a number of papers use ARCH and GARCH techniques to estimate how changes in volatility are transmitted across countries; see for instance Hamao et al. (1990). However, is very difficult to identify the channels through which contagion occurs, and to measure their weight. Many researchers agree that the straightforward approach to test for contagion is to analyse the cross-market correlation coefficient. For example, Butler and Joaquin (2002) analysed the correlation dynamics in bear, calm and bull markets. They found an increase in correlations during bear market periods compared with calm and bullish periods.

We embrace the terminology proposed by Forbes and Rigobon (2002): they asserted that stock markets can exhibit a certain degree of comovements both before and after a shock or a crisis in one market. They define contagion as a significant increase in cross-market linkages after a shock to one country (or group of countries). This means that contagion occurs only if cross-market comovements increase significantly after a shock. On the other hand, if the markets exhibit a high degree of comovement during periods of stability, even if the markets continue to be highly correlated after a shock to one market, this may not constitute contagion. The authors use the term interdependence to refer to this situation. Forbes and Rigobon test for contagion by following a correlation analysis approach. According to their definition, contagion occurs only if there is an increase in the unconditional correlation coefficient. They define the unconditional correlation coefficient as the traditional correlation coefficient adjusted in order to take into account the bias in heteroscedasticity (see Forbes and Rigobon; 2002).

5.2 The Extended Model

We consider an economy with two stock markets $(SM_1 \text{ and } SM_2)$ and one foreign exchange market (FOREX). The two stock markets are perfectly symmetric, except for the accepted trading currency: the stocks of the SM_1 are exchanged using the "dollar", while those of the SM_2 market are exchanged using the "euro". The FOREX determines the exchange rate between the dollar and the euro.

At each time step, each trader trades only within one market. Before trading takes place, each trader, in accordance with an attraction function based on expected gain may decide to leave a market, switching to the other one. Note that the trader converts all her money into the currency of the destination market before leaving the current market. This choice increases the purchasing power of the trader in the just selected market. If a trader decides to buy or sell stocks, she places a limit order on the selected stock market, as described in Chapter 3.

The FOREX market differs from the stock markets in two main respects: the traders issue only market orders, and a market maker is assumed to adjust the exchange rate at the end of each trading period, on the basis of the excess demand, as described in Section 5.2.1. At the end of each simulation step, the exchange orders are collected and the new exchange rate is computed. The FOREX is a closed market, so the total amount of cash cannot vary during the simulations. If all the exchange orders are executed at the new exchange rate, the quantity of both dollars and euro will change. In order to avoid this phenomenon, we randomly choose and discard a number of orders, to equilibrate the amount of exchanged cash.

Also, in the case of the *FOREX* the agents issue market orders, whose size is a random fraction of the current cash owned by the trader herself. In particular, each market order has information about the currency to sell (dollar or euro), the order amount, and the currency to buy (euro or dollars). The order amount cannot exceed the trader's current cash availability, and the amount of currency that the agent will achieve to buy depends on the new exchange rate S(t + 1).

5.2.1 The exchange rate clearing mechanism

The exchange rate is adjusted with respect to excess demand ED(t). In particular, the exchange rate S(t+1) for the simulation step t+1 is given by equation 5.1:

$$S(t+1) = S(t) \cdot (1 + c \cdot ED(t)) \cdot e^{r(t)}, \tag{5.1}$$

where c is a positive normalization coefficient, and r(t) is a draw from a

Gaussian Distribution $N(0, \sigma_s(t))$, whose standard deviation $\sigma_s(t)$ depends on the historical exchange rate standard deviation computed on a time window T, as shown in equation 5.2.

$$\sigma_s(t) = k * \sigma(T), \tag{5.2}$$

where k is a positive coefficient. The value of k is always equal to 1.4, while the window T is 20 steps long.

The excess demand is the sum of the orders issued by the traders. More specifically, excess buying drives exchange rate up and, symmetrically, excess selling drives exchange rate down.

Since that there are only two currencies, we decided that ED represents the excess demand of euro against dollars. It follows that equation 5.1 is the current exchange rate of the euro against the dollar.

5.2.2 The decision making process

We extended the four basic kinds of the agents' behaviour (random, fundamentalist, momentum and contrarian) by allowing them to issue market orders in the FOREX. The only difference between limit orders and market orders is that market orders do not have a limit price, so they are executed at the current price set by the market maker. Each kind of traders issues market orders using exactly the same strategy used for issuing limit orders. The extension is obvious, except for fundamentalists, which require further explanation. Let be S_f the fundamental exchange rate $\mathbf{\epsilon}/\$$ between the dollar and the euro: if $S(t) > S_f$ ($S(t) < S_f$), the fundamentalist trader will place a market order to sell (buy) euro in exchange for dollars.
5.2.3 Attraction functions

At each simulation step, each trader chooses the most attractive stock market by evaluating the attraction functions described in Chapter 3, but in this case we decided to make some changes. First, not before five simulation steps from her last switching (corresponding roughly to one week of trading), each agent chooses the market to trade in on the basis of her trading strategy. This choice fit better with the case of markets in different countries, and allowed us to remove the constraint $\pi_j^i(t) \ge 0.3$. With regard to random traders, we simplified their function and we decided that, when faced with the possibility of operating in one of two markets, they randomly select one of them. Finally, we decided to allow fundamentalists to switch market. This choice was made in order to give them the possibility to leave the bearish market in the case of a crisis. Let be p_f^j the fundamental price in market j = 1, 2. The fundamental traders will choose the most profitable market on the basis of the difference between the current price and the fundamental price, as given by equation 5.3.

$$A_{j}^{F,i} = e^{\frac{|p^{j}(t) - p_{f}^{j}|}{p_{f}^{j}}}.$$
(5.3)

The superscript F indicates the fundamentalist sub-population, j denotes the j - th stock market, and $p^{j}(t)$ is the j - th stock price.

