## Hybrid Dynamical Systems: Modeling and Optimal Control

 $\begin{array}{c} {\rm Short\ Course} \\ {\rm Presented\ at\ the\ WODES\ 2000} \\ {\rm by} \\ {\rm Christos\ G.\ Cassandras^1\ and\ Yorai\ Wardi^2} \end{array}$ 

*Rationale.* We view Hybrid Dynamical Systems (HDS) as bi-layer dynamical systems with time-driven dynamics at the lower layer and event-driven dynamics at the upper layer. Application domains of HDS include manufacturing enterprises, transportation networks and flight-control systems. Common to these kinds of systems is the bi-level structure where, the lower layer describes the evolution of physical processes while the upper layer captures logistical elements. The interaction between the two layers defines the "hybrid" nature of the HDS.

Our main interest is in manufacturing systems, where the physical processes at the lower layer are associated with production processes, and the discrete-event processes correspond to movement and storage of parts and products. Traditionally, design and control procedures at these two layers of the manufacturing hierarchy have been treated separately. For example, process engineers typically are concerned with product quality, whereas plant managers are mainly concerned with delivery due dates, inventories, and other logistical parameters. However, there is a growing recognition of the importance of approaching the two issues from an unified standpoint of systems integration. In particular, a growing number of companies (including ERP system developers) are concerned with controlling the timing of events by physical parameters related to the quality of products.

*Synopsis.* The course will focus on recent developments in modeling and control of HDS. In particular, it will address relevant issues of optimal control and optimization methods.

## Course outline

- 1. Introduction: the HDS paradigm.
- 2. Examples of various application domains.
- 3. Applications in manufacturing, and the importance of timing control.
- 4. Optimal-control formulation.
- 5. Structure of the solution: connection to the classical optimal control theory.
- 6. Special cases: LQR problems over max-plus systems.
- 7. General algorithmic techniques.
- 8. Case study (single-stage systems) of complexity reduction: from exponential to polynomial (and sometimes linear).

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- 9. Implementation issues.
- 10. Software demonstrations of concrete algorithms and problem solutions.
- 11. Conclusions and discussion.