5.3 Results

Here we discuss the results of the computational experiments performed. We analysed the markets behaviour in three steps. In Section 5.3.1 we present the foreign exchange market only, while in Section 5.3.2 we discuss the dynamics of the three markets, first without monetary shocks, and then simulating an inflationary depreciation of the dollar.

5.3.1 Foreign Exchange Market

We first tested the overall behaviour of the FOREX market model, varying the percentage of fundamentalists and of chartists, while fixing the total number of agents to 400. Note that chartists always comprise the same percentage of momentum and contrarian traders. Trader's initial endowment, both in dollar and euro, was obtained by dividing agents into groups of 20 traders, and applying Zipf's law to each group, as described in Chapter 3. Here, each agent is given an average amount of \$50000 and \in 50000.

At the beginning of the simulations, the exchange rate between the two currencies is set at the "equilibrium" value S(f), which depends on the ratio between the total number of dollars and of euro exchanged. Since the total number of euro is exactly equal to the total number of dollars, both the starting exchange rate and the fundamental value used by fundamentalists are set at 1.0. The exchange rate series exhibit the usual "stylized facts", with fat tails of returns and volatility clustering. Figure 5.1 shows both the daily euro-dollar exchange rate (top) and the daily time series of logarithmic price returns (bottom) for a typical simulation 10000 steps long, having a population composed of 10% fundamentalist, 5% momentum and 5% contrarian traders. Figure 5.2 shows the survival probability distribution of the standardized logarithmic return (bold stars) superimposed on the best Gaussian fit (solid line). The deviation from Gaussian distribution shows again a leptokurtic behaviour in the returns tail, with a very well defined power-law behaviour for high values of returns.

The model is capable, to a certain extent, to reproduce the so called disconnected puzzle (Obstfeld and Rogoff; 2000) which states that the exchange rate is usually far from its underlying fundamentals. In our model, which is completely endogenous and thus characterized by the absence of external "news", the fundamental value of the exchange rate is equal to the ratio between the total quantity of dollars and the total quantity of euro owned by the traders, that is 1.0. Figure 5.1 show that the exchange rate can substantially deviate from 1.0 for periods longer than 250 steps, that correspond roughly to one year of trading.



Figure 5.1: Daily time series for euro-dollar exchange rate (top) and returns (bottom).

5.3.2 Two stock markets and the FOREX: putting it all together

Here we discuss the dynamics of the two stock markets combined with the foreign exchange market. We performed extensive simulations, 2000 time steps long, and examined two different cases: the behaviour of the whole economy without any external influence, and the effects of a sudden depreciation of the dollar.

Each simulation was run using 1200 agents divided into two separate groups. The first one was composed of traders acting only in the FOREX market and had 400 agents. The role of this population is to keep the



Figure 5.2: Survival probability distribution of standardized logarithmic returns. The bold stars represent an estimate of the cumulative distribution of returns related to a simulation. The solid line represents the survival probability distribution of the best Gaussian fit.

FOREX alive independently of the two stock markets. The remaining 800 agents form the second group, which trade in the two stock markets following the rules described in Section 5.2.2. It is worth noting that, on average, the agents belonging to the second group are equally distributed between the two stock markets, so all three markets have an average number of traders equal to 400.

Traders' initial stock and currency endowment was obtained as described in Chapter 3. Each trader assigned to SM_1 is given an average \$100000 and 1000 stocks, but no euro or stocks of SM_2 . Symmetrically, each agent that starts to trade in SM_2 is given an average \in 100000 cash and 1000 SM_2 stocks. Finally, the agents populating the *FOREX* is given an average \$100000 and \in 100000 cash, but they do not own any stock.

Both the starting price $p^j(0)$ of the stock j and the fundamental price p_f^j known by the fundamentalists are equal to $j/\sqrt{2j-1} (C_j(0)/A_j(0))$, where $C_j(0)$ is the total cash amount of market j, and $A_j(0)$ is the total number of shares of market j. The value is equal to \$80 for SM_1 , and \in 80 for SM_2 . Similarly, at the beginning of the simulations the exchange rate $\epsilon/$ \$ is set at a value equal to the ratio between the total number of euro and dollars, that is 1.0.

Figures from 5.3(a) to 5.5 show the results of a typical simulation 2000 steps long. The trader population is composed of 10% fundamentalists, 10% momentum, 10% contrarian, and 70% random traders. In particular, figures 5.3(a) and 5.3(b) show the daily prices and daily log-returns for the SM_1 stock and SM_2 stock respectively. Figure 5.4 show the dynamics of the eurodollar exchange rate, and figure 5.5 shows the dynamics of traders' wealth. All three markets of the model exhibit the key stylized facts of financial time series, and the dynamics of wealth distribution is unvaried, with fundamentalist and contrarian traders winning at the expenses of random and momentum traders. The presence of the exchange market does not seem to influence by itself the other markets, either with and without the GARCH effect.



Figure 5.3: Daily time series for stock prices (top) and returns (bottom).



Figure 5.4: Daily time series for euro-dollar exchange rate (top) and returns (bottom).



Figure 5.5: Wealth dynamics of the four trader populations.

5.3.3 The inflationary shock

We studied the consequences of a sudden financial crisis of the dollar currency, due to exogenous factors affecting the dollar market. We modeled this phenomenon by opening the market and increasing the wealth of traders in dollars, simulating a depreciation of dollars against euro. In particular, at the end of the first half of the simulation after the step number 1000, we doubled the amount of dollars owned by each trader.

Regarding price dynamics, the sudden increase of the quantity of dollars lead to a slow increase in the fundamental value of the stock exchanged in dollars, proportional to the quantity of money added. Figure 5.6(a) displays the effect of the cash inflow on the SM_1 : the fundamental value of the stock moves from \$80.0 to \$160.0. On the other hand the price of the SM_2 stock is not affected by the shock, as shown in Figure 5.6(b). Finally, Figure 5.7 points out the consequent appreciation of the euro against the dollar: the euro doubles its value against the dollar currency.

Then we studied the consequences of the inflationary shock on trading volumes. We found an increase in volumes of the *FOREX* (see Figure 5.8), that is due to the cash inflow. The increase in the trading activity does not spread to the stock markets, whose volumes remains substantially unvaried. This behaviour was predictable, because the total number of stocks is kept constants during all simulations, and a change in the total amount of cash is not sufficient to lead to a increase/decrease in the volumes.

There exists a set of *stylized facts* about the spreading of shocks across markets. Corsetti et al. (2001) identified four empirical regularities characterizing periods of financial turmoil:

- 1. periods of financial turmoil favour falls in stock prices;
- 2. volatility of prices increases during crisis periods;

- 3. covariance between stock market returns increases during crisis periods;
- 4. returns correlation is not necessary larger than during tranquil periods.

Figure 5.7 shows that the dollar shock does not influence the exchange rate volatility. On the other hand, both the SM_1 and SM_2 price volatility tend to increase in the days following the shock as presented in Figure 5.9(a) and in Figure 5.9(b) respectively. We computed both weekly and monthly volatility, but for the sake of brevity we report here only the latter. This seems to confirm the second regularity quoted above.

We studied the dynamics of both returns and absolute returns correlations between SM_1 and SM_2 during the tranquil period and during the dollar crisis. Figure 5.10 shows the correlation analysis of a typical simulation 2000 steps long. The correlation coefficients are calculated by considering not overlapping return series 20 days long. Variables computed using weekly and monthly returns gave very similar results. We found that the inflationary shock in not capable to influence the correlation coefficient dynamics, which remain more or less the same during the whole of the simulation. This confirms the fourth regularity quoted above.

On the basis of the definitions given in Section 5.1, we can conclude that this model is not able to reproduce *contagion*, because there is no significant increment in the correlation values of returns. This result rules out the possibility to run further tests on contagion, such as that discussed in a series of papers by Boyer et al. (1997) and by Loretan and English (2000), which require an increase in the correlation coefficients as a necessary and not sufficient condition.

Finally we investigated the covariance dynamics of returns. Figure 5.11 shows that the shock brings to a sharp increase in the covariance of absolute returns (bottom) and to a decrease in the covariance of raw returns (top).

This result seems to confirm the third regularity identified in (Corsetti et al.; 2001).



Figure 5.6: Daily time series for stock prices (top) and returns (bottom). The inflationary shock is applied at the end of the step number 1000.



Figure 5.7: Daily time series for euro-dollar exchange rate (top) and returns (bottom). The inflationary shock is applied at the end of the step number 1000.



Figure 5.8: Daily time series for euro-dollar exchange rate (top) and volumes (bottom). The inflationary shock is applied at the end of the step number 1000.



Figure 5.9: Daily volatility for the stock prices between step 500 and 1500 of the simulation. The inflationary shock is applied at the end of the step number 1000.



Figure 5.10: Cross correlations of returns (top) and of absolute returns (bottom) of the two stock price series. Each point in the horizontal axis represents one month of trading, corresponding to 20 simulation steps.



Figure 5.11: Covariance of returns (top) and of absolute returns (bottom) of the stock price series. Each point in the horizontal axis represents one month of trading, corresponding to 20 simulation steps.

Chapter 6

Conclusions

This thesis presented an agent-based computer simulation framework for building theoretical models in economics and finance. The project was built on the basis of the "Genoa Artificial Stock Market", which was born in the early 2000's at the University of Genoa. The original model has been completely re-engineered, and it has been improved and extended in order to address a wide range of open problems in finance and economics. The current version of the model includes many realistic trading features, and has been validated by showing that the simulated time series exhibit the main empirical properties of real financial markets. This artificial market has been developed using object-oriented software techniques, and is aimed to be easily extended and composed, yielding multi-asset and multi-market simulations. This thesis faces three big open issues that the available literature do not succeed in giving them an answer.

First, we studied the effects of transaction taxes in financial markets. We performed numerous simulations, varying trader composition and tax rate. We found that levying a tax influences market behaviour significantly, even when the rate is low. Price volatility increases consistently with tax rate, but only when chartist traders are present in the market. Then, we analysed the dynamics of two markets, giving each trader the opportunity of choosing the market she prefers to trade in, according to an attraction function. We performed simulations on this market pair with no tax levied, and then taxing one market. Firstly, we observed that, irrespective of trader composition and tax rate, the interplay of markets leads to an increase in price volatility. Secondly, we found that, notwithstanding the small transaction tax (typically 0.1-0.5% of transaction cost) and the simple trader models used, the tax does actually impact heavily on market behaviour, increasing price volatility and reducing trading volumes. This happens only with trader compositions sensitive to the tax, namely those including chartist traders. Despite the low tax rate, introducing the transaction tax increases price volatility, computed for different time horizons, significantly and reduces trading volumes, though to a lesser extent. These results concur with many empirical findings and provide a measure, using a theoretical model, of the impact of a change in market regulation. On the other hand, a part of the literature asserts that speculators tend to leave the taxed markets, that thus become less volatile.

The second case study deals with the impact of margin requirements and of short sale restrictions on stock markets. Considering the closed market, we found that if short selling and margin trading are allowed, volatility will tend to slightly increase. The increase in volatility is substantially unrelated to restriction levels and to debt proneness of traders. We found that, if short selling and margin trading are not banned, some traders could declare bankrupt and leave the market. The number of bankrupts is usually very low and negligible. Also, the wealth distribution of both DPT and of non-DPThave the same trend, but for the first ones the trend is stronger. We showed that generally the open and the closed market have similar features, except for the fact that in an open market the number of bankrupts increases. External factors, such as sudden variations of prices and wealth, damage DPTs much more than non - DPTs, so DPTs tend to fail more easily than non - DPTs. In general, bankrupts favor high volatility of prices and may lead to periods of unsteadiness, so we can assert that in this sense the presence of in debt positions may ease unsteadiness.

Finally, we presented a model made up of two stock markets with different currencies, and of a FOREX market enabling traders to exchange their currencies in order to switch to the other stock market. The markets are completely closed and self-consistent. There is no external influences, and the amount of cash and stocks is kept constant, but for the case of controlled cash inflow simulating an inflationary crisis. Simulating a monetary shock in one market, with a steep, substantial inflow of money, yielded a gradual adaptation to the new fundamental prices in both the affected stock market, and in the FOREX market, and showed, at least to some extent, three of the four stylized facts about the spreading of shocks across markets identified by Corsetti et al. (2001): volatility of prices and covariance between stock market returns increase during crisis periods, and returns correlation is not necessary larger than during tranquil periods. The fourth fact, i.e. that periods of financial turmoil favor falls in stock prices is not observed, but in real markets it depends on traders' risk aversion, and on the ability of real trader to move theirs assets to other investments. This is clearly not possible in a closed model like this one, so the absence of this behaviour is obvious. A similar consideration applies to contagion. We shoved that simply linking two stock markets with an exchange market does not yield an increase in return correlation when there is a crisis in one of the markets. Again, this is not unexpected, because in real world contagion is a consequence of strong economic links between economies, and such links are simply not present in our simple model. It needs to be said also that, while our model is able to get insight on the intrinsic characteristics of linking two stock markets through a FOREX market, showing that interesting behaviours arise simply from the structural properties of the model, in reality FOREX markets are driven by much more than stock markets' needs and by their internal traders. Much more features have to be added to the model to reflect more thoroughly how real markets behave.

These three case studies show how the presented agent-based artificial market can be used to develop theoretical models in order to perform tests and to validate hypotheses in economics and finance. A further extension of the framework is in progress, and its goal is to analyse the impact of stock option trading on the market of the underlying security. Although the current version of the model is able to contribute to open debates which attract public attention and arise the interest of policy makers, much more features have to be added to the model to reflect more thoroughly how real markets behave. Among others, we may quote interest rates of bonds, stock dividends, which could be related to the trend of the economy, also yielding different fundamental values of the stock. Finally, traders' risk aversion has to be modeled, which might affect the switching probability between markets in the case of multi-market models. In future works, we plan to address all these issues by extending the framework again.

Bibliography

- Ackert, L. F., Church, B. K. and Deaves, R. (2002). Bubbles in experimental asset markets: Irrational exuberance no more, *Technical report*.
- Adamic, L. A. (2000). Zipf, power-laws, and pareto a ranking tutorial, *Technical report*, Xerox Palo Alto Research Center.
- Aliber, Z., Chowdhry, B. and Yan, S. (2003). Some evidence that a tobin tax on foreign exchange transactions may increase volatility, *European Finance Review* 7(3): 481–510.
- Arthur, W. B. (1995). Complexity in economic and financial markets, Complexity 1(1): 20-25.
- Arthur, W. B., Holland, J. H., LeBaron, B. D., Palmer, R. and Tayler, P. (1997). Asset pricing under endogeneous expectations in an artificial stock market, in W. Arthur, S. Durlauf and D. Lane (eds), The Economy as an Evolving Complex System II, SFI Studies in the Sciences of Complexity, Addison Wesley Longman.
- Baker, D. (2000). The case for a unilateral speculation tax in the united state, *Technical report*, Center for Economic and Policy Research. Briefing Paper.

- Basu, N., Pryor, R. and Quint, T. (1998). Aspen: a microsimulation model of the economy, *Computational Economics* 12(3): 223-241.
- Beck, K. (1999). Extreme Programming Explained. Embrace Change, Addison-Wesley, Reading, Massachusetts.
- Beck, K. and Andres, C. (2004). Extreme Programming Explained: Embrace Change - Second Edition, Addison-Wesley.
- Beja, A. and Goldman, M. B. (1980). On the dynamic behavior of prices in disequilibrium, *Journal of Finance* 35: 235–247.
- BIS (2005). Bank for international settlements: Triennial central bank survey: Foreign exchange and derivatives market activity in 2004.
- Black, F. (1986). Noise, Journal of Finance 41(3): 529-543.
- Bottazzi, G., Dosi, G. and Rebesco, I. (2005). Institutional architectures and behavioral ecologies in the dynamics of financial markets, *Journal of Mathematical Economics* 41(1-2): 197–228.
- Bouchaud, J.-P. (2000). Power-laws in economics and finance: some ideas from physics, Science & finance (cfm) working paper archive, Science & Finance, Capital Fund Management.
- Bouchaud, J.-P. and Potters, M. (2004). Theory of Financial Risk and Derivative Pricing, 2 edn, Cambridge University Press, Cambridge, UK.
- Boyer, B. H., Gibson, M. S. and Loretan, M. (1997). Pitfalls in tests for changes in correlations, *Technical report*.
- Briys, E. and de Varenne, F. (2000). 1000 years of risk management, Risk pp. 47–48.

- Butler, K. C. and Joaquin, D. C. (2002). Are the gains from international portfolio diversification exaggerated? the influence of downside risk in bear markets, *Journal of International Money and Finance* 21(7): 981–1011.
- Campbell, J. Y. and Shiller, R. J. (1988). The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1(3): 195–228.
- Challet, D., Marsili, M. and Zhang, Y.-C. (2001). Minority games and stylized facts, *Physica A* **299**(1-2): 228–233.
- Challet, D. and Zhang, Y.-C. (1997). On the minority game: Analytical and numerical studies, *Physica A* **256**(3-4): 514–532.
- Chan, T. (2001). Artificial markets and intelligent agents, PhD thesis, Massachusetts Intitute of Technology.
- Chen, J., Hong, H. and Stein, J. C. (2002). Breadth of ownership and stock returns, *Journal of Financial Economics* **66**(2-3): 171–205.
- Chiarella, C. (1992). The dynamics of speculative behavior, Ann Oper Res **37**: 101–123.
- Cincotti, S., Focardi, S. M., Marchesi, M. and Raberto, M. (2003). Who wins? study of long-run trader survival in an artificial stock market, *Physica A* **324**(1-2): 227–233.
- Cincotti, S., Focardi, S. M., Ponta, L., Raberto, M. and E.Scalas (2005). The waiting-time distribution of trading activity in a double auction artificial financial market, in A. Namatame, T. Kaizouji and Y. Aruka (eds), *Economics and Heterogeneous Interacting Agents*, Springer-Verlag, Berlin.
- Cont, R. (1997). Scaling and correlation in financial data.

- Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues, *Quantitative Finance* 1: 223–236.
- Cont, R. and Bouchaud, J.-P. (2000). Herd behavior and aggregate fluctuations in financial markets, *Macroecon Dyn* 4(2): 170–196.
- Cont, R., Potters, M. and Bouchaud, J.-P. (1997). Scaling in stock market data: stable laws and beyond, Science & finance (cfm) working paper archive, Science & Finance, Capital Fund Management.
- Corsetti, G., Pericoli, M. and Sbracia, M. (2001). Correlation analysis of financial contagion: What one should know before running a test, *Working paper*, Economic Growth Center, Yale University.
- Cutler, D. M., Poterba, J. M. and Summers, L. H. (1989). What moves stock prices?, NBER Working Papers 2538, National Bureau of Economic Research, Inc.
- Day, R. H. and Huang, W. H. (1990). Bulls, bears and market sheep, J Econ Behav Organ 14(3): 299–329.
- De Long, J. B., Shleifer, A., Summers, L. H. and Waldmann, R. J. (1990a).
 Noise trader risk in financial markets, J Polit Econ 98(4): 703-738.
- De Long, J. B., Shleifer, A., Summers, L. H. and Waldmann, R. J. (1990b). Positive feedback investment strategies and destabilizing rational speculation, *Journal of Finance* 45(2): 374–97.
- Diamond, D. and Verrecchia, R. (1987). Constraints on short-selling and asset price adjustments to private information, *Journal of Financial Eco*nomics 18: 277–311.

- Dickey, D. A. and Fuller, W. A. (1979). Distribution of the estimators for autoregressive time in financial markets, *Journal of the American Statistical Association* 74(366): 427–431.
- Dickey, D. A. and Fuller, W. A. (1981). Likelihood ratio statistics for autoregressive time series with a unit root, *Econometrica* **49**(4): 1057–72.
- Dragulescu, A. and Yakovenko, V. M. (2002). Exponential and power-law probability distributions of wealth and income in the united kingdom and the united states, *Computing in economics and finance 2002*, Society for Computational Economics.
- Ecca, S., Marchesi, M. and Setzu, A. (2006). Analysis of stock option market in an artificial financial market. 21-23 September 2006, Urbino, Italy.
- Ehrenstein, G., Westerhoff, F. and Stauffer, D. (2005). Tobin tax and market depth, *Quantitative Finance* 5(2): 213–218.
- Eichengreen, B., Rose, A. K. and Wyplosz, C. (1996). Contagious currency crises, *Cepr discussion papers*, C.E.P.R. Discussion Papers.
- Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation, *Econometrica* 50(4): 987–1007.
- Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work, *Journal of Finance* 25(2): 383-417.
- Farmer, J. D. (1999). Physicists attempt to scale the ivory towers of finance, Computing in Science and Engineering 1(6): 26-39.
- Felix, D. and Sau, R. (1996). On the Revenue Potential and Phasing in of the Tobin Tax, in Haq et al. (1996), pp. 223 – 254.

- Ferris, S. and Chance, D. (1988). Margin requirements and stock market volatility, *Economic Letters* 28(3): 251–254.
- Forbes, K. J. (2004). The asian flu and russian virus: the international transmission of crises in firm-level data, *Journal of International Economics* 63(1): 59–92.
- Forbes, K. J. and Rigobon, R. (2002). No contagion, only interdependence: Measuring stock market comovements, *The Journal of Finance* LVII(5): 2223–2261.
- Frankel, J. A. (1996). How Well do Foreign Exchange Markets Work: Might a Tobin Tax Help?, in Haq et al. (1996), pp. 41 – 81.
- Frankel, J. A. and Froot, K. A. (1986). Understanding the u.s. dollar in the eighties: The expectations of chartists and fundamentalists, *The Economic Record* 0(0): 24–38.
- Friedman, M. (1953). The Case for Flexible Exchange Rates, Essays in Positive Economics, Chicago University Press, Chicago.
- Gilbert, N. and Bankes, S. C. (2002). Platforms and methods for agent-based modeling, P Natl Acad Sci USA 99(Suppl. 3): 7197–7198.
- Gode, D. K. and Sunder, S. (1993). Allocative efficiency of markets with zero intelligence traders. market as a partial substitute for individual rationality, *Journal of Political Economy* 101(1): 119–137.
- Grossman, S. J. (1976). On the efficiency of competitive stock markets when trades have diverse information, *J Financ* **31**: 573–585.
- Grossman, S. J. and Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets, Am Econ Rev **79**(3): 393–408.

- Habermeier, K. and Kirilenko, A. A. (2001). Securities transaction taxes and financial markets, Washington, d.c., imf. working paper.
- Hamao, Y., Masulis, R. W. and Ng, V. (1990). Correlations in price changes and volatility across international stock markets, *Review of Financial Studies* 3(2): 281–307.
- Haq, M. u., Kaul, I. and Grunberg, I. (eds) (1996). The Tobin Tax: Coping with Financial Volatility, Oxford University Press, New York.
- Hardouvelis, G. A. (1988). Margin requirements and stock market volatility, Quarterly Review (Sum): 80–89.
- Hardouvelis, G. A. (1990). Margin requirements, volatility, and the transitory component of stock prices, American Economic Review (80): 736–762.
- Haruvy, E. and Noussair, C. N. (2006). The effect of short selling on bubbles and crashes in experimental spot asset markets, *Journal of Finance* 61(3): 1119–1157.
- Ho, K. W. (1996). Short-sales restrictions and volatility the case of the stock exchange of singapore, *Pacific-Basin Finance Journal* 4(4): 377–391. available at http://ideas.repec.org/a/eee/pacfin/v4y1996i4p377-391.html.
- Hommes, C. H. (2001). Financial markets as nonlinear adaptive evolutionary systems, *Tinbergen Institute Discussion Papers 01-014/1*, Tinbergen Institute.
- Hommes, C. H. (2002). Modeling the stylized facts in finance through simple nonlinear adaptive systems, *Proceedings of the National Academy of Sciences*, Vol. 99, pp. 7221–7228.

- Hommes, C. H. (2005). Heterogeneous agent models in economics and finance, *Tinbergen Institute Discussion Papers 05-056/1*, Tinbergen Institute.
- Hsieh, A., D. and Miller, M. H. (1990). Margin regulation and stock market volatility, *Journal of Finance* 45(1): 3–29.
- Izumi, K. and Ueda, K. (1999a). Analysis of dealers' processing financial news based on an artificial market approach, Journal of Computational Intelligence in Finance (7): 23–33.
- Izumi, K. and Ueda, K. (1999b). Analysis of exchange rate scenarios using an artificial market approach, in C.-H. C. A. Amin and et. al (eds), Proceedings of the International Conference on Artificial Intelligence, CSREA Press, pp. 360-366.
- Izumi, K. and Ueda, K. (2001). Phase transition in a foreign exchange market-analysis based on an artificial market approach, *Evolutionary Computation, IEEE Transactions on* 5(5): 456–470.
- Johnson, N. F., Lamper, D., Jefferies, P., Hart, M. L. and Howison, S. (2001). Application of multi-agent games to the prediction of financial time series, *Physica A Statistical Mechanics and its Applications* 299: 222–227.
- Jones, C. M. and Lamont, O. (2002). Short-sale constraints and stock returns, Journal of Financial Economics 66: 207–239.
- Joshi, S., Parker, J. and Bedau, M. A. (1998). Technical trading creates a prisoner's dilemma: Results from an agent-based model, *Research in Economics 98-12-115e*, Santa Fe Institute.
- Kahneman, D. and Tversky, A. (1973). On the psychology of prediction, Psychological Review (80): 237–251.

- Keynes, J. (1936). The General Theory of Employment, Interest, and Money, Macmillan:London.
- Kim, K. A. (2002). Initial margin requirements, volatility, and the individual investor: Insights from japan, *The Financial Review* 37(1): 1–15. available at http://ideas.repec.org/a/bla/finrev/v37y2002i1p1-15.html.
- King, R., R., Smith, V. L., Williams, A. W. and Boening, M. V. (1993).
 The robustness of bubbles and crashes in experimental stock markets, *in*R. Dall and P. Chen (eds), *Nonlinear Dynamics and Evolutionary Economics*, Oxford University Press, pp. 183–200.
- Kraus, A. and Rubin, A. (2003). The effect of short sale constraint removal on volatility in the presence of heterogeneous beliefs, *International Review* of Finance 4.
- Kupiec, P. H. (1995). A securities transactions tax and capital market efficiency, Contemporary Economic Policy 13(1): 101–12.
- Largay, J. and West, R. (1973). Margin changes and stock price behavior, Journal of Political Economy 81(2): 328-339.
- LeBaron, B. (2006). Agent-based computational finance, in L. Tesfatsion and K. L. Judd (eds), Handbook of Computational Economics, Vol. 2 of Handbook of Computational Economics, Elsevier, chapter 24, pp. 1187– 1233.
- LeBaron, B. D. (2000). Agent-based computational finance: Suggested readings and early research, *J Econ Dyn Control* **24**(5-7): 679–702.
- LeRoy, S. F. and Porter, R. D. (1981). The present-value relation: Tests based on implied variance bounds, *Econometrica* **49**(3): 555–74.

- Levy, M., Levy, H. and Solomon, S. (2000). Microscopic Simulation of Financial Markets, Academic Press, New York.
- LiCalzi, M. and Pellizzari, P. (2002). Fundamentalists clashing over the book: A study of order-driven stock markets, *Computational economics*, EconWPA.
- LiCalzi, M. and Pellizzari, P. (2006). The allocative effectiveness of market protocols under intelligent trading, *Working papers*, Department of Applied Mathematics, University of Venice.
- Liu, Y., Gopikrishnan, P., Cizeau, P., Meyer, M., Peng, C.-K. and Stanley,
 H. E. (1999). The statistical properties of the volatility of price fluctuations, *Physical Review E* 60: 1390.
- Loretan, M. and English, W. B. (2000). Evaluating "correlation breakdowns" during periods of market volatility, *Technical report*.
- Lucas, R. E. (1972). Expectations and the neutrality of money, Journal of Economic Theory 4(2): 103-124.
- Lux, T. (1997). Time variation of second moments from a noise trader infection model, J Econ Dyn Control 22(1): 1–38.
- Lux, T. (1999). The socio-economic dynamics of speculative markets: interacting agents, chaos, and the fat tails of return distributions, J Econ Behav Organ 33(2): 143-165.
- Lux, T. and Marchesi, M. (1999). Scaling and criticality in a stochastic multi-agent model of a financial market, *Nature* **397**(6718): 498–500.
- Lux, T. and Marchesi, M. (2000). Volatility clustering in financial markets: a

microsimulation of interacting agents, Int J Theor Appl Finance **3**(4): 675–702.

- Lux, T. and Marchesi, M. (2002). Journal of economic behavior and organization: special issue on heterogeneous interacting agents in financial markets, *Journal of Economic Behavior & Organization* 49(2): 143-147.
- Mandelbrot, B. B. (1963). The variation of certain speculative prices, J. Business 36: 394–413.
- Mannaro, K., Marchesi, M. and Setzu, A. (2005). The impact of transaction taxes on traders' behaviour and wealth: a microsimulation. WEHIA 2005.
- Mannaro, K., Marchesi, M. and Setzu, A. (2006). Using an artificial financial market for assessing the impact of tobin-like transaction taxes, Submitted to Journal of Economic Behavior & Organization.
- Mantegna, R. N. and Stanley, E. H. (1999). An Introduction to Econophysics: Correlations and Complexity in Finance, Cambridge University Press.
- Marchesi, M., Cincotti, S., Focardi, S. M. and Raberto, M. (2003). The Genoa artificial stock market: microstructure and simulation, Vol. 521 of Lecture Notes in Economics and Mathematical Systems, Springer, pp. 277– 289.
- Milgrom, P. and Stokey, N. (1982). Information, trade and common knowledge, *Journal of Economic Theory* **26**(1): 17–27.
- Miller, E. M. (1977). Risk, uncertainty, and divergence of opinion, Journal of Finance 32: 1151–1168.
- Moore, T. (1966). Stock market margin requirements, Journal of Political Economy 74(2): 158–167.

- Muth, J. F. (1961). Rational expectations and the theory of price movements, Econometrica 29: 315–335.
- Obstfeld, M. and Rogoff, K. (2000). The six major puzzles in international macroeconomics: Is there a common cause?, *Nber working papers*, National Bureau of Economic Research, Inc.
- Ofek, E. and Richardson, M. (2001). Dotcom mania: A survey of market efficiency in the internet section. NYU working paper.
- Officer, R. R. (1973). The variability of the market factor of the new york stock exchange, *Journal of Business* **46**(3): 434–53.
- Pagan, A. (1996). The econometrics of financial markets, Journal of Empirical Finance 3(1): 15–102.
- Palley, T. I. (1999). Speculation and tobin taxes: Why sand in the wheels can increase economic efficiency, *Journal of Economics* 69: 113 – 26.
- Palley, T. I. (2003). Debating the Tobin Tax, New Rules for Global Finance Coalition, chapter 2.
- Palmer, R. G., Arthur, W. B., Holland, J. H., LeBaron, B. D. and Tayler, P. (1994). Artificial economic life: a simple model of a stock market, *Physica D* 75: 264–274.
- Pareto, V. (1897). Course d'Economie Politique, Lausanne, CH.
- Pellizzari, P. (2005). Complexities and simplicity: a review of agent-based artificial markets. Giornata di studi in onore di Giovanni Castellani, 26th September 2005, Venice, Italy.
- Raberto, M. (2003). Modelling and implementation of an artificial financial

market using object oriented technology: the Genoa artificial stock market, PhD thesis, University of Genoa, Italy.

- Raberto, M., Cincotti, S., Focardi, S. M. and Marchesi, M. (2001). Agentbased simulation of a financial market, *Physica A* 219: 319–327.
- Raberto, M., Cincotti, S., Focardi, S. and Marchesi, M. (2003). Traders' long-run wealth in an artificial financial market, *Computational Economics* 22(2-3): 255–272.
- Salinger, M. (1989). Stock market margin requirements and volatility: Implications for regulation of stock index futures, Journal of Financial Services Research 3(2/3): 121–138.
- Samuelson, P. A. (1965). Proof that properly anticipated prices fluctuate randomly, *Industrial Management Review* 6(2): 41-49.
- Sargent, R. G. (1994). Verification and validation of simulation models, WSC
 '94: Proceedings of the 26th conference on Winter simulation, Society for
 Computer Simulation International, San Diego, CA, USA, pp. 77–87.
- Schwert, G. (1989). Stock market margin requirements, Journal of Financial Services Research 3(2/3): 153-164.
- Setzu, A. and Marchesi, M. (2006). The effects of short-selling and margin trading: a simulation analysis. WEHIA 2006,.
- Shiller, R. J. (1981). Do stock prices move too much to be justified by subsequent changes in dividends?, NBER Working Papers 0456, National Bureau of Economic Research, Inc.
- Shiller, R. J. (1989). Market Volatility, MIT press, Cambridge.

- Smith, V. L., Suchanek, G. L. and Williams, A. W. (1988). Bubbles, crashes, and endogenous expectations in experimental spot asset markets, *Econometrica* 56(5): 1119–51.
- Summers, L. H. (1986). Does the stock market rationally reflect fundamental values?, Journal of Finance 41(3): 591-601.
- Tay, N. S. P. and Linn, S. C. (2001). Fuzzy inductive reasoning, expectation formation and the behavior of security prices, *Journal of Economic Dynamics and Control* 25(3-4): 321–361.
- Tesfatsion, L. (2001a). Agent-based modeling of evolutionary economic systems, *IEEE T Evolut Comput* 5(5): 437–441.
- Tesfatsion, L. (2001b). Introduction to the special issue on agent-based computational economics, J Econ Dyn Control 25(3-4): 281-293.
- Tesfatsion, L. (2002). Economic agents and markets as emergent phenomena, P Natl Acad Sci USA 99(Suppl. 3): 7191–7192.
- Tesfatsion, L. (2006). Agent-based computational economics: A constructive approach to economic theory, in L. Tesfatsion and K. L. Judd (eds), Handbook of Computational Economics, Vol. 2 of Handbook of Computational Economics, Elsevier, chapter 16, pp. 831–880.
- Tesfatsion, L. and Judd, K. (2006). Agent-Based Computational Economics, Vol. 2 of Handbook of Computational Economics, North Holland.
- Tobin, J. (1978). A proposal for international monetary reform, Eastern Economic Journal IV(3-4): 153-159.
- Tveit, A. (2001). A survey of agent-oriented software engineering, Proc. of the First NTNU CSGS Conference (http://www.amundt.org).

- Umlauf, S. R. (1993). Transaction taxes and the behavior of the swedish stock market, Journal of Financial Economics 33(2): 227-240.
- Werner, I. M. (2003). Comment on 'some evidence that a tobin tax on foreign exchange transactions may increase volatility', *Review of Finance* 7(3): 511-514.
- Westerhoff, F. (2003). Heterogeneous traders and the tobin tax, Journal of Evolutionary Economics 13: 53-70.
- Westerhoff, F. (2004a). The effectiveness of keynes-tobin transaction taxes when heterogeneous agents can trade in different markets: A behavioral finance approach, *Computing in Economics and Finance 2004 14*, Society for Computational Economics.
- Westerhoff, F. H. (2004b). Multiasset market dynamics, Macroeconomic Dynamics 8(5): 596-616.
- Zeeman, E. C. Z. (1974). The unstable behavior of stock exchange, *Journal* of Mathematical Economics.
- Zipf, G. K. (1949). Human Behavior and the Principle of Least Effort, Addison-Wesley (Reading MA).